

AIEEE 2010

Physics Solution

PART A – PHYSICS

Directions: Questions number 1 – 3 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

- The initial shape of the wavefront of the beam is
 - Planar
 - Convex
 - Concave
 - Convex near the axis and concave near the periphery
- (1)** Wavefront is planar for parallel beam.
- The speed of light in the medium is
 - Maximum on the axis of the beam
 - Minimum on the axis of the beam
 - The same everywhere in the beam
 - Directly proportional to the intensity I
- (2)** μ_{axis} is the maximum, $\therefore v_{axis}$ is minimum.
- As the beam enters the medium, it will
 - Travel as a cylindrical beam
 - Diverge
 - Converge
 - Diverge near the axis and converge near the periphery
- (1)** All rays are entering normally (to interface), therefore in medium also they will move normally.

Directions: Questions number 4 – 5 are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is c .

- The speed of daughter nuclei is:
 - $c\sqrt{\frac{\Delta m}{M + \Delta m}}$
 - $c\frac{\Delta m}{M + \Delta m}$
 - $c\sqrt{\frac{2\Delta m}{M}}$
 - $c\sqrt{\frac{\Delta m}{M}}$
- (3)** $\Delta m \cdot c^2 = 2 \left\{ (1/2) \cdot (M/2) \cdot v^2 \right\} \Rightarrow v = c\sqrt{\frac{2\Delta m}{M}}$
- The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2 . Then
 - $E_1 = 2E_2$
 - $E_2 = 2E_1$
 - $E_1 > E_2$
 - $E_2 > E_1$
- (4)** $M + \Delta m \rightarrow \{(M/2) + (M/2) + E\}$, where, $E = \Delta m \cdot c^2$
 \therefore Applying conservation of Energy on both sides, $-E_1 = -E_2 + E \Rightarrow E_2 > E_1$

Directions: Questions number 6 – 7 contain Statement-1 and Statement - 2. Of the four choices given after the statements, choose the one that best describes the two statements.

6. **Statement-1:** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max} . When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2: Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

- (1) Statement-1 is true, Statement-2 is false.
- (2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
- (3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** the correct explanation of Statement-1.
- (4) Statement-1 is false, Statement-2 is true.

6. (1) $hc / \lambda_{incident} = W + KE_{max}$, where $KE_{max} = eV_s$,
 \therefore On decreasing $\lambda_{incident}$, KE_{max} & therefore V_s increases.
 Speeds of different photoelectrons are different due to electrons emitted from different orbits.

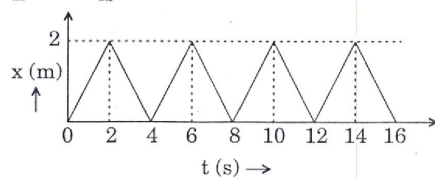
7. **Statement-1:** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2: Principle of conservation of momentum holds true for all kinds of collisions.

- (1) Statement-1 is true, Statement-2 is false.
- (2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1.
- (3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** the correct explanation of Statement-1.
- (4) Statement-1 is false, Statement-2 is true.

7. (2) $V_f = \{(m_1v_1 + m_2v_2) / (m_1 + m_2)\}$
 since collision is inelastic \Rightarrow There is some final KE.

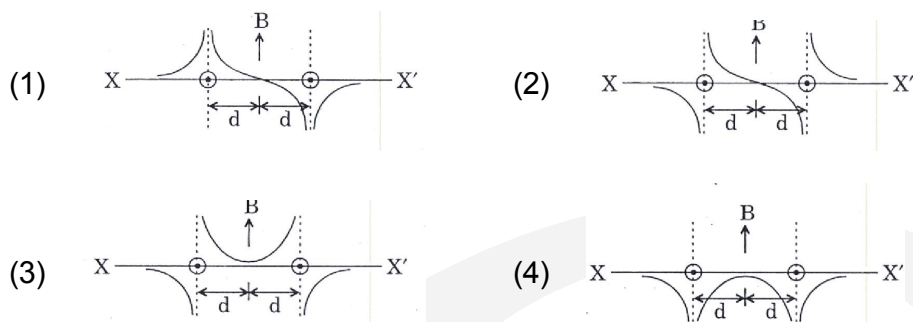
8. The figure shows the position – time ($x - t$) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is:



