02 - COMPLEX NUMBERS

(Answers at the end of all questions)

(1) If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$ are

(a) $-1, \ -1 + 2\omega, \ -1 - 2\omega^2$ (b) $-1, \ -1, \ -1$,
(c) $-1, \ 1 - 2\omega, \ 1 - 2\omega^2$ (d) $-1, \ 1 + 2\omega, \ 1 + 2\omega^2$ [AIEEE 2005]

(2) If z_1 and z_2 are two non-zero complex numbers such that $| z_1 + z_2 | = | z_1 | + | z_2 |$, then $\arg z_1 - \arg z_2$ is equal to

(a) $\frac{\pi}{2}$ (b) $-\pi$ (c) 0 (d) $-\frac{\pi}{2}$ [AIEEE 2005]

(3) If $w = \dfrac{z}{z - \frac{1}{3}i}$ and $| w | = 1$, then z lies on

(a) an ellipse (b) a circle (d) a straight line (d) a parabola [AIEEE 2005]

(4) Let z, w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and $\arg zw = \pi$. Then arg z equals

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{5\pi}{4}$ [AIEEE 2004]

(5) If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\dfrac{\left(\frac{x}{p} + \frac{y}{q} \right)}{p^2 + q^2}$ is equal to

(a) 1 (b) -1 (c) 2 (d) -2 [AIEEE 2004]

(6) If $| z^2 - 1 | = | z |^2 + 1$, then z lies on

(a) the real axis (b) the imaginary axis
(c) a circle (c) an ellipse [AIEEE 2004]

(7) Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further assume that the origin, z_1 and z_2 form an equilateral triangle. Then

(a) $a^2 = b$ (b) $a^2 = 2b$ (c) $a^2 = 3b$ (d) $a^2 = 4b$ [AIEEE 2003]

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(Answers at the end of all questions)

(8) If z and w are two non-zero complex numbers such that $|zw| = 1$ and $\text{Arg}(z) - \text{Arg}(w) = \frac{\pi}{2}$, then $\bar{z}\,w$ is equal to

(a) 1 (b) - 1 (c) i (d) - i [AIEEE 2003]

(9) If $\left(\frac{1+i}{1-i}\right)^x = 1$, then the value of smallest positive integer n is given by

(a) x = 4n (b) x = 2n (c) x = 4n + 1 (d) x = 2n + 1 [AIEEE 2003]

(10) If 1, ω, ω^2 are the cube roots of unity, then the value of $\Delta = \begin{vmatrix} 1 & \omega & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is

(a) 1 (b) 0 (c) ω (d) ω^2 [AIEEE 2003]

(11) If $\frac{c+i}{c-i} = a\ ib$, where a, b, c are real, then the value of $a^2 + b^2$ is

(a) 1 (b) - 1 (c) c^2 (d) $-c^2$ [AIEEE 2002]

(12) If z = x + y, then $|3z - 1| = 3|z - 2|$ represents

(a) x-axis (b) y-axis (c) a circle (d) line parallel to y-axis [AIEEE 2002]

(13) If the cube roots of unity are 1, ω and ω^2, then the value of $\left(\frac{1+\omega}{\omega^2}\right)^3$ is

(a) 1 (b) - 1 (c) ω (d) ω^2 [AIEEE 2002]

(14) If $a = \cos\alpha + i\sin\alpha$ and $b = \cos\beta + i\sin\beta$, then the value of $\frac{1}{2}\left(ab + \frac{1}{ab}\right)$ is

(a) $\sin(\alpha + \beta)$ (b) $\cos(\alpha + \beta)$ (c) $\sin(\alpha - \beta)$ (d) $\cos(\alpha - \beta)$ [AIEEE 2002]

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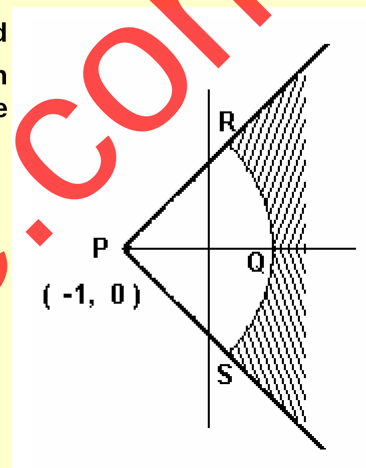
(Answers at the end of all questions)

