Scheme of Examination for M.Sc. Mathematics
w.e.f 2011-12

Semester – I

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-401	Advanced Abstract Algebra – I	80	20	100	3 Hours
MM-402	Real Analysis – I	80	20	100	3 Hours
MM-403	Topology	80	20	100	3 Hours
MM-404	Complex Analysis – I	80	20	100	3 Hours
MM-405	Differential Equations – I	80	20	100	3 Hours
MM-406	Practical-I	--	--	100	4 Hours

Semester – II

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-407	Advanced Abstract Algebra – II	80	20	100	3 Hours
MM-408	Real Analysis – II	80	20	100	3 Hours
MM-409	Computer Programming (Theory)	80	20	100	3 Hours
MM-410	Complex Analysis – II	80	20	100	3 Hours
MM-411	Differential Equations – II	80	20	100	3 Hours
MM-412	Practical-II	--	--	100	4 Hours

Scheme of Examination for M.Sc. Mathematics

Semester – III

Compulsory Papers:

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-501	Functional Analysis	80	20	100	3 Hours
MM-502	Analytical Mechanics and Calculus of Variations	80	20	100	3 Hours

Optional Papers: A student can opt one optional paper from MM-503 opt (i) to opt (iv). Similarly one paper will be opted each from MM-504 opt (i) to opt (iv) and MM-505 opt (i) to (iv)

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-503 (Opt. (i))	Elasticity	80	20	100	3 Hours
MM-503 **(Opt. (ii)**	Difference Equations-I	80	20	100	3 Hours
MM-503 **(Opt. (iii)**	Analytic Number Theory	80	20	100	3 Hours
MM-503 **(Opt. (iv)**	Number Theory	80	20	100	3 Hours
MM-504 (Opt. (i)	Fluid Mechanics – I	80	20	100	3 Hours
MM-504 **(Opt. (ii)**	Mathematical Statistics	80	20	100	3 Hours
MM-504 **(Opt. (iii)**	Algebraic Coding Theory	80	20	100	3 Hours
MM-504 **(Opt. (iv)**	Commutative Algebra	80	20	100	3 Hours

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-505 (Opt. (i)	Integral Equations	80	20	100	3 Hours
MM-505 **(Opt. (ii)**	Mathematical Modeling	80	20	100	3 Hours
MM-505 **(Opt. (iii)**	Linear Programming	80	20	100	3 Hours
MM-505 **(Opt. (iv)**	Fuzzy Sets & Applications –I	80	20	100	3 Hours
MM-506	Practical-III	--	--	100	4 Hours

Semester – IV

Compulsory Papers:

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-507	General Measure and Integration Theory	80	20	100	3 Hours
MM-508	Partial Differential Equations	80	20	100	3 Hours

Optional Papers: A candidate can opt one optional paper from MM-509 opt (i) to opt (iv). Similarly one paper will be opted each from MM-510 opt (i) to opt (iv) and MM-511 opt (i) to opt. (iv)

Paper Code	Nomenclature	External Theory Exam. Marks	Internal Assessment Marks	Max. Marks	Examination Hours
MM-509 (Opt. (i)	Mechanics of Solids	80	20	100	3 Hours
MM-509 **(Opt. (ii)**	Difference Equations-II	80	20	100	3 Hours
MM-509 **(Opt. (iii)**	Algebraic Number Theory	80	20	100	3 Hours
MM-509 **(Opt. (iv)**	Mathematics for Finance & Insurance	80	20	100	3 Hours
MM-510 (Opt. (i)	Fluid Mechanics-II	80	20	100	3 Hours
MM-510 **(Opt. (ii)**	Boundary Value Problems	80	20	100	3 Hours
MM-510 **(Opt. (iii)**	Non-Commutative Rings	80	20	100	3 Hours
MM-510 **(Opt. (iv)**	Advanced Discrete Mathematics	80	20	100	3 Hours
MM-511 (Opt. (i)	Mathematical Aspects of Seismology	80	20	100	3 Hours
MM-511 **(Opt. (ii)**	Dynamical Systems	80	20	100	3 Hours

Paper Code	**Nomenclature**	**External Theory Exam. Marks**	**Internal Assessment Marks**	**Max. Marks**	**Examination Hours**
MM-511 **(Opt. (iii)**	Operational Research	80	20	100	3 Hours
MM-511 **(Opt. (iv)**	Fuzzy Sets & Applications-II	80	20	100	3 Hours
MM-512	Practical-IV	--	--	100	4 Hours

Semester – I

MM-401: Advanced Abstract Algebra-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Questions)

Automorphisms and Inner automorphisms of a group G. The groups Aut(G) and Inn(G). Automorphism group of a cyclic group. Normalizer and Centralizer of a non-empty subset of a group G. Conjugate elements and conjugacy classes. Class equation of a finite group G and its applications. Derived group (or a commutator subgroup) of a group G. perfect groups. Zassenhau's Lemma. Normal and Composition series of a group G. Scheier's refinement theorem. Jordan Holder theorem. Composition series of groups of order p^n and of Abelian groups. Caunchy theorem for finite groups. ∏ - groups and p-groups. Sylow ∏-subgroups and Sylow p-subgroups. Sylow's Ist, IInd and IIIrd theorems. Application of Sylow theory to groups of smaller orders.

Section – II (Two Questions)

Characteristic of a ring with unity. Prime fields Z/pZ and Q. Field extensions. Degree of an extension. Algebraic and transcendental elements. Simple field extensions. Minimal polynomial of an algebraic element. Conjugate elements. Algebraic extensions. Finitely generated algebraic extensions. Algebraic closure and algebraically closed fields. Splitting fields., finite fields.. Normal extensions.

Section – III (Two Questions)

Separable elements, separable polynomials and separable extensions. Theorem of primitive element. Perfect fields. Galois extensions. Galois group of an extension. Dedekind lemma Fundamental theorem of Galois theory. Frobenius automorphism of a finite field. Klein's 4-group and Diheadral group. Galois groups of polynomials. Fundamental theorem of Algebra.

Section – IV (Two Questions)

Solvable groups Derived series of a group G. Simplicity of the Alternating group A_n ($n \geq 5$). Non-solvability of the symmetric group Sn and the Alternating group An ($n \geq 5$). Roots of unity Cyclotomic polynomials and their irreducibility over Q Radicals extensions. Galois radical extensions. Cyclic extensions. Solvability of polynomials by radicals over Q. Symmetric functions and elementary symmetric functions. Construction with ruler and compass only.

Recommended Books:

1. I.D. Macdonald. :The theory of Groups
2. P.B. Bhattacharya
   S.K. Jain & S.R. Nagpal : Basic Abstract Algebra (Cambridge University Press 1995)

Reference Books:

1. Vivek Sahai and Vikas Bist	:	Algebra (Narosa publication House)
2. I.S. Luthar and I.B.S. Passi	:	Algebra Vol. 1 Groups (Narosa publication House)
3. I.N. Herstein	:	Topics in Algebra (Wiley Eastern Ltd.)
4. Surjit Singh and Quazi Zameeruddin	:	Modern Algebra (Vikas Publishing House 1990)

Semester-I

MM-402 : REAL ANALYSIS –I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and existence of Riemann Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, integration of vector-valued functions, Rectifiable curves.
(Scope as in Chapter 6 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Section-II (Two Questions)

Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weirstrass M-test, Abel's test and Dirichlet's test for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann Stieltjes integration, uniform convergence and differentiation, existence of a real continuous nowhere differentiable function, equicontinous families of functions, Weierstrass approximation theorem (Scope as in Sections 7.1 to 7.27 of Chapter 7 of Principles of Mathematical Analysis by Walter Rudin, Third Edition).

Section-III (Two Questions)

Functions of several variables : linear transformations, Derivative in an open subset of R^n, Chain rule, Partial derivatives, directional derivatives, the contraction principle, inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Derivatives of higher order, mean value theorem for real functions of two variables, interchange of the order of differentiation, Differentiation of integrals.

(Scope as in relevant portions of Chapter 9 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition)

Section-IV (Two Questions)

Power Series : Uniqueness theorem for power series, Abel's and Tauber's theorem, Taylor's theorem, Exponential & Logarithm functions, Trigonometric functions, Fourier series, Gamma function
(Scope as in Chapter 8 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Integration of differential forms: Partitions of unity, differential forms, stokes theorem

(scope as in relevant portions of Chapter 9 & 10 of 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition).

Recommended Text:
'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.

Reference Books :

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.
4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
5.Serge Lang, Analysis I & II, Addison-Wesley Publishing Company Inc., 1969.
M.Sc.(P)Mathematics Semester-I

Semester-I

MM-403: TOPOLOGY

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set , closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense sets. A characterization of dense sets.
Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology.
Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using 'properties' of 'Neighbourhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator and 'base'.
First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorem.
Comparison of Topologies on a set, about intersection and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice (scope as in theorems 1-16, chapter 1 of Kelley's book given at Sr. No. 1).

SECTION-II (Two Questions)

Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding.
Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness, Characterisation of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces.

T_0 , T_1, T_2,Regular and T_3 separation axioms, their characterization and basic properties i.e. hereditary property of T_0 , T_1, T_2, Regular and T_3 spaces, and productive property of T_1 and T_2 spaces.
Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space (scope as in theorems 1, 2, 3, 5, 6, 8-11, Chapter 3 and relevant portion of chapter 4 of Kelley's book given at Sr.No.1)

Section-III (Two Questions)

Completely regular and Tychonoff ($T_{3\ 1/2}$), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem.
Normal and T_4 spaces : Definition and simple examples, Urysohn's Lemma, complete regularity of a regular normal space, T_4 implies Tychonoff, Tietze's extension theorem (Statement only). (Scope as in theorems 1-7, Chapter 4 of Kelley's book given at Sr. No. 1).
Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters, Ultra filter (u.f.) and its characterizations, Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.
Convergence of filters: Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters. Hausdorffness and filter convergence.

Section-IV (Two Questions)

Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), continuity and compact sets, compactness and separation properties, Closedness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space.
Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse, compactification and Hausdorff compactification, Stone-Cech compactification, (Scope as in theorems 1,7-11, 13, 14, 15, 22-24, Chapter 5 of Kelley's book given at Sr. No. 1).

Books :

1. Kelley, J.L.	:	General Topology.
2. Munkres, J.R.	:	Topology, Second Edition, Prentice Hall of India/ Pearson.

Semester-I

MM-404: COMPLEX ANALYSIS-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Power series, its convergence, radius of convergence, examples, sum and product, differentiability of sum function of power series, property of a differentiable function with derivative zero. expz and its properties, logz, power of a complex number (z), their branches with analyticity.
Path in a region, smooth path, p.w. smooth path, contour, simply connected region, multiply connected region, bounded variation, total variation, complex integration, Cauchy-Goursat theorem, Cauchy theorem for simply and multiply connected domains.

Section II (Two Questions)

Index or winding number of a closed curve with simple properties. Cauchy integral formula. Extension of Cauchy integral formula for multiple connected domain. Higher order derivative of Cauchy integral formula. Gauss mean value theorem Morera's theorem. Cauchy's inequality. Zeros of an analytic function, entire function, radius of convergence of an entire function, Liouville's theorem, Fundamental theorem of algebra, Taylor's theorem.

