# 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

(1) $\lim_{n \to \infty} \left[ \frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \ldots\ldots + \frac{1}{n} \sec^2 1 \right]$ is

(a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$ (c) $\tan 1$ (d) $\frac{1}{2} \tan 1$ [ AIEEE 2005 ]

(2) The normal to the curve $x = a(\cos\theta + \theta \sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point '$\theta$' is such that

(a) it passes through the origin

(b) it makes angle $\frac{\pi}{2} + \theta$ with the X-axis

(c) it passes through $\left( a\frac{\pi}{2}, -a \right)$

(d) it is at a constant distance from the origin

[ AIEEE 2005 ]

(3) A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched ?

| | Interval | Function | | Interval | Function |
|---|---|---|---|---|---|
| (a) | $(-\infty, \infty)$ | $x^3 - 3x^2 + 3x + 3$ | (b) | $[2, \infty)$ | $2x^3 - 3x^2 + 12x + 6$ |
| (c) | $\left(-\infty, \frac{1}{3}\right)$ | $3x^3 - 2x^2 + 1$ | (d) | $(-\infty, -4)$ | $x^3 - 6x^2 + 6$ |

[ AIEEE 2005 ]

(4) Let $\alpha$ and $\beta$ be the distinct roots of the equation $ax^2 + bx + c = 0$. Then $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0 (c) $-\frac{a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

[ AIEEE 2005 ]

(5) Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals

(a) 3 (b) 4 (c) 5 (d) 6 [ AIEEE 2005 ]

(6) Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(a) $f(6) \geq 8$  (b) $f(6) < 8$  (c) $f(6) < 5$  (d) $f(6) = 5$  [AIEEE 2005]

(7) If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(a) $-1$  (b) 0  (c) 2  (d) 1  [AIEEE 2005]

(8) A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 $cm^3$/min. When the thickness of ice is 5 cm, then the rate at which thickness of ice decreases in cm/min is

(a) $\frac{1}{36\pi}$  (b) $\frac{1}{18\pi}$  (c) $\frac{1}{54\pi}$  (d) $\frac{5}{6\pi}$  [AIEEE 2005]

(9) Let $f : R \rightarrow R$ be a differentiable function having $f(2) = 6$, $f'(2) = \frac{1}{48}$. Then $\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x - 2} dt$ equals

(a) 24  (b) 36  (c) 12  (d) 18  [AIEEE 2005]

(10) If the equation $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$ has a positive root $x = \alpha$, then the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + a_1 = 0$ has a positive root which is

(a) greater than $\alpha$  (b) smaller than $\alpha$
(c) greater than or equal to $\alpha$  (d) equal to $\alpha$  [AIEEE 2005]

(11) If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$, then the values of a and b are

(a) $a \in R$, $b \in R$  (b) $a = 1$, $b \in R$
(c) $a \in R$, $b = 2$  (d) $a = 1$, $b = 2$  [AIEEE 2004]

( 12 ) Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}$, $x \neq \dfrac{\pi}{4}$, $x \in \left[0, \dfrac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is

( a ) 1 ( b ) $\dfrac{1}{2}$ ( c ) $-\dfrac{1}{2}$ ( d ) - 1 [ AIEEE 2004 ]

( 13 ) If $x = e^{y + e^{y + \ldots\ldots \infty}}$, $x > 0$, then $\dfrac{dy}{dx}$ is

( a ) $\dfrac{x}{1 + x}$ ( b ) $\dfrac{1}{x}$ ( c ) $\dfrac{1 - x}{x}$ ( d ) $\dfrac{1 + x}{x}$ [ AIEEE 2004 ]

( 14 ) A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

( a ) ( 2, 4 ) ( b ) ( 2, - 4 ) ( c ) $\left(-\dfrac{9}{8}, \dfrac{9}{2}\right)$ ( d ) $\left(\dfrac{9}{8}, \dfrac{9}{2}\right)$ [ AIEEE 2004 ]