- (1) 0.2 Ns
- (2) 0.4 Ns
- (3) 0.8 Ns
- (4) 1.6 Ns

8. (3) Impulse = $m\Delta v = m \times (-1 - 1) = 0.8 \text{ Ns}$

9. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

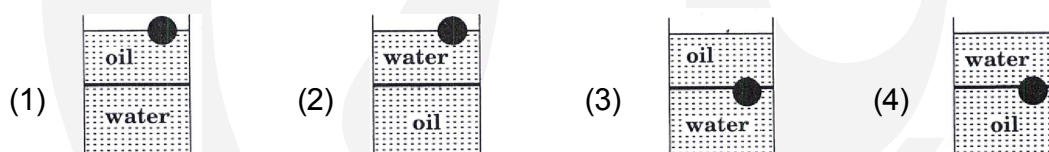


9. (2)

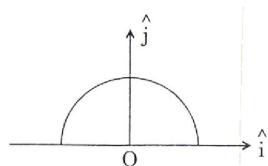
Magnetic field for infinite wire = $(\mu_0 I / 2\pi r)$

- i. At mid point between A & B field is zero.
- ii. In between A to O net field is upward.
- iii. In between O to B net field is downward.
- iv. Left of point A net field is downward
- v. Right of point B net field is upward.

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position?



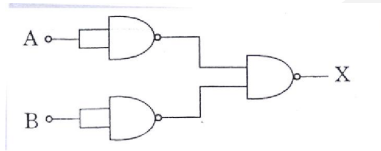
10. (3) i. $\rho_{oil} < \rho_{water} \Rightarrow$ oil will be above water
ii. $\rho_{oil} < \rho < \rho_{water} \Rightarrow$ Ball will be in equilibrium at oil-water interface.
11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field \vec{E} at the centre O is:



- (1) $\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$ (2) $\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$ (3) $-\frac{q}{4\pi^2 \epsilon_0 r^2} \hat{j}$ (4) $-\frac{q}{2\pi^2 \epsilon_0 r^2} \hat{j}$

11. (4) $\vec{E} = \int dE \sin\theta(-\hat{j}) = \int_0^\pi k(r d\theta) \cdot \frac{q}{\pi r} \cdot \frac{1}{r^2} \sin\theta(-\hat{j}) = \frac{2kq}{\pi^2} (-\hat{j}) = \frac{q}{2\pi^2 \epsilon_0 r^2} (-\hat{j})$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32V$, the efficiency of the engine is:
- (1) 0.25 (2) 0.5 (3) 0.75 (4) 0.99
12. (3) $TV^{\gamma-1} = \text{Const.} \Rightarrow T_1 V^{2/5} = T_2 (32V)^{2/5} \Rightarrow T_1 / T_2 = 4$
 $\therefore \eta = 1 - (T_2 / T_1) = 1 - (1/4) = 3/4 = 0.75$
13. The respective number of significant figure for the numbers 23.023, 0.0003 and 2.1×10^{-3} are
- (1) 4, 4, 2 (2) 5, 1, 2 (3) 5, 1, 5 (4) 5, 5, 2
13. (2) Number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are 5, 1, 2 respectively.
14. The combination of gates shown below yields



- (1) NAND gate (2) OR gate (3) NOT gate (4) XOR gate
14. (2) Each element in the circuit is NAND - gate

<u>A</u>	<u>B</u>	<u>X</u>
0	0	0
0	1	1
1	0	1
1	1	1

The truth table obtained is similar to OR-gate.

15. If a source of power 5 kW produces 10^{20} photons / second, the radiation belongs to a part of the spectrum called:
- (1) γ -ray (2) X-rays (3) ultraviolet rays (4) microwaves
15. (2) Energy of one photon = $4 \times 10^3 / 10^{20} = 4 \times 10^{-17}$ J
 $\therefore \lambda = hc / (4 \times 10^{-17}) = \{(6.67 \times 10^{-34} \times 3 \times 10^8) / (4 \times 10^{-17})\}$
 $\approx 5 \times 10^{-9}$ m = 50Å \Rightarrow X-rays
16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α -particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be
- (1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$ (3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$
16. (3) If only 3 α particles emitted, No. of protons = $Z - 6$
 No. of neutrons = $A - Z - 6$. Now, if again 2 positrons emitted,
 No. of protons = $Z - 6 - 2 = Z - 8$ & No. of neutrons = $A - Z - 6 + 2 = A - Z - 4$.

17. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0 \left(\frac{5}{4} - \frac{r}{R} \right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$, where r is the distance from the origin. The electric field at a distance $r (r < R)$ from the origin is given by

(1) $\frac{\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right)$ (2) $\frac{4\pi\rho_0 r}{3\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right)$ (3) $\frac{\rho_0 r}{4\epsilon_0} \left(\frac{5}{3} - \frac{r}{R} \right)$ (4) $\frac{4\rho_0 r}{3\epsilon_0} \left(\frac{5}{4} - \frac{r}{R} \right)$

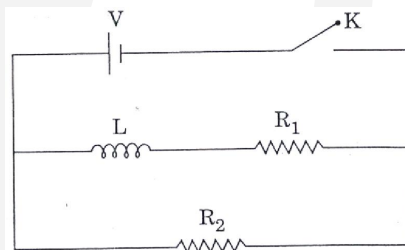
17. (3) $q_{\text{enclosed}}(r) = \int_0^r \rho_0 \left(\frac{5}{4} - \frac{x}{R} \right) 4\pi x^2 dx$
 $= \pi\rho_0 r^3 \left\{ (5/3) - (r/R) \right\}$
 $E(r) = (1/4\pi\epsilon_0) \cdot (q_{\text{enclosed}} / r^2) = (\rho_0 r / 4\epsilon_0) \left\{ (5/3) - (r/R) \right\}$.

18. In a series LCR circuit $R = 200 \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30° . On taking out the inductor from the circuit the current leads the voltage by 30° . The power dissipated in the LCR circuit is

(1) 242 W (2) 305 W (3) 210 W (4) Zero W

18. (1) $\tan 30^\circ = X_L / R = X_C / R \Rightarrow X_L = X_C$
 $\therefore I = 220 / 200 \text{ A} \therefore P = I^2 R = 242 \text{ W}$

19. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(1) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$ (2) $\frac{V R R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$
(3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$ (4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V R R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

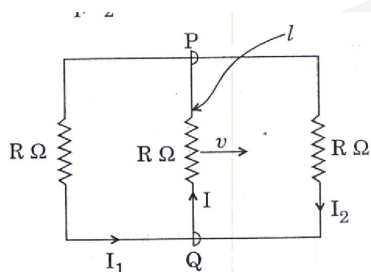
19. (3) At $t = 0^+$; $i_L = 0 \therefore i_{\text{battery}} = V / R_2$
At $t = \infty$; $V_L = 0 \therefore i_{\text{battery}} = \{V(R_1 + R_2) / R_1 R_2\}$

20. A particle is moving with velocity $\vec{v} = K(y\hat{j} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2 + \text{constant}$ (2) $y = x^2 + \text{constant}$
(3) $y^2 = x + \text{constant}$ (4) $xy = \text{constant}$

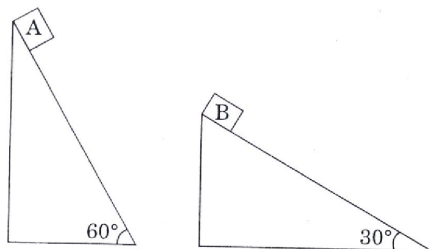
20. (1) $V_x = Ky, V_y = xK \quad (dx/dt) = Ky, \quad (dy/dt) = xK; \quad x = Kyt + C_1 \quad y = xKt + c_2$
 $x = Ky \cdot (y/xK) + c_3 \Rightarrow x^2 - y^2 = c_3$

21. Let C be the capacitance of a capacitor discharging through a resistor R . Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1 / t_2 will be
 (1) 2 (2) 1 (3) 1/2 (4) 1/4
21. (4) $q = q_0 e^{-t/CR}$ $U = 1/2 (q^2 / C) = U_0 e^{-2t/CR}$
 $q_0 / 4 = q_0 e^{-t_2/CR}$ $U_0 / 2 = U_0 e^{-2t_1/CR} \Rightarrow 2t_1 / t_2 = \ln 2 / \ln 4 \Rightarrow t_1 / t_2 = 1 / 4$
22. A rectangular loop has sliding connector PQ of length l and resistance $R \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are



- (1) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$ (2) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$
 (3) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$ (4) $I_1 = I_2 = I = \frac{Blv}{R}$
22. (3) Motional emf = Blv $I + I_2 = Blv / R$...1
 $I + I_1 = Blv / R$...2
 $I = I_1 + I_2$...3
 from 1, 2 and 3 $I_1 = I_2 = Blv / 3R; I = 2Blv / 3R$
23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by
 $y = 0.02(m) \sin \left[2\pi \left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)} \right) \right]$. The tension in the string is
 (1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N
23. (1) $y = 0.02(m) \sin \left[2\pi \left(\frac{t}{0.04} - \frac{x}{0.50} \right) \right]$
 $\therefore v_{\text{wave}} = \text{Coefficient of } t / \text{Coefficient of } x = 50 / 4 \text{ ms}^{-1}$
 $\therefore T = v^2 \mu = (50 / 4)^2 \times 0.04 = 6.25 \text{ N}$

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



- (1) 4.9 ms^{-2} in vertical direction (2) 4.9 ms^{-2} in horizontal direction
 (3) 9.8 ms^{-2} in vertical direction (4) Zero