Section-III (Two Questions)

Maximum modulus principle, Minimum modulus principle. Schwarz Lemma. Singularity, their classification, pole of a function and its order. Laurent series, Cassorati – Weiertrass theorem Meromorphic functions, Poles and zeros of Meromorphic functions. The argument principle, Rouche's theorem, inverse function theorem.

Section-IV (Two Questions)

Residue : Residue at a singularity, residue at a simple pole, residue at infinity. Cauchy residue theorem and its use to calculate certain integrals, definite integral ($\int_0^{2\Pi} f(\cos\theta, \sin\theta)\, d\theta$, $\int_{-\infty}^{\infty} f(x)dx$), integral of the type $\int_0^{\infty} f(x) \sin mx\, dx$ or $\int_0^{\infty} f(x) \cos mx\, dx$, poles on the real axis, integral of many valued functions.

Bilinear transformation, their properties and classification, cross ration, preservance of cross ration under bilinear transformation, preservance of circle and straight line under bilinear transformation, fixed point bilinear transformation, normal form of a bilinear transformation. Definition and examples of conformal mapping, critical points.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-I

MM-405: Differential Equations –I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

<u>Section –I (Two Questions)</u>

Preliminaries: Initial value problem and equivalent integral equation, ε-approximate solution, equicontinuous set of functions.
Basic theorems: Ascoli- Arzela theorem, Cauchy –Peano existence theorem and its corollary. Lipschitz condition. Differential inequalities and uniqueness, Gronwall's inequality. Successive approximations. Picard-Lindelöf theorem. Continuation of solution, Maximal interval of existence, Extension theorem. Kneser's theorem (statement only)
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

<u>Section-II (Two Questions)</u>

Linear differential systems: Definitions and notations. Linear homogeneous systems; Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Non-homogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

<u>Section-III (Two Questions)</u>

Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's Identity, Fundamental set, More Wronskian theory. Reduction of order. Non-homogeneous LDE. Variation of parameters. Adjoint equations, Lagrange's Identity, Green's formula. Linear equation of order n with constant coefficients. (Relevant portions from the books of 'Theory of Ordinary Differential Equations' by Coddington and Levinson and the book 'Differential Equations' by S.L. Ross)

Section –IV (Two Questions)

System of differential equations, the n-th order equation. Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness theorems: Kamke's theorem, Nagumo's theorem and Osgood theorem.
(Relevant portions from the book 'Ordinary Differential Equations' by P. Hartman)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
4. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
5. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
6. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
7. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.
8. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.

Semester-I

Paper MM-406 : Practical-I

Examination Hours : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem-solving techniques based on papers MM-401 to MM-405 will be taught.

Part-B : Implementation of the following programs in ANSI C.

1. Use of nested **if.. .else** in finding the smallest of four numbers.
2. Use series sum to compute **sin(x)** and **cos(x)** for given angle **x** in degrees. Then, check error in verifying $\sin^2 x+\cos^2(x)=1$.
3. Verify $\sum n^3 =\{\sum n \}^2$, (where n=1,2,..,m) & check that prefix and postfix increment operator gives the same result.
4. Compute simple interest of a given amount for the annual rate = .12 if amount >=10,000/- or time >=5 years; =.15 if amount >=10,000/- and time >=5 years; and = .10 otherwise.
5. Use array of pointers for alphabetic sorting of given list of English words.
6. Program for interchange of two rows or two columns of a matrix. Read/write input/output matrix from/to a file.
7. Calculate the eigenvalues and eigenvectors of a given symmetric matrix of order 3.
8. Calculate standard deviation for a set of values {x(j)j=l,2,...,n} having the corresponding frequencies {f(j)j=l,2,...,n}.
9. Find GCD of two positive integer values using pointer to a pointer.
10. Compute GCD of 2 positive integer values using recursion.
11. Check a given square matrix for its positive definite form.
12. To find the inverse of a given non-singular square matrix.

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

Semester – II

MM-407: Advanced Abstract Algebra-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Commutators and higher commutators. Commutators identities. Commutator subgroups. Derived group. Three subgroups Lemma of P.Hall. Central series of a group G. Nilpotent groups. Centre of a nilpotent group. Subgroups and factor subgroups of nilpotent groups. Finite nilpotent groups. Upper and lower central series of a group G and their properties. Subgroups of finitely generated nilpotent groups. Sylow-subgroups of nilpotent groups. (Scope of the course as given in the book at Sr. No. 2).

Section-II (Two Questions)

Similar linear transformations. Invariant subspaces of vector spaces. Reduction of a linear transformation to triangular form. Nilpotent transformations. Index of nilpotency of a nilpotent transformation. Cyclic subspace with respect to a nilpotent transformation. Uniqueness of the invariants of a nilpotent transformation.

Primary decomposition theorem. Jordan blocks and Jordan canonical forms. Cyclic module relative to a linear transformation. Companion matrix of a polynomial f(x). Rational Canonicals form of a linear transformation and its elementary divisior. Uniqueness of the elementary divisior. (Sections 6.4 to 6.7 of the book. Topics in Algebra by I.N. Herstein).

Section-III (Two Questions)

Modules, submodules and quotient modules. Module generated by a non-empty subset of an R-module. Finitely generated modules and cyclic modules. Idempotents. Homomorphism of R-modules. Fundamental theorem of homomorphism of R-modules. Direct sum of modules. Endomorphism rings $End_Z(M)$ and $End_R(M)$ of a left R-module M. Simple modules and completely reducible modules (semi-simple modules). Finitely generated free modules. Rank of a finitely generated free module. Submodules of free modules of finite rank over a PID. (Sections 14.1 to 14.5 of the book Basic Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal)

Section-IV (Two Questions)

Endomorphism ring of a finite direct sum of modules. Finitely generated modules. Ascending and descending chains of sub modules of an R-module. Ascending and Descending change conditions (A.C.C. and D.C.C.). Noetherian modules and Noetherian rings. Finitely co-generated modules. Artinian modules and Artinian rings. Nil and nilpotent ideals. Hilbert Basis Theorem. Structure theorem of finite Boolean rings. Wedeerburn-Artin theorem and its consequences. (sections 19.1 to 19.3 of the book Basic Abstract Algebra by P.B. Bhattacharya S.K. Jain and S.R. Nagpal).

Recommended Books:

1. Basic Abstract Algebra	:	P.B. Bhattacharya S.R. Jain and S.R. Nagpal
2. Theory of Groups	:	I.D. Macdonald
3. Topics in Algebra	:	I.N. Herstein
4. Group Theory	:	W.R. Scott

Semester-II

MM-408 : REAL ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Lebesgue outer measure, elementary properties of outer measure, Measurable sets and their properties, Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets and their measurability, characterization of measurable sets in terms of open, closed, F and G sets, existence of a non-measurable set.

Lebesgue measurable functions and their properties, characteristic functions, simple functions, approximation of measurable functions by sequences of simple functions, measurable functions as nearly continuous functions. Borel measurability of a function.

Section-II (Two Questions)

Almost uniform convergence, Egoroff's theorem, Lusin's theorem, convergence in measure, F.Riesz theorem that every sequence which is convergent in measure has an almost everywhere convergent subsequence.

The Lebesgue Integral :
Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of finite measure and its properties, Lebsegue integral as a generalization of the Riemann integral, Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions.

Section-III (Two Questions)

Integral of a non negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral, Lebesgue convergence theorem.
Differentiation and Integration :
Differentiation of monotone functions, Vitali's covering lemma, the four Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions.

Section-IV (Two Questions)

Differentiation of an integral, absolutely continuous functions, convex functions, Jensen's inequality.

The L^p spaces

The L^p spaces, Minkowski and Holder inequalities, completeness of L^p spaces, Bounded linear functionals on the L^p spaces, Riesz representation theorem.

Recommeded Text :

'Real Analysis' by H.L.Royden (3rd Edition) Prentice Hall of India, 1999.

Reference Books :

1. G.de Barra, Measure theory and integration, Willey Eastern Ltd.,1981.
2. P.R.Halmos, Measure Theory, Van Nostrans, Princeton, 1950.
3. I.P.Natanson, Theory of functions of a real variable, Vol. I, Frederick Ungar Publishing Co., 1961.
4. R.G.Bartle, The elements of integration, John Wiley & Sons, Inc.New York, 1966.
5. K.R.Parthsarthy, Introduction to Probability and measure, Macmillan Company of India Ltd.,Delhi, 1977.

   P.K.Jain and V.P.Gupta, Lebesgue measure and integration, New age International (P) Ltd., Publishers, New Delhi, 1986.

Semester-II

MM-409 : Computer Programming (Theory)

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Numerical constants and variables; arithmetic expressions; input/output; conditional flow; looping.

Section-II (Two Questions)

Logical expressions and control flow; functions; subroutines; arrays.

Section- III(Two Questions)

Format specifications; strings; array arguments, derived data types.

Section- IV(Two Questions)

Processing files; pointers; modules; FORTRAN 90 features; FORTRAN 95 features.

Recommended Text:

V. Rajaraman : Computer Programming in FORTRAN 90 and 95; Printice-Hall of India Pvt. Ltd., New Delhi, 1997.

References :

1. V. Rajaraman : Computer Programming in FORTRAN 77, Printice-Hall of India Pvt. Ltd., New Delhi, 1984.
2. J.F. Kerrigan : Migrating of FORTRAN 90, Orielly Associates, CA, USA, 1993.
3. M.Metcalf and J.Reid : FORTRAN 90/95 Explained, OUP, Oxford, UK, 1996.

Semester-II

MM-410 : COMPLEX ANALYSIS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Spaces of analytic functions and their completeness, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, infinite products, Weierstrass factorization theorem, Factorization of sine function, Gamma function and its properties, functional equation for gamma function, Integral version of gamma function.

Section- II (Two Questions)

Reimann-zeta function, Riemann's functional equation, Runge's theorem, Mittag-Leffler's theorem.
Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation , Schwarz reflection principle.

Section –III (Two Questions)

Monodromy theorem and its consequences. Harmonic function as a disk, Poisson's Kernel. Harnack's inequality, Harnack's theorem, Canonical product, Jensen's formula, Poisson-Jensen formula, Hadamard's three circle theorem. Dirichlet problem for a unit disk. Dirichlet problem for a region, Green's function.

Section –IV (Two Questions)

Order of an entire function, Exponent of convergence, Borel theorem, Hadamard's factorization theorem. The range of an analytic function, Bloch's theorem, Little-Picard theorem, Schottky's theorem, Montel-Carathedory theorem, Great Picard theorem.
Univalent functions, Bieberbach's conjecture (Statement only), and $1^{7}/4$ theorem.