( 15 ) A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point ( 2, 1 ) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

( a ) $(x - 1)^2$ ( b ) $(x - 1)^3$ ( c ) $(x + 1)^3$ ( d ) $(x + 1)^2$ [ AIEEE 2004 ]

( 16 ) The normal to the curve $x = a(1 + \cos\theta)$, $y = a\sin\theta$ at '$\theta$' always passes through the fixed point

( a ) ( a, 0 ) ( b ) ( 0, a ) ( c ) ( 0, 0 ) ( d ) ( a, a ) [ AIEEE 2004 ]

( 17 ) If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

( a ) ( 0, 1 ) ( b ) ( 1, 2 ) ( c ) ( 2, 3 ) ( d ) ( 1, 3 ) [ AIEEE 2004 ]

( 18 ) Let f ( x ) be a polynomial function of second degree. If f ( 1 ) = f ( - 1 ) and a, b, c are in A. P., then f ' ( a ), f ' ( b ) and f ' ( c ) are in

( a ) A. P. ( b ) G. P. ( c ) H. P. ( d ) A. G. P. [ AIEEE 2003 ]

( 19 ) $\lim_{x \to \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right]\left[1 - \sin x\right]}{\left[1 + \tan\left(\frac{x}{2}\right)\right]\left[\pi - 2x\right]^3} =$

( a ) 0 ( b ) $\infty$ ( c ) $\frac{1}{32}$ ( d ) $\frac{1}{8}$ [ AIEEE 2003 ]

( 20 ) If $\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x} = k$ then the value f k is

( a ) 0 ( b ) $-\frac{1}{3}$ ( c ) $\frac{2}{3}$ ( d ) $-\frac{2}{3}$ [ AIEEE 2003 ]

( 21 ) If $f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$ is continuous at x = 0, then the value of f ( 0 ) is

( a ) ab ( b ) a + b ( c ) a - b ( d ) log a - log b [ AIEEE 2003 ]

( 22 ) If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$ then the value of $\frac{dy}{dx}$ is

( a ) 0 ( b ) 1 ( c ) x ( d ) y [ AIEEE 2003 ]

( 23 ) The value of $\lim_{n \to \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5}$ is

( a ) zero ( b ) $\frac{1}{4}$ ( c ) $\frac{1}{5}$ ( d ) $\frac{1}{30}$ [ AIEEE 2003 ]

( 24 ) If $f : R \rightarrow R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then the value of $\sum_{r=1}^{n} f'(r)$ is

( a ) $\frac{7n}{2}$ ( b ) $7n(n + 1)$ ( c ) $\frac{7(n + 1)}{2}$ ( d ) $\frac{7n(n + 1)}{2}$ [ AIEEE 2003 ]

( 25 ) The real number x when added to its inverse gives the minimum value of the sum at x equal to

( a ) 2 ( b ) - 2 ( c ) 1 ( d ) - 1 [ AIEEE 2003 ]

( 26 ) If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$ where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

( a ) 3 ( b ) 1 ( c ) 2 ( d ) 4 [ AIEEE 2003 ]

( 27 ) If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + ... + \frac{(-1)^n f^n(1)}{n!}$ is

( a ) $2^n$ ( b ) $2^{n-1}$ ( c ) 1 ( d ) 0 [ AIEEE 2003 ]

( 28 ) If $x = t^2 + t + 1$ and $y = \sin\left(\frac{\pi}{2}t\right) + \cos\left(\frac{\pi}{2}t\right)$, then at $t = 1$, the value of $\frac{dy}{dx}$ is

( a ) $\frac{\pi}{2}$ ( b ) $-\frac{\pi}{6}$ ( c ) $\frac{\pi}{3}$ ( d ) $-\frac{\pi}{4}$ [ AIEEE 2002 ]

( 29 ) If $x = 3\cos\theta - 2\cos^3\theta$ and $y = 3\sin\theta - 2\sin^3\theta$, then the value of $\frac{dy}{dx}$ is

