IES/ISS EXAM,2014

Sl. **A–HDR/HRR–N–TUD**

STATISTICS—IV

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are EIGHT questions divided under TWO Sections.

Candidate has to attempt FIVE questions in ALL.

Question Nos. 1 and 5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE from each Section.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Candidates should attempt questions/parts as per the instructions given in the Section.

All parts and sub-parts of a question are to be attempted together in the answer-book.

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the answer-book must be clearly struck off.

Answers must be written in ENGLISH only.

Section—A

1. Answer *all* of the following : 8×5=40

(a) Define *(i)* persistent, *(ii)* transient and *(iii)* ergodic states. Show that if $i \leftrightarrow j$ and if state i is persistent, then state j is also persistent. 8

(b) Define random walk and explain gambler's ruin problem. 8

(c) Using graphical method, solve the following problems :

(i) Maximize $Z = 6x_1 - 4x_2$

subject to

$$2x_1 - 4x_2 \leq 4$$

$$4x_1 - 8x_2 \geq 16$$

$$x_1, x_2 \geq 0$$ 4

(ii) Maximize $Z = 10x_1 - 6x_2$

subject to

$$5x_1 + 3x_2 \leq 30$$

$$x_1 + 2x_2 \leq 18$$

$$x_1, x_2 \geq 0$$ 4

(d) A shop produces three items in lots. The demand rate for each item is constant and can be assumed to be deterministic. No back orders are allowed. The pertinent data for the items is given as follows :

Item	*I*	*II*	*III*
Carrying cost (₹ per unit per year)	20	20	20
Setup cost (₹ per setup)	50	40	60
Cost per unit (₹)	6	7	5
Yearly demand (units)	10000	12000	7500

Determine approximately the economic order quantity for three items subject to the condition that the total value of average inventory levels of these items does not exceed ₹ 1,000. 8

(e) Explain the procedure of generating random numbers from $N(\mu, \sigma^2)$, (where μ and σ are specified) using Box-Muller formula. 8

2\. (a) Prove that a discrete parameter stochastic process $\{X_n, n \geq 0\}$ is called a martingale if $E\{|X_n|\} < \infty$ and $E\{X_{n+1} / X_n, X_{n-1}, ..., X_0\} = X_n$. 10

(b) Define branching process and state the properties of its generating function. 10

(c) Explain the additive property of Poisson process and also the difference of two independent Poisson processes. 10

(d) Describe birth and death processes, and show that it is a particular case of Poisson process. 10

3. (a) *XYZ* Tobacco Company purchases tobacco and stores in warehouses located in the following four cities :

Warehouse Location	*Capacity* (in tonnes)
City *A*	90
City *B*	50
City *C*	80
City *D*	60

The warehouses supply tobacco to cigarette companies in three cities that have the following demand :

Cigarette Company	*Demand* (in tonnes)
Bharat	120
Janata	100
Re Lamp	110

Because of railroad construction, shipments are temporarily prohibited from warehouse at city *A* to Bharat Cigarette Company.

(i) Find the optimum distribution for *XYZ* Tobacco Company.

(ii) Are there any multiple optimum solutions? If yes, identify them.

(iii) Write the dual of the given transportation problem and use it for checking the optimum solution. 10

(b) Some of the spare parts of a ship cost ₹ 50,000. These parts can only be ordered together with the ship. If not ordered at the time the ship is constructed, these parts are not available as and when needed. Suppose that a loss of ₹ 4,50,000 is incurred for each spare part that is needed when none is available in the stock and the probability distribution that the spares will be needed for replacement during lifetime is

Spares required	:	0	1	2	3	4	5	≥ 6
Probability	:	0·90	0·04	0·025	0·02	0·01	0·05	0

Find the optimum order quantity. 10

(c) Machine *A* costs ₹ 45,000 and the operating costs are estimated at ₹ 1,000 for the first year, increasing by ₹ 10,000 per year in the second and subsequent years. Machine *B* costs ₹ 50,000 and the operating costs are ₹ 2,000 for the first year, increasing by ₹ 4,000 per year in the second and subsequent years. If we now have a machine of type A, should we replace it with B? If so, when? Assume that both machines have no resale value and future costs are not discounted. 10

(d) A bakery keeps stock of a popular brand of cakes. Previous experience shows that the daily demand pattern for the item with associated probabilities is as given below :

Daily demand (in numbers)	:	0	10	20	30	40	50
Probability	:	0·01	0·20	0·15	0·50	0·12	0·02

Use the following sequence of random numbers to simulate the demand for the next 10 days :

25 39 65 76 12 05 73 89 19 49

Estimate the daily average demand for the cakes on the basis of simulated data.