<u>Books recommended</u> :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

Semester-II

MM-411: DIFFERENTIAL EQUATIONS-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all, taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section –I (Two Questions)

Linear second order equations: Preliminaries, self adjoint equation of second order, Basic facts, superposition principle, Riccati's equation, Prüffer transformation, zero of a solution, Oscillatory and non-oscillatory equations. Abel's formula. Common zeros of solutions and their linear dependence.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Section –II (Two Questions)

Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and its corollaries. Elementary linear oscillations.
Autonomous systems: the phase plane, paths and critical points, Types of critical points; Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Section-III (Two Questions)

Critical points and paths of non-linear systems: basic theorems and their applications. Liapunov function. Liapunov's direct method for stability of critical points of non-linear systems.
Limit cycles and periodic solutions: Limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence criterion. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. Index of a critical point.

(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Section-IV (Two Questions)

Second order boundary value problems(BVP): Linear problems; periodic boundary conditions, regular linear BVP, singular linear BVP; non-linear BVP. Sturm-Liouville BVP: definitions, eigen value and eigen function. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values. Green's function. Applications of boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.
(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.
4. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
5. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
6. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
7. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
8. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.

Semester-II

Paper MM-412 : Practical-II

Examination Hours : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-407 to MM-411 will be taught.

Part-B : Implementation of the following programs in FORTRAN-90

1. Calculate the area of a triangle with given lengths of its sides.
2. Given the centre and a point on the boundary of a circle, find its perimeter and area.
3. To check an equation $ax^2+ by^2+2cx+2dy+e=0$ in (x, y) plane with given coefficients for representing parabola/ hyperbola/ ellipse/ circle or else.
4. For two given values x and y, verify g*g=a*h, where a, g and h denote the arithmetic, geometric and harmonic means respectively.
5. Use IF..THEN…ELSE to find the largest among three given real values.
6. To solve a quadratic equation with given coefficients, without using COMPLEX data type.
7. To find the location of a given point (x,y) i) at origin, ii) on x-axis or y-axis iii) in quadrant I, II, III or IV.
8. To find if a given 4-digit year is a leap year or not.
9. To find the greatest common divisor (gcd) of two given positive integers.
10. To verify that sum of cubes of first m positive integers is same as the square of the sum of these integers.
11. Find error in verifying sin(x+y)= sin(x) cos(y)+cos(x)sin(y), by approximating the sin(x) and cos(x) functions from the finite number of terms in their series expansions.
12. Use SELECT…CASE to calculate the income tax on a given income at the existing rates.

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

SEMESTER-III

MM-501 Functional Analysis

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Normed linear spaces, Banach spaces and examples, subspace of a Banach space, completion of a normed space, quotient space of a normed linear space and its completeness, product of normed spaces, finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, F.Riesz's lemma.

Bounded and continuous linear operators, differentiation operator, integral operator, bounded linear extension, linear functionals, bounded linear functionals, continuity and boundedness, definite integral, canonical mapping, linear operators and functionals on finite dimensional spaces, normed spaces of operators, dual spaces with examples. (Scope of this section is as in relevant parts of Chapter 2 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

SECTION-II (Two Questions)

Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces, application to bounded linear functionals on C[a,b], Riesz-representation theorem for bounded linear functionals on C[a,b], adjoint operator, norm of the adjoint operator.

Reflexive spaces, uniform boundedness theorem and some of its applications to the space of polynomials and fourier series. (Scope of this section is as in relevant parts of sections 4.1 to 4.7 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

SECTION-III (Two Questions)

Strong and weak convergence, weak convergence in l^p, convergence of sequences of operators, uniform operator convergence, strong operator convergence, weal operator convergence, strong and weak* convergence of a sequence of functionals. Open mapping theorem, bounded inverse theorem, closed linear operators, closed graph theorem, differential operator, relation between closedness and boundedness of a linear operator. (Scope of this section is as in relevant parts of sections 4.8, 4.9, 4.12 and 4.13 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Inner product spaces, Hilbert spaces and their examples, pythagorean theorem, Apolloniu's identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct sums, projection theorem, characterization of sets in Hilbert spaces whose space is dense. (Scope as in relevant parts of sections 3.1, 3.2 and 3.3 of Chapter 3 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

SECTION-IV (Two Questions)

Orthonormal sets and sequences, Bessel's inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval's identity, separable Hilbert spaces.Representation of functionals on Hilbert spaces, Riesz representation theorem for bounded linear functionals on a Hilbert space, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space.
Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self adjoint, unitary, normal, positive and projection operators. (Scope of this section is as in relevant parts of sections 3.4 to 3.6 and 3.8 to 3.10 of Chapter 3 and sections 9.3 to 9.6 of Chapter 9 of 'Introductory Functional Analysis with Applications' by E.Kreyszig.

Recommended Text:
E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1978.

References:

1. G.F.Simmons: Introduction to Topology and Modern Analysis, McGraw Hill Book Co.,New York, 1963.
2. C.Goffman and G.Pedrick: First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
3. G.Bachman and L.Narici, Functional Analysis, Academic Press, 1966.
4. L.A.Lustenik and V.J.Sobolev, Elements of Functional Analysis, Hindustan Publishing Corporation, New Delhi, 1971.
5. J.B.Conway: A Course in Functional Analysis, Springer-Verlag, 1990.
6. P.K.Jain, O.P.Ahuja and Khalil Ahmad: Functional Analysis, New Age International(P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.

SEMESTER- III

MM-502 Analytical Mechanics and Calculus of Variations

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Motivating problems of calculus of variations: shortest distance, Minimum surface of revolution, Brachistochrone problem, Isoperimetric problem, Geodesic. Fundamental Lemma of calculus of variation. Euler's equation for one dependent function of one and several independent variables, and its generalization to (i) Functional depending on 'n' dependent functions, (ii) Functional depending on higher order derivatives. Variational derivative, invariance of Euler's equations, natural boundary conditions and transition conditions, Conditional extremum under geometric constraints and under integral constraints . Variable end points.

SECTION-II (Two Questions)

Free and constrained systems, constraints and their classification. Generalized coordinates. Holonomic and Non-Holonomic systems. Scleronomic and Rheonomic systems. Generalized Potential, Possible and virtual displacements,ideal constraints. . Lagrange's equations of first kind, Principle of virtual displacements D'Alembert's principle, HolonomicSystems independent coordinates, generalized forces, Lagrange's equations of second kind. Uniqueness of solution. Theorem on variation of total Energy. Potential, Gyroscopic and dissipative forces, Lagrange's equations for potential forces equation for conservative fields.

SECTION-III (Two Questions)

Hamilton's variables. Don kin's theorem. Hamilton canonical equations. . Routh's equations. Cyclic coordinates Poisson's Bracket. Poisson's Identity. Jacobi-Poisson theorem. Hamilton's Principle, second form of Hamilton's principle. Poincare-Carton integral invariant. Whittaker's equations. Jacobi's equations. Principle of least action

SECTION-IV (Two Questions)

Canonical transformations, free canonical transformations, Hamilton-Jacobi equation. Jacobi theorem. Method of separation of variables for solving Hamilton-Jacobi equation. Testing the Canonical character of a transformation. Lagrange brackets. Condition of canonical character of a transformation in terms of Lagrange brackets and Poisson brackets. Simplicial nature of the Jacobian matrix of a canonical transformations. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

Books:

1. F. Gantmacher, Lectures in Analytic Mechanics, Khosla Publishing House, New Delhi.

2. H. Goldstein, Classical Mechanics (2nd edition), Narosa Publishing House, New Delhi.
3. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
4. Francis B. Hilderbrand, Methods of applied mathematics , Prentice Hall,
5. Narayan Chandra Rana & Pramod Sharad Chandra Joag. Classical Mechanics, Tata McGraw Hill, 1991.
6. Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University Press, 1998.

SEMESTER-III

MM-503 (opt. i) Elasticity

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Tensor Algebra: Coordinate-transformation, Cartesian Tensor of different order.
Properties of tensors, Isotropic tensors of different orders and relation between them, Symmetric and skew symmetric tensors. Tensor invariants, Deviatoric tensors, Eigen-values and eigen-vectors of a tensor.
Tensor Analysis: Scalar, vector, tensor functions, Comma notation, Gradient, divergence and curl of a vector / tensor field. (Relevant portions of Chapters 2 and 3 of book by D.S. Chandrasekharaiah and L Debnath)

SECTION-II (Two Questions)

Analysis of Strain : Affine transformation, Infinitesimal affine deformation, Geometrical Interpretation of the components of strain. Strain quadric of Cauchy. Principal strains and invariance, General infinitesimal deformation. Saint-Venant's equations of compatibility. Finite deformations
Analysis of Stress : Stress Vecotr, Stress tensor, Equations of equilibrium, Transformation of coordinates.
(Relevant portion of Chapter I & II of book by I.S. Sokolnikoff).

SECTION-III (Two Questions)

Stress quadric of Cauchy, Principal stress and invariants. Maximum normal and shear stresses. Mohr’s circles, examples of stress. Equations of Elasticity : Generalised Hooks Law, Anisotropic symmetries, Homogeneous isotropic medium.
(Relevant portion of Chapter II & III of book by I.S. Sokolnikoff).

SECTION-IV (Two Questions)

Elasticity moduli for Isotropic media. Equilibrium and dynamic equations for an isotropic elastic solid. Strain energy function and its connection with Hooke’s Law, Uniqueness of solution. Beltrami-Michell compatibility equations. Clapeyrom’s theorem. Saint-Venant's principle.
(Relevant portion of Chapter III of book by I.S.Sokolnikoff).

Books:

1. I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata-McGraw Hill Publishing Company Ltd., New Delhi, 1977.
2. A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
3. Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
4. D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.
5. Shanti Narayan, Text Book of Cartesian Tensor, S. Chand & Co., 1950.
6. S. Timeshenki and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
7. I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delhi, 1975.

SEMESTER-III

MM-503 (opt. ii) Difference Equations-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Introduction,the difference calculus: The difference operator,falling factorial power $t^{\underline{r}}$,binomial coefficient $\binom{t}{r}$, summation, definition, properties and examples, Abel's summation formula, Generating functions, Euler's summation formula, Bernoulli polynomials and examples, approximate summation.

SECTION-II (Two Questions)

Linear Difference Equation: First order linear equations, general results for linear equations, solution of linear difference equation with constant coefficients and with variable coefficients, Non-Linear Equations that can be linearized, applications.

SECTION-III (Two Questions)

Stability Theory : Initial value Problems for Linear systems, eigen values, eigen vectors and spectral radius, Caylay-Hamilton Theorem, Putzer algorithm. Solution of nonhomogeneous system with initial conditions, Stability of linear systems, stable subspace theorem and example. Stability of non-linear system, Chaotic behaviour.

SECTION-IV (Two Questions)

The Z-Transform, definition, Properties, initial and final value Theorem, Convolation Theorem, Solving the initial value problems, Volterra summation equation and Fredholm summation equation by use of Z-Transform.

Asymptotic Methods : Introduction, Asymptotic Analysis of Sums, and examples. Asymptotic behaviour of solutions of homogeneous linear equations, Poincare's Theorem, Perron Theorem (Statement only), non-linear equations.

Recommended Text:

W.G. Kelley and A.C. Peterson: Difference Equations; An introduction with Applications, Academic Press, Harcourt, 1991. (Relevant portions of chapters 1-5.)