( a ) $\sin\theta$ ( b ) $\cos\theta$ ( c ) $\tan\theta$ ( d ) $\cot\theta$ [ AIEEE 2002 ]

( Answers at the end of all questions )

(30) Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim_{x \to a} \dfrac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$, then the value of k is

(a) 4 (b) 2 (c) 1 (d) 0 [ AIEEE 2002 ]

(31) The value of $\lim_{x \to 0} \dfrac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$ is

(a) $\dfrac{10}{3}$ (b) $\dfrac{3}{10}$ (c) $\dfrac{6}{5}$ (d) $\dfrac{5}{6}$ [ AIEEE 2002 ]

(32) The value of $\lim_{\alpha \to \beta} \left[ \dfrac{\sin^2 \alpha - \sin^2 \beta}{\alpha^2 - \beta^2} \right]$ is

(a) 0 (b) 1 (c) $\dfrac{\sin \beta}{\beta}$ (d) $\dfrac{\sin 2\beta}{2\beta}$ [ AIEEE 2002 ]

(33) The value of $\lim_{x \to 0} \dfrac{\sqrt{1 - \cos 2x}}{x}$ is

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) does not exist [ AIEEE 2002 ]

(34) $f(x) = 2x^3 - 3x^2 - 12x + 5$ on $[-2, 4]$, then relative maximum occurs at x =

(a) - 2 (b) - 1 (c) 2 (d) 4 [ AIEEE 2002 ]

(35) If $f(x) = \begin{cases} x e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0, \\ 0, & x = 0 \end{cases}$ then $f(x)$ is

(a) discontinuous everywhere
(b) continuous as well as differentiable for all x
(c) neither differentiable nor continuous at x = 0
(d) continuous at all x but not differentiable at x = 0 [ AIEEE 2002 ]

( 36 ) If y is a twice differentiable function and $x \cos y + y \cos x = \pi$, then $y''(0) =$

(a) $\pi$  (b) $-\pi$  (c) 0  (d) 1  [ IIT 2005 ]

( 37 ) $f(x) = \left| |x| - 1 \right|$ is not differentiable at $x =$

(a) $0, \pm 1$  (b) $\pm 1$  (c) 0  (d) 1  [ IIT 2005 ]

( 38 ) If f is a differentiable function such that $f : R \to R$, $f\left(\frac{1}{n}\right) = 0 \;\; \forall \; n \in I, \; n \geq 1$, then

(a) $f(x) = 0 \;\; \forall \; x \in [0, 1]$  (b) $f(0) = 0$, but $f'(0)$ may or may not be 0
(c) $f(0) = 0 = f'(0)$  (d) $|f(x)| \leq 1 \;\; \forall \; x \in [0, 1]$  [ IIT 2005 ]

( 39 ) f is a twice differentiable polynomial function of x such that $f(1) = 1$, $f(2) = 4$ and $f(3) = 9$, then

(a) $f''(x) = 2, \;\; \forall \; x \in R$  (b) $f''(x) = f'(x) = 5, \; x \in [1, 3]$
(c) $f''(x) = 2$ for only $x \in [1, 3]$  (d) $f''(x) = 3, \; x \in (1, 3)$  [ IIT 2005 ]

[ Note: This question should have been better put as 'polynomial function of degree two rather than twice differentiable function'. ]

( 40 ) S is a set of polynomial of degree less than or equal to 2, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0, \; \forall \; x \in [0, 1]$, then set S =

(a) $ax + (1 - a)x^2, \; a \in R$  (b) $ax + (1 - a)x^2, \; 0 < a < 2$
(c) $ax + (1 - a)x^2, \; 0 < a < \infty$  (d) $\phi$  [ IIT 2005 ]

( 41 ) Let y be a function of x, such that $\log(x + y) = 2xy$, then $y'(0)$ is

(a) 0  (b) 1  (c) $\frac{1}{2}$  (d) $\frac{3}{2}$  [ IIT 2004 ]

