Q. 1. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 . Then which one of the following gives possible values a and b?

i. a = 1, b = 6
ii. a = 3, b = 4
iii. a = 0, b = 7
iv. a = 5, b = 2

Sol.

$$Mean = \frac{\sum x}{n} = 6$$

$$Variance = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = 6.8$$

$$= \frac{a^2 + b^2 + 64 + 25 + 100}{5} - 36 = 6.8$$

$$\Rightarrow a^2 + b^2 + 189 - 180 = 34$$

$$\Rightarrow a^2 + b^2 = 25$$

Possible values of a and b is given by (2)

Q. 2. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $+ \vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β?

i. $\alpha = 2, \beta = 1$
ii. $\alpha = 1, \beta = 1$
iii. $\alpha = 2, \beta = 2$
iv. $\alpha = 1, \beta = 2$

Sol.

As $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$\therefore \left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

Or, $\alpha+\beta=2$ (i)

Also $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$

$\therefore \vec{a}=\lambda\left(\hat{b}+\hat{c}\right)$

or, $\vec{a}=\lambda\left(\dfrac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{2}}\right)$ (ii)

But $\vec{a}=\alpha\hat{i}+2\hat{j}+\beta\hat{k}$

Hence $\lambda=\sqrt{2}$ and $\alpha=1, \beta=1$

Which also satisfy (i)

$\therefore$ Correct answer is (2)

Q. 3.

The non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are related by $\vec{a}=8\vec{b}$ and $\vec{c}=-7\vec{b}$

Then the angle between $\vec{a}$ and $\vec{c}$ is

i. $\frac{\pi}{2}$
ii. π
iii. 0
iv. $\frac{\pi}{4}$

Sol. The sign of $\vec{a}$ and $\vec{c}$ are opposite. Hence they are parallel but directions are opposite. Therefore angle between $\vec{a}$ and $\vec{c}$ is π

$\therefore$ correct answer is (2)

Q. 4. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then

i. a = 6, b = 4
ii. a = 8, b = 2
iii. a = 2, b = 8
iv. a = 4, b = 6

Sol. Equation of line through (5, 1, a) and (3, b, 1) is

$$\frac{x-5}{-2}=\frac{y-1}{b-1}=\frac{z-a}{1-a}=\lambda$$

any point on (i) is

$$\{5-2\lambda, 1+(b-1)\lambda, a+(1-a)\lambda\} \qquad (ii)$$

As $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$ lies on (i)

$$5-2\lambda=0 \Rightarrow a=\frac{5}{2} \qquad (iii)$$

$$1+(b-1)\times\frac{5}{2}=\frac{17}{2}$$

or, $2+5b-5=17$

or, $b=4$

and $a+(1-a)\times\frac{5}{2}=-\frac{13}{2}$

or, $2a+5-5a=-13$

or, $a=6$

$\therefore$ Correct answer is (1)

Q. 5. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to

i. 2
ii. 2
iii. 5
iv. 5

Sol. As the given lines intersect

$$\therefore \begin{vmatrix} 2-1 & 3-2 & 1-3 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

$$\text{Or, } \begin{vmatrix} 1 & 1 & 2 \\ k & 2 & 3 \\ 3 & k & 2 \end{vmatrix}=0$$

or, $k=-5, \frac{5}{2}$

Integer is -5 only

$\therefore$ Correct answer is (3)

Q. 6. The differential of the family of circles with fixed radius 5 units and centre on the line y = 2 is

i. $(y-2)^2 y'^2 = 25-(y-2)^2$
ii. $(x-2)^2 y'^2 = 25-(y-2)^2$
iii. $(x-2) y'^2 = 25-(y-2)^2$
iv. $(y-2) y'^2 = 25-(y-2)^2$

Sol. The required equation of circle is

$$(x-a)^2+(y-2)^2=25 \qquad (i)$$

differentiating we get

$$2(x-a)+2(y-2)y'=0$$

$$or,\ a=x+(y-2)y' \qquad (ii)$$

putting a in (i)

$$(x-x-(y-2)y')^2+(y-2)^2=25$$

$$or,\ (y-2)^2 y'^2=25-(y-2)^2$$

∴ The correct answer is (1)

Q. 7. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz,\ y = az + cx$ and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to

i. 0
ii. 1
iii. 2
iv. - 1

Sol.

$$x = cy + bz \Rightarrow x - cy - bz = 0 \qquad (i)$$

$$y = az + bx \Rightarrow bx - y + az = 0 \qquad (ii)$$

$$z = bx + ay \Rightarrow bx + ay - z = 0 \qquad (iii)$$

Eliminating x, y, z from (i), (ii) and (iii) we get

$$\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$$or,\ a^2+b^2+c^2+2abc=1.$$

∴ The correct answer is (2)

Q. 8. Let A be a square matrix all of whose entries are integers. Then which one of the following is true?

i. If $\det A = \pm 1$, then A^{-1} exists and all its entries are integers

ii. If $\det A = \pm 1$, then A^{-1} need not exist

iii. If $\det A = \pm 1$, then A^{-1} exist but all its entries are not necessarily integers

iv. If $\det A = \pm 1$, then A^{-1} exist and all its entries are non-integers

Sol. The obvious answer is (1).