(15) If α is cube root of unity, then for $n \in N$, the value of $\alpha^{3n+1} + \alpha^{3n+5}$ is

(a) - 1 (b) 0 (c) 1 (d) 3 [AIEEE 2002]

(16) Four points P (- 1, 0), Q (1, 0), R ($\sqrt{2}$ - 1, $\sqrt{2}$) and S ($\sqrt{2}$ - 1, - $\sqrt{2}$) are given on a complex plane, equation of the locus of the shaded region excluding the boundaries is given by



(a) $|z + 1| > 2$ and $|\arg(z + 1)| < \frac{\pi}{4}$

(b) $|z + 1| > 2$ and $|\arg(z + 1)| < \frac{\pi}{2}$

(c) $|z - 1| > 2$ and $|\arg(z - 1)| < \frac{\pi}{4}$

(d) $|z - 1| > 2$ and $|\arg(z - 1)| < \frac{\pi}{2}$ [IIT 2005]

(17) If ω is cube root of unity ($\omega \neq 1$), then the least value of n where n is a positive integer such that $(1 + \omega^2)^n = (1 + \omega^4)^n$ is

(a) 2 (b) 3 (c) 5 (d) 6 [IIT 2004]

(18) The complex number z is such that $|z| = 1$, $z \neq -1$ and $\omega = \frac{z - 1}{z + 1}$, then real part of ω is

(a) $\frac{1}{|z + 1|^2}$ (b) $\frac{-1}{|z + 1|^2}$ (c) $\frac{\sqrt{2}}{|z + 1|^2}$ (d) 0 [IIT 2003]

(19) Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is

(a) 3ω (b) $3\omega(\omega - 1)$ (c) $3\omega^2$ (d) $3\omega(1 - \omega)$ [IIT 2002]

(20) For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is

(a) 0 (b) 2 (c) 7 (d) 17 [IIT 2002]

(21) The complex numbers z_1, z_2 and z_3 satisfying $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is

(a) of area zero (b) right-angled isosceles
(c) equilateral (d) obtuse-angled isosceles [IIT 2001]

(22) If z_1 and z_2 be nth roots of unity which subtend a right angle at the origin, then n must be of the form

(a) 4k + 1 (b) 4k + 2 (c) 4k + 3 (d) 4k [IIT 2001]

(23) If arg (z) < 0, then arg (- z) - arg (z) =

(a) π (b) $-\pi$ (c) $-\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{2}$ [IIT 2000]

(24) If z_1, z_2 and z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left| \dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3} \right| = 1$, then $|z_1 + z_2 + z_3|$ is

(a) 1 (b) < 1 (c) > 3 (d) 3 [IIT 2000]

(25) If $i = \sqrt{-1}$, then $4 + 5\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2} \right)^{334} + 3\left(-\dfrac{1}{2} + \dfrac{i\sqrt{3}}{2} \right)^{365}$ is equal to

(a) $1 - i\sqrt{3}$ (b) $-1 + i\sqrt{3}$ (c) $i\sqrt{3}$ (d) $-i\sqrt{3}$ [IIT 1999]

(26) If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals

(a) $128\,\omega$ (b) $-128\,\omega$ (c) $128\,\omega^2$ (d) $-128\,\omega^2$ [IIT 1998]

(27) The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals

(a) i (b) i - 1 (c) - i (d) 0 [IIT 1998]

(28) If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$
(c) $x = 0, y = 3$ (d) $x = 0, y = 0$ [IIT 1998]

(29) For positive integers n_1, n_1, the value of the expression $(1+i)^{n_1} + (1+i^3)^{n_1} + (1+i^5)^{n_2} + (1+i^7)^{n_2}$, where $i = \sqrt{-1}$ is a real number if and only if

(a) $n_1 = n_2 + 1$ (b) $n_1 = n_2 - 1$ (c) $n_1 = n_2$ (d) $n_1 > 0$, $n_2 > 0$ [IIT 1996]

(30) If ω ($\neq 1$) is a cube root of unity and $(1 + \omega)^7 = A + B\omega$, then A and B are respectively the numbers