( 42 ) Let $f(x) = x^{\alpha} \log x$ for $x >$ and $f(0) = 0$ follows Rolle's theorem for $x \in [0, 1]$, then $\alpha$ is

(a) $-2$  (b) $-1$  (c) 0  (d) $\frac{1}{2}$  [ IIT 2004 ]

# 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

( 43 ) If $f(x)$ is strictly increasing and differentiable, then $\lim\limits_{x \to 0} \dfrac{f(x^2) - f(x)}{f(x) - f(0)}$ is

( a ) 1 ( b ) - 1 ( c ) 0 ( d ) 2 [ IIT 2004 ]

( 44 ) Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$, then $f(x)$

( a ) is strictly increasing ( b ) has local maxima
( c ) has local minima ( d ) is a bounded curve [ IIT 2004 ]

( 45 ) If $f(x)$ is a differentiable function, $f'(1) = 1$, $f'(2) = 6$, where $f'(c)$ means the derivative of the function at $x = c$, then

$$\lim_{h \to 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$$

( a ) does not exist ( b ) - 3 ( c ) 3 ( d ) $\dfrac{3}{2}$ [ IIT 2003 ]

( 46 ) If $\lim\limits_{x \to 0} \dfrac{\sin nx[(a - n)nx - \tan x]}{x} = 0$, where n is a non-zero positive integer, then a is equal to

( a ) $\dfrac{n+1}{n}$ ( b ) $n^2$ ( c ) $\dfrac{1}{n}$ ( d ) $n + \dfrac{1}{n}$ [ IIT 2003 ]

( 47 ) Which function does not obey Mean Value Theorem in [ 0, 1 ] ?

( a ) $f(x) = \begin{cases} \dfrac{1}{2} - x, & x < \dfrac{1}{2} \\ \left(\dfrac{1}{2} - x\right)^2, & x \geq \dfrac{1}{2} \end{cases}$ ( b ) $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

( c ) $f(x) = x\,|x|$ ( d ) $f(x) = |x|$ [ IIT 2003 ]

( 48 ) The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \leq 1 \\ \dfrac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases}$ is

( a ) R - { 0 } ( b ) R - { 1 } ( c ) R - { - 1 } ( d ) R - { - 1, 1 } [ IIT 2002 ]

# 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

( 49 ) The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is

( a ) 1 ( b ) 2 ( c ) 3 ( d ) 4 [ IIT 2002 ]

( 50 ) If f : R → R be such that f ( 1 ) = 3 and f ' ( 1 ) = 6, then $\lim_{x \to 0} \left( \frac{f(1 + x)}{f(1)} \right)^{\frac{1}{x}}$ equals

( a ) 1 ( b ) $e^{\frac{1}{2}}$ ( c ) $e^2$ ( d ) $e^3$ [ IIT 2002 ]

( 51 ) The point ( s ) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is / ( are )

( a ) $\left( \pm \frac{4}{\sqrt{3}}, -2 \right)$ ( b ) $\left( \pm \sqrt{\frac{11}{3}}, 0 \right)$ ( c ) ( 0, 0 ) ( d ) $\left( \pm \frac{4}{\sqrt{3}}, 2 \right)$ [ IIT 2002 ]

( 52 ) Let f : R → R be a function defined by f ( x ) = { x, $x^3$ }. The set of all points where f ( x ) is not differentiable is

( a ) { - 1, 1 } ( b ) { - 1, 0 } ( c ) { 0, 1 } ( d ) { - 1, 0, 1 } [ IIT 2001 ]

( 53 ) The left hand derivative of f ( x ) = [ x ] sin ( πx ) at x = k, where k is an integer, is

( a ) $(-1)^k (k - 1) \pi$ ( b ) $(-1)^{k-1} (k - 1) \pi$

( c ) $(-1)^k k \pi$ ( d ) $(-1)^{k-1} k \pi$ [ IIT 2001 ]

( 54 ) The left hand derivative of f ( x ) = [ x ] sin ( πx ) at x = k, where k is an integer, is