Reference Book:

Calvin Ahlbrandt & Allan C. Peterson, Discreet Hamiltonian systems, Difference Equations, Continued Fractions & Ricati Equation, Kluwer Botson, 1996

SEMESTER-III

MM-503 (opt.iii) Analytic Number Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Arithmetical functions, Mobius function, Euler totient function, relation connecting Mobius function and Euler totient function, Product formula for Euler totient function, Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, Mangoldt function, multiplicative functions, Multiplicative functions and Dirichlet multiplication. Inverse of completely multiplicative function, Liouville's function, divisor function, generalized convolutions, Formal power-series, Bell series of an arithmetical function, Bell series and Dirichlet multiplication, Derivatives of arithmetical functions, Selberg identity. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas, average order of divisor functions, average order of Euler totient function.

SECTION-II (Two Questions)

Application to the distribution of lattice points visible from the origin, average order of Mobius function and Mangoldt function, Partial sums of a Dirichlet Product, applications to Mobius function and Mangoldt function, Legendre's identity, another identity for the partial sums of a Dirichlet product. Chebyshev's functions, Abel's identity, some equivalent forms of the prime number theorem. Inequalities for $\pi(n)$ and P_n .

SECTION-III (Two Questions)

_Shapiro's Tauberian theorem. Applications of Shapiro's theorem. An asymptotic formula for the partial sums $\sum_{p\le x}\left(\frac{1}{p}\right)$. Partial sums of the Mobius function. Brief sketch of an elementary proof of the prime number theorem; Selberg's asymptotic formula.

Elementary properties of groups, construction of subgroups, characters of finite abelian groups, the character group, orthogonality relations for characters, Dirichlet characters, Sums-involving Dirichlet characters, Nonvanishing of $L(1,\chi)$ for real nonprincipal χ.

SECTION-IV (Two Questions)

Dirichlet's theorem for primes of the form 4n-1 and 4n+1. Dirichlet's theorem. Functions periodic modulo K, Existence of finite Fourier series for periodic arithmetical functions. Ramanujan's sum and generalizations, multiplicative properties of the sums $S_k(n)$. Gauss sums associated with Dirichlet characters. Dirichlet characters with nonvanishing Gauss sums. Induced moduli and primitive characters, properties of induced moduli conductor of a character. Primitive characters and separable Gauss sums.

Finite fourier series of the Dirichlet characters. Polya's inequality for the partial sums of primitive characters.

Recommended Book:

Tom M. Apostol	Introduction to Analytic Number Theory

SEMESTER-III

MM-503 (opt. iv) Number Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

The equation ax+by = c, simultaneous linear equations, Pythagorean triangles, assorted examples, ternary quadratic forms, rational points on curves.

SECTION-II (Two Questions)

Elliptic curves, Factorization using elliptic curves, curves of genus greater than 1. Farey sequences, rational approximations, Hurwitz theorem, irrational numbers, Geometry of Numbers, Blichfeldt's principle, Minkowski's Convex body theorem Lagrange's four square theorem.

SECTION-III (Two Questions)

Euclidean algorithm, infinite continued fractions, irrational numbers, approximations to irrational numbers, Best possible approximations, Periodic continued fractions, Pell's equation.

SECTION-IV (Two Questions)

Partitions, Ferrers Graphs, Formal power series, generating functions and Euler's identity, Euler's formula, bounds on P(n), Jacobi's formula, a divisibility property.

Recommended Text:

An Introduction to the Theory of Numbers	Ivan Niven Herbert S. Zuckerman Hugh L: Montgomery John Wiley & Sons(Asia)Pte.Ltd. (Fifth Edition)

SEMESTER- III

MM-504 (opt. i) Fluid Mechanics-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Kinematics of fluid in motion:Velocity at a point of a fluid. Lagrangian and Eulerian methods. Stream lines, path lines and streak liens, vorticity and circulation, Vortex lines, Acceleration and Material derivative, Equation of continuity (vector or Cartesian form). Reynolds transport Theorem. General analysis of fluid motion. Properties of fluids- static and dynamic pressure. Boundary surfaces and boundary surface conditions. Inotational and rotational motions. Velocity potential.

SECTION-II (Two Questions)

Equation of Motion : Lagrange's and Euler's equations of Motion (vector or in Cartesian form). Bernculli's theorem. Applications of the Bernoulli Equation in one –dimensional flow problems. Kelvins circulation theorem, vorticity equation. Energy equation for incompressible flow. Kinetic energy of irrotational flow. Kelvins minimum energy theorem ,mean potential over a spherical surface. Kinetic energy of infinite liquid. Uniqueness theorems.

SECTION –III (Two Questions)

Stress components in a real fluid. Relations between rectangular components of stress. Connection between stresses and gradients of velocity.Navier- Stoke's equations of motion. Steady flows between two parallel plates, Plane Poiseuille and Couette flows.

SECTION –IV (Two Questions)

Reduction of Navier-Stock equations in flows having axis of symmetry, steady flow in circular pipe: the Hagen-Poiseuille flow, steady flow between two coaxial cylinders, flow between two concentric rotating cylinders. Steady flows through tubes of uniform cross-section in the form (i) Ellipse, (ii) equilateral triangle, (iii) rectangle, under constant pressure gradient, uniqueness theorem.

Books :

1. W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers, Delhi, 1988.
2. F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
3. Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, 1986.
4. G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New Delhi, 1994.
5. A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics Springer-Verlag, New York, 1993.
6. L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
7. H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1979.
8. R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi. 1976.9
9.. A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.
10. S. w. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New Delhi, 1976.

Semester-III

MM : 504 (opt. ii) Mathematical Statistics

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

Random distribution: preliminaries, Probability density function, Probability models, Mathematical Expectation, Chebyshev's Inequality; Conditional probability, Marginal and conditional distributions, Correlation coefficient, Stochastic independence.

Section-II (Two Questions)

Frequency distributions: Binomial, Poissson, Gamma, Chi-square, Normal, Bivariate normal distributions.

Distributions of functions: Sampling, Transformations of variables: discrete and continuous; t & F distributions; Change of variable technique; Distribution of order; Moment-generating function technique; other distributions and expectations.

Section-III (Two Questions)

Limiting distributions: Stochastic convergence, Moment generating function, Related theorems.

Intervals: Random intervals, Confidence intervals for mean, differences of means and variance; Bayesian estimation.

Section-IV (Two Questions)

Estimation & sufficiency: Point estimation, sufficient statistics, Rao-Blackwell Theorem, Completeness, Uniqueness, Exponential PDF, Functions of parameters; Stochastic independence.

Books:

1. R.V. Hogg & A.T. Craig: Introduction to Mathematical Statistics, Amerind Pub. Co. Pvt. Ltd. New Delhi, 1972. (Chapters 1 to 7)
2. SC Gupta, VK Kapoor: Fundamentals of **Mathematical Statistics**, Sultan Chand & Sons (2007)

Semester – III

MM- 504 (opt. iii) Algebraic Coding Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION – I (Two Questions)

Block Codes. Minimum distance of a code. Decoding principle of maximum likelihood. Binary error detecting and error correcting codes. Group codes. Minimum distance of a group code (m, m+1) parity check code. Double and triple repition codes. Matrix codes. Generator and parity check matrices. Dual codes. Polynomial codes. Exponent of a polynomial over the binary field. Binary representation of a number. Hamming codes. Minimum distance of a Hamming code. (Chapter 1, 2, 3 of the book given at Sr. No. 1).

SECTION – II (Two Questions)

Finite fields. Construction of finite fields. Primitive element of a finite field. Irreducibility of polynomials over finite fields. Irreducible polynomials over finite fields. Primitive polynomials over finite fields. Automorphism group of $GF(q^n)$. Normal basis of $GF(q^n)$. The number of irreducible polynomials over a finite field. The order of an irreducible polynomial. Generator polynomial of a Bose-Chaudhuri-Hocqhenghem codes (BCH codes) construction of BCH codes over finite fields. (Chapter 4 of the book given at Sr. No. 1 and Section 7.1 to 7.3 of the book given at Sr. No. 2).

SECTION – III (Two Questions)

Linear codes. Generator matrices of linear codes. Equivalent codes and permutation matrices. Relation between generator and parity-check matrix of a linear codes over a finite field. Dual code of a linear code. Self dual codes. Weight distribution of a linear code. Weight enumerator of a linear code. Hadamard transform. Macwilliams identity for binary linear codes.

Maximum distance separable codes. (MDS codes). Examples of MDS codes. Characterization of MDS codes in terms of generator and parity check matrices. Dual code of a MDS code. Trivial MDS codes. Weight distribution of a MDS code. Number of code words of minimum distance d in a MDS code. Reed solomon codes. (Chapter 5 & 9 of the book at Sr. No. 1).

SECTION – IV (Two Questions)

Hadamard matrices. Existence of a Hadamard matrix of order n. Hadamard codes from Hadamard matrices Cyclic codes. Generator polynomial of a cyclic code. Check polynomial of a cyclic code. Equivalent code and dual code of a cyclic code. Idempotent generator of a cyclic code. Hamming and BCH codes as cyclic codes. Perfect codes. The Gilbert-varsha-move and Plotkin bounds. Self dual binary cyclic codes. (Chapter 6 & 11 of the book given at Sr. No. 1).

Recommended Text :

1. L.R. Vermani : Elements of Algebraic Coding Theory (Chapman and Hall Mathematics)
2. Steven Roman : Coding and Information Theory (Springer Verlag)

SEMESTER-III

MM-504 (opt. iv) Commutative Algebra

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Zero divisors, nilpotent elements and units, Prime ideals and maximal ideals, Nil radical and Jacobson radical, Comaximal ideals, Chineese remainder theorem, Ideal quotients and annihilator ideals. Extension and contraction of ideals. Exact sequences. Tensor product of module Restriction and extension of scalars. Exactness property of the tensor product. Tensor products of algebras.

SECTION-II (Two Questions)

Rings and modules of sections. Localization at the prime ideal P. Properties of the localization. Extended and contracted ideals in rings of fractions.

Primary ideals, Primary decomposition of an ideal, Isolated prime ideals, Multiplicatively closed subsets.

SECTION-III (Two Questions)

Integral elements, Integral closure and integrally closed domains, Going-up theorem and the Going-down theorem, valuation rings and local rings, Noether's normalization lemma and weak form of nullstellensatz Chain condition, Noetherian and Artinian modules, composition series and chain conditions.

SECTION-IV (Two Questions)

Noetherian rings and primary decomposition in Noetherian rings, radical of an ideal. Nil radical of an Artinian ring, Structure Theorem for Artinian rings, Discrete valuation rings, Dedekind domains, Fractional ideals.

(Scope of the course is as given in Chapter 1 to 9 of the recommended text).