(a) 0, 1 (b) 1, 1 (c) 1, 0 (d) - 1, 1 [IIT 1995]

(31) If ω ($\neq 1$) is a cube root of unity, then $\begin{vmatrix} 1 & 1 + i\omega^2 & \omega^2 \\ 1 - i & -1 & \omega^2 - 1 \\ -i & i + \omega - 1 & -1 \end{vmatrix}$ equals

(a) 0 (b) 1 (c) i (d) ω [IIT 1995]

(32) If z and ω be two non-zero complex numbers such that $|z| = |\omega|$ and $\text{Arg}\ z^{+} - \text{Arg}\ \omega = \pi$, then z equals

(a) ω (b) $-\omega$ (c) $\overline{\omega}$ (d) $-\overline{\omega}$ [IIT 1995]

(33) If z and w be two complex numbers such that $|z| \leq 1$, $|w| \leq 1$ and $|z + iw| = |z - iw| = 2$, then z equals

(a) 1 or i (b) i or - i (c) 1 or - 1 (d) i or - 1 [IIT 1995]

(34) The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for

(a) $x = n\pi$ (b) $x = 0$ (c) $x = (n + 1/2)\pi$ (d) no value of x [IIT 1988]

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(Answers at the end of all questions)

(35) If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to

(a) $-\pi$ (b) $-\frac{\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$ (e) π [IIT 1987]

(36) The value of $\sum_{k=1}^{6} \left(\sin \frac{2\pi k}{7} - i \cos \frac{2\pi k}{7} \right)$ is

(a) - 1 (b) 0 (c) - i (d) i (e) none of these [IIT 1987]

(37) Let z_1 and z_2 be complex numbers such that $z_1 \neq z_2$ and $|z_1| = |z_2|$. If z_1 has positive real part and z_2 has negative imaginary part, then $\frac{z_1 + z_2}{z_1 - z_2}$ may be

(a) zero (b) real and positive (c) real and negative
(d) purely imaginary (e) none of these [IIT 1986]

(38) If a, b, c and u, v, w are complex numbers representing the vertices of two triangles such that c = (1 - r) a + rb and w = (1 - r) u + rv, where r is a complex number, then the two triangles

(a) have the same area (b) are similar
(c) are congruent (d) none of these [IIT 1985]

(39) If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}(z_1 \overline{z_2}) = 0$, then the pair of complex numbers $w_1 = a + ic$ and $w_2 = b + id$ satisfies

(a) $|w_1| = 1$ (b) $|w_2| = 1$
(c) $\text{Re}(w_1 \overline{w_2}) = 0$ (d) none of these [IIT 1985]

(40) If $z = x + iy$ and $w = \frac{1 - iz}{z - i}$, then $|w| = 1$ implies that, in the complex plane,

(a) z lies on the imaginary axis (b) z lies on the real axis
(c) z lies on the unit circle (d) None of these [IIT 1983]

(41) The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if

(a) $z_1 + z_4 = z_2 + z_3$ (b) $z_1 + z_3 = z_2 + z_4$

(c) $z_1 + z_2 = z_3 + z_4$ (d) None of these [IIT 1983]

(42) The inequality $|z - 4| < |z - 2|$ represents the region given by

(a) $Re(z) > 0$ (b) $Re(z) < 0$

(c) $Re(z) > 2$ (d) none of these [IIT 1982]

(43) If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$, then

(a) $Re(z) = 0$ (b) $Im(z) = 0$

(c) $Re(z) > 0$, $Im(z) > 0$ (d) $Re(z) > 0$, $Im(z) < 0$ [IIT 1982]

(44) If the cube roots of unity are 1, ω, ω^2, then the roots of the equation $(x - 1)^2 + 8 = 0$ are

(a) -1, $1 + 2\omega$, $1 + 2\omega^2$ (b) -1, $1 - 2\omega$, $1 - 2\omega^2$

(c) -1, -1, -1 (d) none of these [IIT 1979]

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	c	c	c	d	b	c	d	a	b	a	d	b	b	b	a	b	d	b	b

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	d	a	a	c	d	b	d	d	b	a	b	c	d	a,e	d	a,d	b	a,b,c	b

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	d	b	b																