24. (1) $|\bar{a}_{AB(v)}| = |\bar{a}_{A(v)} - \bar{a}_{B(v)}| = g(\sin^2 60^\circ - \sin^2 30^\circ) = g/2 = 4.9 \text{ ms}^{-2}$

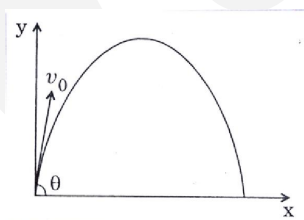
25. For a particle in uniform circular motion, the acceleration \bar{a} at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x -axis)

- (1) $\frac{v^2}{R} \hat{i} + \frac{v^2}{R} \hat{j}$ (2) $-\frac{v^2}{R} \cos \theta \hat{i} + \frac{v^2}{R} \sin \theta \hat{j}$
 (3) $-\frac{v^2}{R} \sin \theta \hat{i} + \frac{v^2}{R} \cos \theta \hat{j}$ (4) $-\frac{v^2}{R} \cos \theta \hat{i} - \frac{v^2}{R} \sin \theta \hat{j}$

25. (4) Resolving the v^2/R along x & y -axes at point $(R \cos \theta, R \sin \theta)$

$$\frac{v^2}{R} \cos \theta (-\hat{i}) + \frac{v^2}{R} (-\hat{j})$$

26. A small particle of mass m is projected at an angle θ with the x -axis with an initial velocity v_0 in the $x - y$ plane as shown in the figure. At a time $t < \frac{v_0 \sin \theta}{g}$, the angular momentum of the particle is



- (1) $\frac{1}{2} mg v_0 t^2 \cos \theta \hat{i}$ (2) $-mg v_0 t^2 \cos \theta \hat{j}$
 (3) $mg v_0 t \cos \theta \hat{k}$ (4) $-\frac{1}{2} mg v_0 t^2 \cos \theta \hat{k}$

Where \hat{i}, \hat{j} and \hat{k} are unit vectors along x, y and z -axis respectively.

26. (4) Only possible answer is last option since direction for angular momentum will be only in $-ve$ z -direction.

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3} , the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3} , the dielectric constant of the liquid is

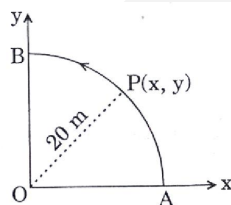
- (1) 1 (2) 4 (3) 3 (4) 2

27. (4) $\tan 30^\circ = V\rho g / F_e$

When system immersed in liquid $\tan 30^\circ = \frac{\epsilon_r V (\rho - \rho_{liq}) g}{F_e}$

$\therefore \epsilon_r (\rho - \rho_{liq.}) = \rho \Rightarrow \epsilon_r = \{\rho / (\rho - \rho_{liq.})\} = 2$

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



- (1) 14 ms^{-2} (2) 13 ms^{-2} (3) 12 ms^{-2} (4) 7.2 ms^{-2}

28. (1) $a_T = d^2s / dt^2 = 6t$ & $a_N = v^2 / R = (3t^2)^2 / 20$
 \therefore At 2 sec, $a_T = 12$, $a_N = 7.2 \text{ ms}^{-2}$, $\therefore a = \sqrt{(12^2 + 7.2^2)} \approx 14 \text{ ms}^{-2}$

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U_{(x=\infty)} - U_{\text{at equilibrium}}] D$

- (1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$ (3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

29. (4) $U_{(x=\infty)} = 0$ At Equilibrium, $F = -dU / dx = -[(a \cdot (-12) / x^{13}) - (b \cdot (-6) / x^7)] = 0$;
 $x^6 = 2a / b$, $\therefore D = [0 - \{(a / (2a / b)^2) - (b / (2a / b))\}] = b^2 / 4a$

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2 . The respective temperature coefficients of their series and parallel combinations are nearly

- (1) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ (2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$
 (3) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ (4) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

30. (1) $\alpha_{eq} = \frac{R_{final} - R_{initial}}{R_{initial} \Delta\theta}$. In series : $R_{initial} = 2R, R_{final} = 2R + R(\alpha_1 + \alpha_2) \Delta\theta$.

$\therefore \alpha_{eq} = \frac{R(\alpha_1 + \alpha_2) \Delta\theta}{2R \Delta\theta} = \frac{\alpha_1 + \alpha_2}{2}$. In parallel $R_{initial} = R / 2$

$\frac{R}{2} (1 + \alpha_{eq} \Delta\theta) = \frac{R(1 + \alpha_1 \Delta\theta) R(1 + \alpha_2 \Delta\theta)}{R(1 + \alpha_1 \Delta\theta) + (1 + \alpha_2 \Delta\theta)}$ $\therefore \alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$