Recommended Text:

M.F.Atiyah, FRS and I.G.Macdonald — Introduction to Commutative Algebra (Addison-Wesley Publishing Company)

Reference Books:

1. N.S.Gopal Krishnan, Oxonian Press Pvt. Ltd. — Commutative Algebra
2. Zariski, Van Nostrand Princeton(1958) — Commutative Algebra(Vol. I)

SEMESTER-III

MM-505 (opt. i) Integral Equations

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Definition of Integral Equations and their classifications. Eigen values and Eigen functions. Special kinds of Kernel Convolution Integral. The inner or scalar product of two functions. Reduction to a system of algebraic equations. Fredholm alternative, Fredholm theorem, Fredholm alternative theorem, An approximate method.
(Relevant portions from the Chapters 1 & 2 of the book "Linear Integral Equations, Theory & Techniques by R.P.Kanwal").

SECTION-II (Two Questions)

Method of successive approximations, Iterative scheme for Fredholm and Volterrra Integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution. Some results about the resolvent Kernel. Application of iterative scheme to Volterra integral equations of the second kind.
Classical Fredholm's theory, the method of solution of Fredholm equation, Fredholm's First theorem, Fredholm's second theorem, Fredhom's third theorem.
(Relevant portions from the Chapter 3 & 4 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-III (Two Questions)

Symmetric Kernels, Introduction, Complex Hilbert space. An orthonormal system of functions, Riesz-Fisher theorem, A complete two-Dimensional orthonormal set over the rectangle $a \le s \le b, c \le t \le d$. Fundamental properties of Eigenvalues and Eigenfunctions for symmetric Kernels. Expansion in eigen functions and Bilinear form. Hilbert-Schmidt theorem and some immediate consequences.
Definite Kernels and Mercer's theorem. Solution of a symmetric Integral Equation. Approximation of a general ℓ_2-Kernel(Not necessarily symmetric) by a separable Kernel. The operator method in the theory of integral equations. Rayleigh-Ritz method for finding the first eigenvalue.
(Relevant portions from the Chapter 7 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-IV (Two Questions)

The Abel Intergral Equation. Inversion formula for singular integral equation with Kernel of the type h(s)-h(t), $0<\alpha<1$, Cauchy's principal value for integrals solution of the Cauchy-type singular integral equation, closed contour, unclosed contours and the Riemann-Hilbert problem. The Hilbert-Kernel, solution of the Hilbert-Type singular Intergal equation.
(Relevant portions from the Chapter 8 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

References:

1. R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.
2. S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book Agency, 1960.
3. I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North Holland, 1966.
4. I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II, Mac.Millan, 1969.
5. Pundir and Pundir, Integral Equations and Boundary value problems, Pragati Prakashan, Meerut.

Semester-III

MM 505 : (opt. ii) Mathematical Modeling

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section-I (Two Questions)

The process of Applied Mathematics; mathematical modeling: need, techniques, classification and illustrative; mathematical modeling through ordinary differential equation of first order; qualitative solutions through sketching.

Section-II (Two Questions)

Mathematical modeling in population dynamics, epidemic spreading and compartment models; mathematical mode1ing through systems of ordinary differential equations; mathematical mode1ing in economics, medicine, arm-race, battle.

Section-III (Two Questions)

Mathematical modeling through ordinary differential equations of second order. Higher order (linear) models. Mathematical modeling through difference equations: Need, basic theory; mathematical modeling in probability theory, economics, finance, population dynamics and genetics.

Section-IV(Two Questions)

Mathematical modeling through partial differential equations: simple models, mass-balance equations, variational principles, probability generating function, traffic flow problems, initial & boundary conditions.

Book recommended :

J.N. Kapur: Mathematical Modeling, Wiley Eastern Limited, 1990 (Relevant portions, mainly from Chapters 1 to 6.)

Semester – III

MM-505 (opt. iii) LINEAR PROGRAMMING

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Questions)

Simultaneous linear equations, Basic solutions, Linear transformations, Point sets, Lines and hyperplanes, Convex sets, Convex sets and hyperplanes, Convex cones, Restatement of the Linear Programming problem, Slack and surplus variables, Preliminary remarks on the theory of the simplex method, Reduction of any feasible solution to a basic feasible solution, Definitions and notations regarding linear programming problems. Improving a basic feasible solution, Unbounded solutions, Optimality conditions, Alternative optima, Extreme points and basic feasible solutions.

Section-II (Two Questions)

The simplex method, Selection of the vector to enter the basis, Degeneracy and breaking ties, Further development of the transformation formulas, The initial basic feasible solution-----artificial variables, Inconsistency and redundancy, Tableau format for simplex computations, Use of the tableau format, Conversion of a minimization problem to a maximization problem, Review of the simplex method.

The two-phase method for artificial variables, Phase I, Phase II, Numerical examples of the two-phase method, Requirements space, Solutions space, Determination of all optimal solutions, Unrestricted variables, Charnes' perturbation method regarding the resolution of the degeneracy problem.

Section-III (Two Questions)

Selection of the vector to be removed, Definition of b(€). Order of vectors in b(€), Use of perturbation technique with simplex tableau format, Geometrical interpretation of the perturbation method. The generalized linear programming problem, The generalized simplex method, Examples pertaining to degeneracy, An example of cycling.

Revised simplex method: Standard Form I, Computational procedure for Standard Form I, Revised simplex method: Standard Form II, Computational procedure for Standard Form II, Initial identity matrix for Phase I, Comparison of the simplex and revised simplex methods, The product form of the inverse of a non-singular matrix. Alternative formulations of linear programming problems,

Section-IV (Two Questions)

Dual linear programming problems, Fundamental properties of dual problems, Other formulations of dual problems,Complementary slackness, Unbounded solution in the primal, Dual simplex algorithm, Alternative derivation of the dual simplex algorithm, Initial solution for dual simplex algorithm, The dual simplex algorithm; an example, geometric interpretations of the dual linear programming problem and the dual simplex algorithm. A primal dual algorithm, Examples of the primal-dual algorithm. Transportation problem, its formulation and simple examples.

Books :

1. G.Hadley : Linear Programming Narosa publishing House (1995)
2. S.I. Gauss : Linear Programming : Methods and Applications (4th Edition) McGraw Hill, New York 1975

SEMESTER-III

MM 505 (opt. iv) Fuzzy Sets and Applications-I

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Fuzzy Sets: Basic definitions, α-cuts, strong α-cuts, level set of a fuzzy set, support of a fuzzy set, the core and height of a fuzzy set, normal and subnormal fuzzy sets, convex fuzzy sets, cutworthy property, strong cutworthy property, standard fuzzy set operations, standard complement, equilibrium points, standard intersection, standard union, fuzzy set inclusion, scalar cardinality of a fuzzy set, the degree of subsethood (Scope as in relevant parts of sections 1.3-1.4 of Chapter 1 of the book given at Sr.No.1).

Additional properties of α-cuts involving the standard fuzzy set operators and the standard fuzzy set inclusion, Representation of fuzzy sets, three basic decomposition theorems of fuzzy sets Extension principle for fuzzy sets: the Zedah's extension principle, Images and inverse images of fuzzy sets, proof of the fact that the extension principle is strong cutworthy but not cutworthy (Scope as in relevant parts of Chapter 2 of the book mentioned at the end).

SECTION-II (Two Questions)

Operators on fuzzy sets: types of operations, fuzzy complements, equilibrium of a fuzzy complement, equilibrium of a continuous fuzzy complement, first and second characterization theorems of fuzzy complements, fuzzy intersections (t-norms), standard fuzzy intersection as the only idempotent t-norm, standard intersection, algebraic product, bounded difference and drastic intersection as examples of t-norms, decreasing generator, the Pseudo-inverse of a decreasing generator, increasing generators and their Pseudo-inverses, convertion of decreasing generators and increasing generators to each other, characterization theorem of t-norms(statement only). Fuzzy unions(t-conorms), standard union, algebraic sum, bounded sum and drastic union as examples of t-conorms, characterization theorem of t-conorms (Statement only) (Scope as in relevant parts of sections 3.1 to 3.4 of Chapter 3 of the book mentioned at the end).

SECTION-III (Two Questions)

Fuzzy numbers, relation between fuzzy number and a convex fuzzy set, characterization of fuzzy numbers in terms of its membership functions as piecewise defined functions, fuzzy cardinality of a fuzzy set using fuzzy numbers, arithmetic operators on fuzzy numbers, extension of standard arithmetic operations on real numbers to fuzzy numbers.

lattice of fuzzy numbers, (R, MIN, MAX) as a distributive lattice, fuzzy equations, equation A+X = B, equation A.X = B (Scope as in relevant parts of sections Chapter 4 of book mentioned at the end).

SECTION-IV (Two Questions)

Fuzzy Relations: Crisp and fuzzy relations, projections and cylindrical extensions, binary fuzzy relations, domain, range and height of a fuzzy relation, membership matrices, sagittal diagram, inverse of a fuzzy relation, composition of fuzzy relations, standard composition, max-min composition, relational join, binary relations on a single set, directed graphs, reflexive irreflexive, antireflexive, symmetric, asymmetric, antisymmetric, transitive (max-min transtitive), non transitive, antitransitive fuzzy relations.

Fuzzy equivalence relations, fuzzy compatibility relations, α-compatibility class, maximal α-compatibles, complete α-cover, reflexive undirected graphs, fuzzy ordering relations, fuzzy upper bound, fuzzy pre ordering, fuzzy weak ordering, fuzzy strict ordering, fuzzy morphisms.

(Scope of this section is as in the relevant parts of sections 5.1 to 5.8 of Chapter 5 of the book mentioned at the end).

Recommended Text:

G.J.Klir and B.Yuan: Fuzzy Sets and Fuzzy Logic; Theory and Applications, Sixth Indian Reprint, Prentice Hall of India, New Delhi, 2002.

Semester-III

Paper MM- 506 : Practical-III

Examination Hours : 4 hours

Max. Marks : 100

Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-501 to MM-505 will be taught.

Part-B : Implementation of the following programs in FORTRAN-90/95

1. Use a function program for simple interest to display year-wise compound interest and amount, for given deposit, rate and time.
2. Use logical operators in computing the compound interest on a given amount for rate of interest varying with amount as well as time of deposit.
3. Write a subroutine program to check (logical output) whether the three given points in a plane are collinear.
4. Use subroutine program to multiply two given matrices and use resource files in main program to read input and write output.
5. Use ALLOCATABLE size declaration for given set of points in a plane and fit a straight line through these points.
6. Write a program to display the use of whole-array operations on non-conformable arrays.
7. Write a program to display the procedure of format-rescan-rule and the action of tab-edit descriptors.
8. Use string operations to find if a given string is a palindrome or not.
9. Compute a given definite integral (as summation) in a subroutine using integrand as a dummy argument.
10. Explain the use of MODULE in defining an abstract (derived) data type for complex arithmetic.
11. Use of pointers in manipulating a linked-list.
12. To solve a quadratic equation with given (complex-valued) coefficients, using COMPLEX data type

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

SEMESTER-IV

MM-507 General Measure and Integration Theory

Examination Hours : 3 Hours

Max. Marks : 100

(External Theory Exam. Marks:80 + Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Measures, some properties of measures, outer measures, extension of measures, uniqueness of extension, completion of a measure, the LUB of an increasingly directed family of measures.(Scope as in the Sections 3-6, 9-10 of Chapter 1 of the book 'Measure and Integration' by S.K.Berberian).

