IES / ISS EXAM, 2010

Serial No. 1801

C–HLR–K–TD

STATISTICS—IV

Time Allowed : Three Hours

Maximum Marks : 200

INSTRUCTIONS

*Candidates should attempt **FIVE** questions in all including Question nos. 1 and 5 which are compulsory and attempt remaining **THREE** questions by choosing at least **ONE** each from Sections A and B.*

The number of marks carried by each question is indicated at the end of the question.

*Answers must be written in **ENGLISH**.*

Symbols and abbreviations are as usual.

If any data is required to be assumed for answering a question, it may be suitably assumed, indicating this clearly.

SECTION—A

1. Attempt any **FIVE** parts :—

(a) Give a brief critical account of Poisson process. 8

(Contd.)

(b) Let φ designate the set of all feasible solution to the linear program in standard form. Then prove that every extreme point of φ has at least n-m zero components and is a basic feasible solution. 8

(c) Obtain differential-difference equations for a birth-death process. 8

(d) Describe Vogel's method for solving a transportation problem. 8

(e) Explain how the theory of replacement is used in :

(i) replacement of items whose maintenance cost varies with time,

(ii) replacement of items that fail completely.

8

(f) Explain the terms :

(i) Simulation of queues

(ii) Average inventory level

(iii) Group replacement. 8

2. (a) Show that in a simple Random walk with two reflecting barriers at 'o' and 'a', the stationary distribution π_j is given by :

$$\pi_j = \frac{1 - p/q}{1 - (p/q)^{a+1}} (p/q)^j, \quad j = 0, 1, \ldots\ldots, a$$

where $\pi_j = \lim\limits_{n \to \infty} p_{ij}^{(n)}$; and $p_{ij}^{(n)}$ has the usual interpretation that it represents the probability that the particle occupies the state j at time n having started in the state i; what will be the value of π_j when the Random walk is homogeneous (i.e., $p = q$) ? 10

(b) Investigate the M | M | 1 queueing model by the Markov chain technique. 10

(c) Let $\{X_t\}$ be a stochastic process of independent increments. If $P(X_{t_o} = \alpha) = 1$ for some epoch t_o and some constants α, then show that $\{X_t\}$ is a Markov process. 10

(d) Let X_n denote the size of n^{th} generation, $n = 0, 1, 2, \ldots\ldots,$ and $X_0 = 1$. Then the r.v. $y_n = \sum_{i=0}^{n} X_i$, denotes the total number of progeny. Then, show that the p.g.f. $R_n(s)$ of Y_n satisfy the recurrence relation :

$$R_n(s) = sP\,(R_{n-1}(s)),$$

where P(s) being the p.g.f. of the offspring distribution. 10

3. (a) If either the primal or the dual problem has a finite optimum solution, then show that the other problem has a finite optimum solution. 10

(b) Define a transportation problem. Obtain the necessary and sufficient condition for the existence of a feasible solution to a transportation problem. 10

(c) For a given homogeneous M | M | K queueing system, if the steady state probabilities exist, then prove that the traffic intensity of the system is less than unity. 10

(d) Solve the following LP problem by two-phase Simplex method :

Minimize :

$$Z = 80x_1 + 60x_2$$

subject to :

$$0{\cdot}20x_1 + 0{\cdot}32x_2 \le 0{\cdot}25$$

$$x_1 + x_2 = 1$$

with x_1 and x_2 non-negative. 10

4. (a) The men's department of a large store employs one tailor for customer fittings. The number of customers requiring fittings appears to follow a Poisson distribution with mean arrival rate 24 per hour. Customers are fitted on a first-come, first-served basis, and they are always willing to wait for the tailor's service, because alterations are free.

The time it takes to fit a customer appears to be exponentially distributed, with a mean of 2 min.

(i) What is the average number of customers in the fitting room ?

(ii) How much time should a customer expect to spend in the fitting room ?

(iii) What percentage of the time is the tailor idle ? 10

(b) If $X_n = \max\{Y_1, Y_2, \ldots\ldots, Y_n\}$, where Y_i denotes the number on the face turning up in the i^{th} toss of a die with faces 1, 2, ..., 6. Show that $\{X_n\}$ is a Markov chain. Obtain its transition matrix. 10

(c) A large number of special light bulbs are used in a precision assembly shop all of which must be kept in working order. If a bulb fails in service it costs Rs. 20 to replace it. If we replace all the bulbs in the same operation, we can do it only for Rs. 7 a bulb. The probability distribution of lives are as follows :

Failure week :	1	2	3	4	5	6
Probability :	0·09	0·16	0·24	0·36	0·12	0·03

Calculate the week of group replacement so that the average cost is minimum. 10

(d) Describe the ABC inventory classification system. 10

(Contd.)

SECTION—B

5. Attempt any **FIVE** parts :—

(a) Describe the structure of a complete life table. Explain how the different columns of a life table may be computed on the basis of observed age-specific mortality rates. 8

(b) Define CBR, GFR and ASFR, and indicate why each is considered an improvement on the preceding measure of fertility. 8

(c) Derive Makeham's formula, starting from suitable assumptions. 8

(d) What do you mean by intrinsic growth rate ? Derive an expression for intrinsic growth rate from Lotka's renewal equation. 8

(e) (i) Find the binary equivalent of $(23)_{10}$.

(ii) Find the hexadecimal equivalent of $(41819)_{10}$.

(iii) Find the decimal equivalent of the hexadecimal number $(0{\cdot}4c)_{16}$. 8

(f) Describe different types of databases. 8

6. (a) Discuss the relative importance of logistic and component methods of population projections. 10

(b) Describe the different methods of computing the infant mortality rate. 10

(c) The following table relates to the all India female population (1951), the number of female live births classified according to the age of mothers and the survival rate for females. Calculate the GRR and NRR.

Age	Female Population (100)	Total number of female live births (100)	Survival rate (per 100,000)
15—19	157670	4632	58065
20—24	147624	14443	55870
25—29	124200	14058	52981
30—34	105865	8329	48963
35—39	89264	4036	44146
40—44	77887	2158	39154
45—49	61161	689	34198

10

(d) Explain the following :—

(i) Age pyramid

(ii) Standardization of vital rates

(iii) Migration models. 10

7. (a) Discuss migration analysis based on place of birth data. What are the drawbacks of this method ? 10

(Contd.)

(b) What is age heaping ? How can you detect its presence in census age data ? Explain Wipple's index. 10

(c) What is the purpose of memory in a computer ? What are the main characteristics of a memory cell ? 10

(d) Explain the following :—

(i) Data entry database

(ii) Data manipulation language

(iii) On-line real time processing. 10

8. (a) What criteria are used in selecting a computer network topology ? 10

(b) Outline the uses of SQL (Structured Query Language) for the data administration. 10

(c) Describe the nature and properties of stable and stationary populations. 10

(d) Define a Gompertz curve and explain a method of fitting the curve to a given set of population figures. 10