Can you estimate the daily average demand analytically? If so, find the bias in estimating daily average demand. 10

4. (a) Obtain Chapman-Kolmogorov equation with transition probabilities for a Markov chain. 10

(b) Discuss M/M/1 queuing model with steady-state behaviour. 10

(c) Use simplex method to solve the following problem :

A company makes two kinds of leather belts. Belt *A* is of high quality and belt *B* is of low quality. The respective profits are ₹ 4 and ₹ 3 per belt. The production of each of type *A* requires as much time as belt of type *B* and if all belts were of type *B*, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both *A* and *B* put together). Belt *A* requires fancy buckles and only 400 per day are available. There are only 700 buckles a day available for belt *B*. What should be the daily production of each type of belt to maximize the total profit? 10

(d) Let the value of the money be assumed to be depreciated @ 10% per year and suppose that machine *A* is replaced after every three years whereas machine *B* is replaced every six years. The yearly costs (in ₹) of both the machines are given as under :

Year	:	1	2	3	4	5	6
Machine A	:	1,000	200	400	1,000	200	400
Machine B	:	1,700	100	200	300	400	500

Determine which machine should be purchased. 10

Section—B

5. Answer *all* of the following : 8×5=40

(a) Briefly outline the uses of life tables. 8

(b) Mention how logistic curve fitting is used for population projection. 8

(c) Outline and describe the inter-censal and post-censal estimates. 8

(d) Write short notes on : 8

(i) Magnetic Ink Character Recognition

(ii) Plotters

(e) Use 2's complement to—

(i) subtract 3 from 5;

(ii) subtract (–3) from (–5);

(iii) add (–5) and (–2);

(iv) add 5 and 4. 8

6. *(a)* Compute *(i)* GFR, *(ii)* SFR, *(iii)* TFR and *(iv)* gross production rate from the data given below : 10

Age group of child-bearing females	:	15–19	20–24	25–29	30–34
No. of women ('000)	:	16·0	16·4	15·8	15·2
Total births	:	260	2244	1894	1320

Age group of child-bearing females	:	35–39	40–44	45–49
No. of women ('000)	:	14·8	15·0	14·5
Total births	:	916	280	145

Assume that the proportion of female births is 46·2%.

(b) Outline the uses of Makeham and Gompertz curves in life tables. 10

(c) Describe stable and stationary populations. 10

(d) Explain various mortality rates and standardized death rates. 10

7. *(a)* Write binary multiplication algorithm using register(s) and accumulator. 10

(b) Discuss the following types of operating system : 10

(i) Batch

(ii) Multiprogramming

(iii) Time-sharing

(iv) Real-time

(c) Given (X_i, Y_i) where $i = 1, n$. Write a flow chart to find a and b in fitting $Y = a + bX$. 10

(d) Discuss error detecting and error correcting codes, and illustrate single error detecting code in detail. 10

8. (a) State the general procedure and steps for the construction of life tables. 10

(b) From the data given below, calculate the gross reproduction rate and net reproduction rate : 10

Age group	*No. of children born to 1000 women passing through the age group*	*Mortality rate* (per 1000)
16–20	150	120
21–25	1500	180
26–30	2000	150
31–35	800	200
36–40	500	220
41–45	200	230
46–50	100	250

Sex ratio being males : females = 52 : 48.

(c) Illustrate mail merge application in detail describing the common features available in any word processing package. 10

(d) Discuss various features of graphics wizard in any spreadsheet package in connection with statistical data processing. 10

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