Measurable functions, combinations of measurable functions, limits of measurable functions, localization of measurability, simple functions (Scope as in Chapter 2 of the book 'Measure and Integration' by S.K.Berberian).

SECTION-II (Two Questions)

Measure spaces, almost everywhere convergence, fundamental almost everywhere, convergence in measure, fundamental in measure, almost uniform convergence, Egoroff's theorem, Riesz-Weyl theorem (Scope as in Chapter 3 of the book 'Measure and Integration' by S.K.Berberian).

Integration with respect to a measure: Integrable simple functions, non-negative integrable functions, integrable functions, indefinite integrals, the monotone convergence theorem, mean convergence (Scope as in Chapter 4 of the book 'Measure and Integration' by S.K.Berberian)

SECTION-III (Two Questions)

Product Measures: Rectangles, Cartesian product of two measurable spaces, measurable rectangle, sections, the product of two finite measure spaces, the product of any two measure spaces, product of two σ - finite measure spaces; iterated integrals, Fubini's theorem, a partial converse to the Fubini's theorem (Scope as in Chapter 6 (except section 42) of the book 'Measure and Integration' by S.K.Berberian)

Signed Measures: Absolute continuity, finite singed measure, contractions of a finite signed measure, purely positive and purely negative sets, comparison of finite measures, Lebesgue decomposition theorem, a preliminary Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, upper variation, lower variation, total variation, domination of finite signed measures, the Radyon-Nikodym theorem for a finite measure space, the Radon-Nikodym theorem for a σ - finite measure space (Scope as in Chapter 7 (except Section 53) of the book 'Measure and Integration' by S.K.Berberian).

SECTION-IV (Two Questions)

Integration over locally compact spaces: continuous functions with compact support, G_δ 's and F_σ's, Baire sets, Baire function, Baire-sandwich theorem, Baire measure, Borel sets, Regularity of Baire measures, Regular Borel measures, Integration of continuous functions with compact support, Riesz-Markoff's theorem (Scope as in relevant parts of the sections 54-57,60,62,66 and 69 of Chapter 8 of the book 'Measure and Integration' by S.K.Berberian)

Recommended Text:

S.K.Berberian: Measure and Integration, Chelsea Publishing Company, New York, 1965.

References:

1. H.L.Royden: Real Analysis, Prentice Hall of India, 3rd Edition, 1988.
2. G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd.,1981.
3. P.R.Halmos: Measure Theory, Van Nostrand, Princeton, 1950.
4. I.K.Rana: An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.
5. R.G.Bartle: The Elements of Integration, John Wiley and Sons, Inc. New York, 1966.

SEMESTER- IV

MM-508 Partial Differential Equations

Examination
Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

PDE of k^{th} order: Definition, examples and classifications. Initial value problems. Transport equations homogeneous and non-homogeneous, Radial solution of Laplace's Equation: Fundamental solutions, harmonic functions and their properties, Mean value Formulas, Poisson,s equation and its solution, strong maximum principle, uniqueness, local estimates for harmonic functions, Liouville,s theorem, Harnack's inequality.

SECTION-II (Two Questions)

Green's function and its derivation, representation formula using Green's function, symmetry of Green's function, Green's function for a half space and for a ball. Energy methods: uniqueness, Drichlet's principle. Heat Equations: Physical interpretation, fundamental solution. Integral of fundamental solution , solution of initial value problem, Duhamel's principle, non-homogeneous heat equation, Mean value formula for heat equation, strong maximum principle and uniqueness. Energy methods.

SECTION-III (Two Questions)

Wave equation- Physical interpretation, solution for one dimentional wave equation, d'Alemberts formula and its applications, reflection method, Solution by spherical means Euler-Poisson_Darboux equation, Kirchhoff's and Poisson's formulas (for n=2, 3 only), Solution of non –homogeneous wave equation for n=1,3. Energy method. Uniqueness of solution, finite propagation speed of wave equation. Non-linear first order PDE- complete integrals, envelopes, Characteristics of (i) linear, (ii) quasilinera, (iii) fully non-linear first order partial differential equations. Hamilton Jacobi equations (calculus of variations Hamilton's ODE, Legendre Transform, Hopf-Lax formula, weak solutions, Uniqueness).

SECTION-IV (Two Questions)

Conservative Laws (Shocks, entropy condition, Lax-Oleinik formula., weak solutions uniqueness. Riemann's problem, long time behaviuor).

Representation of Solutions- Separation of variables, Similarity solutions (Plane and traveling waves, solitones, similarity under Scaling). Fourier Transform, Laplace Transform, Converting non linear into linear PDE, Cole-Hop Transform, Potential functions, Hodograph and Legendre transforms.

Books:

1L.C. Evans, Partial Differential Equations, Graduate Studies in

2 Books with the above title by I.N. Snedden, F. John, P. Prasad and R. Ravindran, Amarnath etc.

SEMESTER-IV

MM-509 (opt. i) Mechanics of Solids

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Two dimensional problems : Plane stress. Generalized plane stress. Airy stress function. General solution of biharmonic equation, Stresses and displacements in terms of complex potentials. The structure of functions of $\varphi(z)$ and $\psi(z)$. First and second boundary-value problems in plane elasticity. Existence and uniqueness of the solutions.

(Section 65-74 of the book by I.S. Sokolnikoff).

SECTION -II (Two Questions)

Waves : Propagation of waves in an isotropic elastic solid medium. Waves of dilatation and distortion. Plane waves. Elastic surface waves : Rayleigh waves and Love waves.

Extension : Extension of beams, bending of beams by own weight and terminal couples,; bending of rectangular beams

(Section 204 of A.E.H. Love, Sections 7,7-8, 10 of Y.C. Fung; Chapter 4, Sections 30 to 32 and 57 of book by I.S. Sokolnikoff).

SECTION -III (Two Questions)

Torsion : Torsion of cylindrical bars; Torsional rigidity. Torsion and stress functions. Lines of shearing stress. Torsion of anisotropic beams; Simple problems related to circle, ellipse and equilateral triangle.

(Chapter 4: Sections 33 to 38 and 51 of the book; I.S. Sokolnikoff, Section 221 of A.E.H. Love).

SECTION -IV(Two Questions)

Variational methods : Theorems of minimum potential energy. Theorems of minimum complementary energy. Reciprocal theorem of Betti and Rayleigh. Deflection of elastic string cental line of a beam and elastic membrane. Solution of Euler's equation by Ritz, Galerkin and Kantorovich methods.

(Chapter 7: Sections 107-110, 112, 113, 115 & 117 of I.S. Sokolnikoff).

Books:

1. I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw Hill Publishing Company Ltd., New Delhi, 1977.
2. A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
3. Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
4. D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.
5. S. Timeshenki and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
6. I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delh, 1975.

SEMESTER-IV

MM-509 (opt. ii) Difference Equations-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

The self-adjoint second order linear equations: Introduction, Lagrange identity, Green's Theorem, Liouville's formula, Polya Factorization Theorem and application, Cauchy function, variation of constants formula.

Sturmian Theory : Sturm separation theorem and examples. The Riccati Equation.

SECTION-II (Two Questions)

Sturm comparison Theorem. Oscillation.

The Sturm-Liouville problem : Introduction, eigen functions and eigen values of Sturm-Liouville problem, Finite Fourier analysis, Non-homogeneous problem. Rayleigh's inequality.

SECTION-III (Two Questions)

Green's functions and Boundary Value Problems, Disconjugacy. B.V.P. for non-linear equation : Inrtroduction, contraction mapping theorem. Lipschitz condition & examples. Existence of solutions, some basic theorem and examples. B.V.P. for Differential Equations.

SECTION-IV (Two Questions)

Discrete calculus of variation, Introduction, Necessary condition for the simplest variational problem of local extremum, Euler- Lagrange equation, Sufficient condition and Disconjugacy, Sturm comparison Theorem, Weiesstrass Summation formula.

Partial Differential Equations, Discretization of Partial Differential Equations, Solution of Partial Differential Equation.

Recommend Text:

W.G. Kelley and A.C. Peterson: Difference Equations; An introduction with Applications, Academic Press, Harcourt, 1991. (Relevant portions of chapters 6-10.)

Reference Book:

Calvin Ahlbrandt & Allan C. Peterson, Discreet Hamiltonian systems, Difference Equations, Continued Fractions & Ricati Equation, Kluwer Botson, 1996.

SEMESTER-IV

MM-509 (opt. iii) Algebraic Number Theory

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Algebraic numbers and algebraic integers. Transcendental Numbers. Liouville's Theorem for real Algebraic numbers. Thue Theorem and Roth's theorem (statement only). Algebraic numberfield K. Theorem of Primitive elements. Liouville's Theorem for complex algebraic numbers. Minimal polynomial of an algebraic integer. Primitive m-th roots of unity. Cyclotomic Polynomials. Norm and trace of algebraic numbers and algebraic integers. Bilinear form on algebraic number field K.

SECTION-II (Two Questions)

Integral basis and discriminant of an algebraic number field. Index of an element of K. Ring O_k of algebraic integers of an algebraic number field K. Ideals in the ring of algebraic number field K. Integrally closed domains. Dedekind domains. Fractional ideals of K. Factorization of ideals as a product of prime ideals in the ring of algebraic integers of an algebraic number field K. G.C.D. and L.C.M. of ideals in O_k. Chinese Remainder theorem.

SECTION-III (Two Questions)

Different of an algebraic number field K. Dedekind theorem. Euclidean rings. Hurwitz Lemma and Hurwitz constant. Equivalent fractional ideals. Ideal class group. Finiteness of the ideal class group. Class number of the algebraic number field K. Diophantine equations Minkowski's bound.

SECTION-IV (Two Questions)

Quadratic reciprocity Legendre Symbol. Gauss sums. Law of quadratic reciprocity. Quadratic fields. Primes in special progression.

Recommended Text:

Jody Esmonde and M.Ram Murty	Problems in Algebraic Number Theory (Springer Verlag, 1998)

Reference Books:

1. Paulo Ribenboim	Algebraic Numbers
2. R. Narasimhan and S. Raghavan	Algebraic Number Theory Mathematical Pamphlets-4. Tata Institute of Fundamental Research(1966).

Semester-IV

MM-509 Option (iv): Mathematics for Finance & Insurance

Examination Hours : 3 Hours
Max. Marks : 100

(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Questions)

Normal Random Variables : Continuous Random Variables, Normal Random Variables & their properties, Central Limit Theorem.

Geometric Brownian Motion : Basic concepts & simple Models, Brownian Motion. Interest Rates, Present Value Analysis, Rate of Return, continuously varying Interest Rates.

Section – II (Two Questions)

Financial Derivatives – An Introduction, Types of Financial Derivatives, Forwards and Futures, Options and its kinds and SWAPS

The Arbitrage Theorem and Introduction to Portfolio Selection and Capital Market Theory: Static and Continuous-Time Model.

Section – III (Two Questions)

Pricing by Arbitrage-A Single-Period option Pricing Model; Multi-Period Pricing Model, Cox-Ross-Rubinstein Model; Bounds on Option Prices.