Q. 9. The quadratic equations $x^2 - 6x a = 0$ and $x^2 - cx + 6 = 0$ and have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

i. 3
ii. 2
iii. 1
iv. 4

Sol.

Let the roots of $x^2 - 6x + a = 0$

be α *and* 4β *and that of* $x^2 - cx + 6 = 0$ *be* α *and* 3β

$$\therefore \alpha + 4\beta = 6 \quad (i)$$

$$4\alpha\beta = a \quad (ii)$$

$$\alpha + 3\beta = c \quad (iii)$$

$$3\alpha\beta = 6 \quad (iv)$$

Using (ii) & (iv)

$$\frac{4}{3} = \frac{a}{6} \Rightarrow a = 8$$

Then $x^2 - 6x + a = 0$

reduces to

$$x^2 - 6x + 8 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$= \frac{6 \pm 2}{2} = 4, 2$$

$$\therefore \alpha = 2, \ \beta = 1$$

$\therefore$ *Correct answer is* (2)

Q. 10. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?

i. $6.8.{}^{7}C_4$

ii. $7.{}^{6}C_4.{}^{8}C_4$

iii. $8.{}^{6}C_4.{}^{7}C_4$

iv. $6.7.{}^{8}C_4$

Sol. M = 1, I = 4, P = 2

These letters can be arranged by

$$\frac{(1+4+2)!}{1!4!2!} = 7\ {}^{6}C_4 \text{ ways}$$

The remaining 8 gaps can be filled by 4 S by ${}^{8}C_4$ ways

$\therefore$ *Total no. of ways* $= 7\ {}^{6}C_4 \quad {}^{8}C_4$

$\therefore$ *Correct answer is (2)*

Q. 11.

Let $i = \int_0^1 \frac{\cos x}{\sqrt{x}}\, dx$. *Then which one of the following is true?*

i. $I < \frac{2}{3}$ *and* $J > 2$

ii. $I < \frac{2}{3}$ *and* $J < 2$

iii. $I > \frac{2}{3}$ *and* $J > 2$

iv. $I < \frac{2}{3}$ *and* $J > 2$

Sol.

We Know $\frac{\sin x}{x} < 1$, *when* $x \in (0, 1)$

$\therefore \frac{\sin x}{\sqrt{x}} < \sqrt{x}$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \int_0^1 \sqrt{x}\, dx$

$\Rightarrow \int_0^1 \frac{\sin x}{\sqrt{x}} dx < \frac{2}{3}$

Also, $\cos x < 1$, *when* $x \in (0,1)$

$\therefore \frac{\cos x}{\sqrt{x}} < \frac{1}{\sqrt{x}}$

$\Rightarrow \int_0^1 \frac{\cos x}{\sqrt{x}} dx < \int \frac{1}{\sqrt{x}}\, dx$

$\int_0^1 \frac{\cos x}{\sqrt{x}} dx < 2$

$\therefore I < \frac{2}{3}$ *and* $J < 2$

$\therefore$ *Correct answer is (4)*

Q. 12. The area of the plane region bounded by the curve $x + 2y^2 = 0$ *and* $3y^2 = 1$ *is equal to*

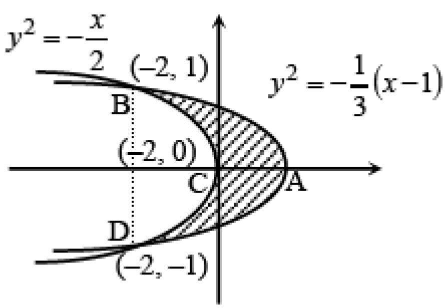
i. $\frac{2}{3}$

ii. $\frac{4}{3}$

iii. $\frac{5}{3}$

iv. $\frac{1}{3}$

Sol.



$x + 2y^2 = 0 \Rightarrow y^2 = -\frac{x}{2}$

$x + 3y^2 = 1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

$\therefore -\frac{x}{2} = -\frac{1}{3}(x-1)$

or, $\quad -\frac{x}{2} = -\frac{x}{3} + \frac{1}{3}$

or, $\quad \frac{x}{3} - \frac{x}{2} = \frac{1}{3}$

or, $\quad -\frac{x}{6} = \frac{1}{3}$

or, $\quad x = -2$

$\therefore y^2 = 1 \Rightarrow y = \pm 1$

Area of the region BCA

$$= \left|\int_0^1 \left((-2y^2) - (1-3y^2)\right) dy\right|$$

$$= \left|\int_0^1 (y^2 - 1) dy\right|$$

$$= \left|\left[\frac{y^3}{3} - y\right]_0^1\right|$$

$$= \left|\frac{1}{3} - 1\right| = \frac{2}{3}$$

Hence area of the region bounded by the curve is equal to $2 \times \frac{2}{3} = \frac{4}{3}$

∴ Correct answer is (2)