The Ito's Lemma and the Ito's Integral. Concepts from Insurance: Introduction; The Claim Number Process; The Claim Size Process; Solvability of the Portfolio; Reinsurance and Ruin Problem.

Section – IV (Two Questions)

Premium and Ordering of Risks-Premium Calculation Principles and Ordering Distributions.

Distribution of Aggregate Claim Amount-Individual and Collective Model; Compound Distributions; Claim Number of Distributions; Recursive Computation Methods; Lundberg Bounds and Approximation by Compound Distributions.

References:

1. John C.Hull, Options, Futures, and Other Derivatives, Prentice-Hall of India Private Limited.
2. Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge University Press.
3. Salih N. Neftci, An Introduction to the Mathematics of Financial Derivatives, Academic Press, Inc.
4. Robert J. Elliott and P. Ekkehard Kopp. Mathematics of Financial Markets, Springer-Verlag, New York Inc.
5. Robert C. Marton, Continuous-Time Finance, Basil Blackwell Inc.
6. Daykin C.D., Pentikainen T. and Pesonen M., Practical Risk Theory for Actuaries, Chapman & Hall.

SEMESTER- IV

MM-510 (opt. i) Fluid Mechanics –II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Fundamental Equations: Derivation of the equations of continuity and equation of motion in cylindrical and spherical coordinates.

Two–dimentional inviscid incompressible flows, Stream function : Irrotatonal motion in two dimensions. Complex velocity potential. Sources, sinks, doublets and their images. Thomson circle theorem. Two- dimensional irrotational motion produced by motion of circular cylinder.

SECTION-II (Two Questions)

Two dimensional motion : Motion due to elliptic cylinder in an infinite mass of liquid, Kinetic energy of liquid contained in rotating elliptic cylinder, circulation about elliptic cylinder. Theorem of Blasius. Theorem of Kutta and Joukowski. Kinetic energy of a cyclic and acyclic irrotational motion. Axisymmetric flows, Stoke's stream function ,Stoke's stream functions of some basic flows.

SECTION-III (Two Questions)

Three –dimential motion : Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion a sphere. Alembert's paradox, impulsive motion, initial motion of liquid contained in the intervening space between two concentric spheres. Vortex motion and its elementary properties. Kelvin's proof of permanence. Motions due to circular and rectilinear vortices. Infinite rowes of line vortices.

SECTION-IV (Two Questions)

Dynamical similarity . Buckingham pi- theorem , Reynolds number. Prandtl's boundary layer, boundary layer equations in two dimensions. Blasius solution Boundary layer thickness. Displacement thickness, Karman integral conditions, separation of boundary layer.

Books :

1. W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers, Delhi, 1988.
2. F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
3. Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, 1986.
4. G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New Delhi, 1994.
5. A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics Springer-Verlag, New York, 1993.
6. L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
7. H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1979.
8. R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi. 1976.
9. A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.
10. S. w. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New Delhi, 1976.event

SEMESTER-IV

MM-510 (opt.ii) Boundary Value Problems

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Applications to Ordinary Differential Equations; Initial value problems, Boundary Value Problems. Dirac Delta functions. Green's function approach to reduce boundary value problems of a self-adjoint-differential equation with homogeneous boundary conditions to integral equation forms. Green's function for N^{th}-order ordinary differential equation. Modified Green's function.

(Relevant portions from the Chapter 5 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-II (Two Questions)

Applications to partial differential equations: Integral representation formulas for the solution of the Laplace and Poisson Equations. The Newtonian, single-layer and double-layer potentials, Interior and Exterior Dirichlet problems, Interior and Exterior Neumann problems. Green's function for Laplace's equation in a free space as well as in a space bounded by a ground vessel. Integral equation formulation of boundary value prolems for Laplace's equation. Poisson's Integral formula. Green's function for the space bounded by grounded two parallel plates or an infinite circular cylinder. The Helmholtz equation.

(Relevant portions from the Chapter 6 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

SECTION-III (Two Questions)

Integral Transform methods: Introduction, Fourier transform. Laplace transform. Convolution Integral. Application to Volterra Integral Equations with convolution-type Kernels. Hilbert transform.

Applications to mixed Boundary Value Problems: Two-part Boundary Value problems, Three-part-Boundary Value Problems, Generalized Three-part Boundary Value problems.

(Relevant portions from the Chapter 9 and 10 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

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SECTION-IV (Two Questions)

Integral equation perturbation methods: Basic procedure, Applications to Electrostatics, Low-Reynolds-Number Hydrodynamics: Steady stokes Flow, Boundary effects on Stokes flow, Longitudnal oscillations of solids in stokes Flow, Steady Rotary Stokes

Flow, Rotary Oscillations in Stokes Flow, Rotary Oscillation in Stokes Flow, Oseen Flow-Translation Motion, Oseen Flow-Rotary motion Elasticity, Boundary effects, Rotation, Torsion and Rotary Oscillation problems in elasticity, crack problems in elasticity, Theory of Diffraction.
(Relevant portions from the Chapter 11 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

References:

1. R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.
2. S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book Agency, 1960.
3. I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North Holland, 1966.
4. I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II, Mac.Millan, 1969.
5. Pundir and Pundir, Integral equations and Boundary value problems, Pragati Prakashan, Meerut.

SEMESTER-IV

MM-510 (opt. iii) Non-Commutative Rings

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Basic terminology and examples of non-commutative rings i.e. Hurwitz's ring of integral quaternions, Free k-rings. Rings with generators and relations. Hilbert's Twist, Differential polynomial rings, Group rings, Skew group rings, Triangular rings, D.C.C. and A.C.C. in triangular rings. Dedekind finite rings. Simple and semi-simple modules and rings. Spliting homomorphisms. Projective and Injective modules. (Section 1.1 to 1.26 and Section 2.1 to 2.9 of the book given at Sr. No. 1).

SECTION-II (Two Questions)

Ideals of matrix ring $M_n(R)$. Structure of semi simple rings. Wedderburn-Artin Theorem Schur's Lemma. Minimal ideals. Indecomposable ideals. Inner derivation δ. δ-simple rings. Amitsur Theorem on non-inner derivations. Jacobson radical of a ring R.Annihilator ideal of an R-module M. Jacobson semi-simple rings. Nil and Nilpotent ideals. Hopkins-Levitzki Theorem. Jacobson radical of the matrix ring $M_n(R)$. Amitsur Theorem on radicals. Nakayama's Lemma. Von Neumann regular rings. E. Snapper's Theorem. Amitsur Theorem on radicals of polynomial rings. (Section 3.1 to 3.19, Sections 4.1 to 4.27, Section 5.1 to 5.10 of the book given at Sr. No. 1).

SECTION-III (Two Questions)

Prime and semi-prime ideals. m-systems. Prime and semi-prime rings. Lower and upper nil radical of a ring R Amitsur theorem on nil radical of polynomial rings. Brauer's Lemma. Levitzki theorem on nil radicals. Primitive and semi-primitive rings. Left and right primitive ideals of a ring R. Density Theorem. Structure theorem for left primitive rings. (Section 10.1 to 10.30, Section 11.1 to 11.20 of the book given at Sr. No. 1).

SECTION-IV (Two Questions)

Sub-direct products of rings. Subdirectly reducible and irreducible rings. Birchoff's Theorem. Reduced rings. G.Shin's Theorem. Commutativity Theorems of Jacobson, Jacobson-Herstein and Herstein Kaplansky. Division rings. Wedderburn's Little Theorem. Herstein's Lemma. Jacobson and Frobenius Theorem. Cartan-Brauer-Hua Theorem. Herstein's Theorem. (Sections 12.1 to 12.11 and Sections 13.1 to 13.26 of the book given at Sr. No. 1).

Recommended Book:

1. T.Y.Lam A First Course in Noncommutive Rings, (Springer Verlag 1990)
2. I.N.Herstein Non-Commutative Rings carus monographs in Mathematics Vol.15. Math Asso. of America 1968.

SEMESTER-IV

MM-510 (opt. iv) Advanced Discrete Mathematics

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

Graphs, Konisberg seven bridges problem. Finite and infinite graphs. Incidence vertex. Degree of a vertex. Isolated and pendant vertices. Null graphs. Isomosphism of graphs. Subgraphs, walks, paths and circuits. Connected and disconnected graphs. Components of a graph. Euler graphs. Hamiltonian paths and circuits. The traveling salesman problem. Trees and their properties. Pendant vertices in a tree. Rooted and binary tree. Spanning tree and fundamental circuits. Spanning tree in a weighted graph. (Chapter 1,2,3 of the book given at Sr. No. 1).

SECTION-II (Two Questions)

Cutsets and their properties. Fundamental circuits and cutsets. Connectivity and separability. Network flows. Planner graphs. Kuratowski's two graphs. Representation of planner graphs. Euler formula for planner graphs. Vector space associated with a graph. Basis vectors of a graph. Circuit and cutset subspaces. Intersection and joins of W_C and W_S. Incidence matrix A(G) of a graph G, Submatrices of A(G), Circuit matrix, Fundamental circuit matrix, and its rank, Cutset matrix, path matrix and adjacency matrix of a graph. (Chapter 4, Theorems 5.1 to 5.6 of chapter 5, chapter 6 & 7 of the book given at Sr. No. 1).

SECTION-III (Two Questions)

Partially ordered sets and lattices. Lattice as an algebraic system. Sublattices. Isomorphism of lattices. Distributive and modular lattices. Lattices as intervals. Similar and projective intervals. Chains in lattices. Zassenhaus's Lemma and Schreier Theorem, Composition chain and Jordan Holder Theorem. Chain conditions. Fundamental dimensionality relation for modular lattices. Decomposition theory for lattices with ascending chain conditions, i.e. reducible and irreducible elements. Independent elements in lattices. (Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

SECTION-IV (Two Questions)

Points (atoms) of a lattice. Complemented lattices. Chain conditions and complemented lattices. Boolean algebras. Conversion of a Boolean algebra into a Boolean ring with unity and vice-versa. Direct product of Boolean algebras. Uniqueness of finite Boolean algebras. Boolean functions and Boolean expressions. Application of Boolean algebra to switching circuit theory. (Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

Recommended Texts:

1. Narsingh Deo — Graph Theory with application to Engineering and Computer Science, Prentice Hall of India.
2. Nathan Jacobson — Lectures in Abstract Algebra Vol.I, D.Van Nostrand Company, Inc.
3. L.R. Vermani and Shalini — A course in discrete Mathematical structures(Imperial College Press London 2011)

SEMESTER-IV

MM-511 (opt. i) Mathematical Aspects of Seismology

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80 + Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two Questions)

General form of progressive waves, Harmonic waves, Plane waves, the wave equation. Principle of superposition. Special types of solutions: Progressive and Stationary type solutions of wave equation. Equation of telegraphy. Exponential form of harmonic waves. D' Alembert's formula. Inhomogeneous wave equation. Dispersion: Group velocity, relation between phase velocity and group velocity.

(Relevant articles from the book "*Waves*" by Coulson & Jefferey)

SECTION-II (Two Questions)

Reduction of equation of motion to wave equations. P and S waves and their characteristics. Polarisation of plane P and S waves. Snell's law of reflection and refraction. Reflection of plane P and SV waves at a free surface. Partition of reflected energy. Reflection at critical angles.

Reflection and reflection of plane P,SV and SH waves at an interface. Special cases of Liquid-Liquid interface, Liquid-Solid interface and Solid-Solid interface. Rayleigh waves, Love waves and Stoneley waves. (Relevant articles from the book, "*Elastic waves in Layered Media*" by Ewing et al).

SECTION-III (Two Questions)

Two dimensional Lamb's problems in an isotropic elastic solid: Area sources and Line Sources in an unlimited elastic solid. A normal force acts on the surface of a semi-infinite elastic solid, tangential forces acting on the surface of a semi-infinite elastic solid.

Three dimensional Lamb's problems in an isotropic elastic solid: Area sources and Point sources in an unlimited elastic solid, Area source and Point source on the surface of semi-infinite elastic solid.

Haskell matrix method for Love waves in multilayered medium.

(Relevant articles from the book "*Mathematical Aspects of Seismology*" by Markus Bath).

SECTION-IV (Two Questions)

Spherical waves. Expansion of a spherical wave into plane waves: Sommerfield's integral. Kirchoff's solution of the wave equation, Poissons's formula, Helmholtz's formula.
(Relevant articles from the book "*Mathematical Aspects of Seismology*" by Markus Bath).
Introduction to Seismology: Location of earthquakes, Aftershocks and Foreshocks, Earthquake magnitude, Seismic moment, Energy released by earthquakes, observation of earthquakes, interior of the earth.
(Relevant articles from the book "*The Solid Earth*" by C.M.R.Fowler)

References:

1. P.M.Shearer, *Introduction to Seismology*, Cambridge University Press,(UK) 1999.
2. C.M.R.Fowler, *The Solid Earth*, Cambridge University Press, 1990.
3. C.A.Coulson and A.Jefferey, *Waves*, Longman, New York, 1977.
4. M.Bath, *Mathematical Aspects of Seismology*, Elsevier Publishing Company, 1968.
5. W.M.Ewing, W.S.Jardetzky and F.Press, *Elastic Waves in Layered Media*, McGraw Hill Book Company, 1957.

SEMESTER-IV

MM-511 (opt. ii) Dynamical Systems

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to two questions from each section and one compulsory question consisting of eight parts and distributed over the whole syllabus. An examinee is required to attempt one question from each section and the compulsory question.

Section-I

Orbit of a map; fixed point; Periodic point; Circular map, Configuration space & phase space.

Section-II

Origin of bifurcation; Stability of a fixed point, equilibrium point; Concept of limit cycle & torus; Hyperbolicity; Quadratic map; Feigenbaum's universal constant.

Section-III

Turning point, transcritical, pitch work; Hopf bifurcation; Period doubling phenomenon. Non-linear oscillators

Section-IV

Conservative system; Hamiltonian system; Various types of oscillators; Solutions of non-linear differential equations.

Books :

1. D.K. Arrowosmith, Introduction to Dynamical Systems, CUP, 1990.
2. R.L Davaney, An Introduction to Chaotic Dynamical Systems, Addison-Wesley, 1989.
3. P.G. Drazin, Nonlinear System, CUP, 1993.
4. V.I Arnold, Nonlinear Systems III-Mathematical Aspects of Classical and Celestial Mechanics, Springer-Verlag, 1992.
5. V.I Arnold, Nonlinear Systems V-Bifurcation Theory and Catastrophe Theory, Springer-Verlag, 1992.

Semester-IV

MM-511 (opt. iii) Operational Research

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory question will consist of eight parts and will be distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

Section – I (Two Question)

Dynamic Programming – Nature of Dynamic Programming (DP), Bellman's principle of optimality in DP, DP algorithm, mathematical formulation of multistage model, the recursive operation approach, Application of DP in Linear Programming.

Integer Programming : types of integer programming problem, cutting plane method (Gomory technique), construction of Gomory's constraints, Graphical interpretation of cutting plane method, cutting plane algorithm, Fractional cut methodthe branch and bound method.

Section – II (Two Question)

Game theory : Definition, characteristics of games, two person, zero sum game, pay of matrix strategy & its types, Saddle point, solution of rectangular game with saddle point, solution method of rectangular game in terms & strategy, strategy of mixed optimal strategy, concept of Dominance, Graphical method of solving (2xn) and (mx2) games, Algebraic method for the solution of general game, equivalence of the rectangular matrix games and linear programming, fundamental theory of game theory, limitation of game theory, solution of rectangular game by singular method, matrix method for (nxn) games.

Section –III (Two questions)

Nonlinear Programming-Definition and examples of non-linear programming, Mobi-Tucker theory: Kuhn-Tucker (K-T) optimality conditions, K-T first order necessary optimality conditions, K-T, second order optimality conditions, Lagrange's method, Economic interpretation of multipliers-Wolf duality theorem on non-linear programming, Quadratic programming, K-T conditions for Quadratic programming problems, Wolf modified simplex method, Beale's method, separable, convex and non-convex programming.

Section –IV (Two questions)

Inventory model : classification of inventory models, Determinsitc inventory model (DIM), Basic Economic-order quantity (EOQ) models, EOQ model with uniform rate of demand infinite production rate and having no shortage EOQ model with uniform rate of demand in different production cycles, infinite production rate & having non shortage, EOQ with finite replenishment DIM with shortage, Fixed Time Model, EOQ with finite production, EOQ with price break, EOQ with one price break, single multi-item deterministic inventory model, Queuing models: classification of queuing models, solution of queue models, model I (M/M/1) : (∞/FCFS) model II (General Erlong queuing model, model III M/M/1): (N/FCFS). Network (PERT/CPM), schedule chart (Gantt Bar Chart), difference between CPM and PERT, Network components, construction of the Network diagram, CPM analysis.

Books :

1. G.Hadley	:	Linear Programming
2. C.W. Churchman et.al.	:	Introduction to Operations Research
3. B.S. Goel & S.K. Mittal	:	Operations research
4. D. Gross & C.M. Harris	:	Fundamentals of Queuing Theory
5. A.O. Allen	:	Probability Statistics & Queuing Theory with
Computer		
		Science Applications

SEMESTER-IV

MM-511 (opt. iv) Fuzzy Sets and Applications-II

Examination Hours : 3 Hours
Max. Marks : 100
(External Theory Exam. Marks:80
+ Internal Assessment Marks:20)

NOTE : The examiner is requested to set nine questions in all taking two questions from each section and one compulsory question. The compulsory questions will consist of eight parts and distributed over the whole syllabus. The candidate is required to attempt five questions selecting at least one from each section and the compulsory question.

SECTION-I (Two questions)

Possibility Theory : Fuzzy measures, continuity from below and above, semicontinuous fuzzy measures, examples and simple properties; Evidence Theory, belief measure, superadditivity, monotonicity, plausibility measure, subadditivity, basic assignment, its relation with belief measure and plausibility measure, focal element of basic assignment, body of evidence, total ignorance, Dempster`s rule of combination, examples; Possibility Theory, necessity measure, possibility measure, implications, possibility distribution function, lattice of possibility distributions, joint possibility distribution.

Fuzzy sets and possibility theory, degree of compatibility, degree of possibility, relation with possibility distribution function and possibility measure, example of possibility distribution for fuzzy proposition. Possibility theory versus probability theory, characterization of relationship between belief measures and probability measures, probability distribution function, joint probability distribution function, marginal probability distributions, noninteractive, independent marginal distributions (Scope as in the relevant parts of Chapter 7 of the book mentioned at the end.)

SECTION-II (Two questions)

Fuzzy Logic: An overview of classical logic, about logic functions of two variables, Multivalued logics, about three-valued logic, n-valued logic, degrees of truth, definition of primitives, Fuzzy propositions, classification, canonical forms, relation with possibility distribution function, Fuzzy Quantifiers, their two kinds, relation with possibility distribution function, Linguistic hedges, as a unary operation and modifiers, properties, Inference from conditional fuzzy propositions, relations with characteristic and membership functions, Compositional rule of inference, modus ponens and tollens, hypothetical syllogism, inference from conditional and qualified propositions, equivalence of the method of truth-value restrictions to the generalized modus ponens. (Scope as in the relevant parts of sections 8.1 to 8.7 of Chapter 8 of the book mentioned at the end.)

SECTION-III (Two questions)

Approximate reasoning: An overview of fuzzy expert system, Fuzzy implications as functions and operators, S-implications, R-implications, Gödel implication, QL-implications, Zadeh implication, examples, properties, combinations, axioms of fuzzy implications and characterization (only statement).

Selection of fuzzy implications, selection of approximate fuzzy implications to reasoning with unqualified fuzzy propositions, relation with compositional rule of inference, modus ponens and tollens,hypothetical syllogism Multiconditional approximate reasoning, method of interpolation, an illustration of the method for two if-then rules, as special case of compositional rule of inference and related results of fuzzy sets involved, The role of fuzzy relation equations, necessary and sufficient condition for a solution of the system of fuzzy relation equations for a fuzzy relation, its implications.(Scope as in the relevant parts of sections 11.1 to 11.5 of Chapter 11 of the book mentioned at the end .)

SECTION-IV (Two questions)

An introduction to fuzzy control: Fuzzy controllers, its modules, Fuzzy rule base, Fuzzy inference enzine, fuzzification and defuzzifications, steps of design of fuzzy controllers, defuzzification method, center of area method, center of maxima method and mean of maxima method. (Scope as in the relevant part of section 12.2 of chapter 12 of the book mentioned at the end .)

Decision –making in Fuzzy environment: Individual decision-making, fuzzy decision, simple examples, idea of weighting coefficients, Multiperson decision-making, fuzzy group decision, examples, Multicriteria decision-making, matrix representation of fuzzy relation, convertion to single-criterion decision, examples, Multistage decision-making, idea of principle of optimality, Fuzzy ranking methods, Hamming distance, priority set, examples, Fuzzy linear programming, two different methods one with only one side involving fuzzy numbers and other where only the coefficients of constraint matrix are fuzzy numbers . (Scope as in the relevant parts of Chapter 15 of the book mentioned at the end.)

Book :

G. J. Klir and B. Yuan : Fuzzy Sets and Fuzzy Logic Theory and Appplications.

Semester-IV

Paper MM-512 : Practical-IV

Time : 4 hours
Max. Marks : 100

Part-A : Problem Solving

In this part, problem solving techniques based on papers MM-507 to MM-511 will be taught.

Part-B : Problem solving through MATLAB

Computer programs based on following Numerical Methods:

1. Solutions of simultaneous linear equations.
2. Solution of algebraic / transcendental equations.
3. Inversion of matrices
4. Numerical differentiation and integration
5. Solution of ordinary differential equations
6. Statistical problems on central tendency and dispersion
7. Fitting of curves by least square method.

Note :- Every student will have to maintain practical record on a file of problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. The question paper will consists of questions on problem solving techniques/algorithm and computer programs. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) one or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.