( a ) $(-1)^k (k - 1) \pi$ ( b ) $(-1)^{k-1} (k - 1) \pi$

( c ) $(-1)^k k \pi$ ( d ) $(-1)^{k-1} k \pi$ [ IIT 2001 ]

( 55 ) $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals

( a ) - π ( b ) π ( c ) π / 2 ( d ) 1 [ IIT 2001 ]

( 56 ) If $f(x) = x e^{x(1 - x)}$, then $f(x)$ is

( a ) increasing on $\left[-\frac{1}{2}, 1\right]$ ( b ) decreasing on R

( c ) increasing on R ( d ) decreasing on $\left[-\frac{1}{2}, 1\right]$ [ IIT 2001 ]

( 57 ) Which of the following functions is differentiable at $x = 0$ ?

( a ) $\cos(|x|) + |x|$ ( b ) $\cos(|x|) - |x|$
( c ) $\sin(|x|) + |x|$ ( d ) $\sin(|x|) - |x|$ [ IIT 2001 ]

( 58 ) If $x^2 + y^2 = 1$, then

( a ) $yy'' - 2(y')^2 + 1 = 0$ ( b ) $yy'' + (y')^2 + 1 = 0$
( c ) $yy'' + (y')^2 - 1 = 0$ ( d ) $yy'' + 2(y')^2 + 1 = 0$ [ IIT 2000 ]

( 59 ) For $x \in R$, $\lim_{x \to \infty} \left(\frac{x - 3}{x + 2}\right)^x =$

( a ) $e$ ( b ) $e^{-1}$ ( c ) $e^{-5}$ ( d ) $e^5$ [ IIT 2000 ]

( 60 ) Consider the following statements in S and R:

S: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$

R: If a differentiable function decreases in an interval ( a, b ), then its derivative also decreases in ( a, b ).

Which of the following is true ?

( a ) Both S and R are wrong.
( b ) Both S and R are correct, but R is not the correct explanation of S.
( c ) S is correct and R is correct explanation of S.
( d ) S is correct and R is wrong. [ IIT 2000 ]

( 61 ) If the normal to the curve $y = f(x)$ at the point ( 3, 4 ) makes an angle $\frac{3\pi}{4}$ with the positive X-axis, then $f'(3) =$

( a ) - 1 ( b ) - 3 / 4 ( c ) 4 / 3 ( d ) 1 [ IIT 2000 ]

( 62 ) If $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$, then at x = 0, f has

( a ) a local maximum ( b ) no local maximum
( c ) a local minimum ( d ) no extremum [ IIT 2000 ]

( 63 ) For all $x \in (0, 1)$, which of the following is true ?

( a ) $e^x < 1 + x$ ( b ) $\log_e(1 + x) < x$
( c ) $\sin x > x$ ( d ) $\log_e x > x$ [ IIT 2000 ]

( 64 ) The function $f(x) = \sin^4 x + \cos^4 x$ increases if

( a ) $0 < x < \frac{\pi}{8}$ ( b ) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
( c ) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ ( d ) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$ [ IIT 1999 ]

( 65 ) The function $f(x) = [x]^2 - [x^2]$ where [ y ] is the greatest integer less than or equal to y, is discontinuous at

( a ) all integers ( b ) all integers except 0 and 1
( c ) all integers except 0 ( d ) all integers except 1 [ IIT 1999 ]

( 66 ) The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at

( a ) - 1 ( b ) 0 ( c ) 1 ( d ) 2 [ IIT 1999 ]

( 67 ) $\lim\limits_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} =$

( a ) 2 ( b ) - 2 ( c ) $\frac{1}{2}$ ( d ) $-\frac{1}{2}$ [ IIT 1999 ]

( 68 ) The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3 (t - 3)^5 \, dt$ has a local minimum at x =

( a ) 0 ( b ) 1 ( c ) 2 ( d ) 3 [ IIT 1999 ]

(69) $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$

(a) exists and is equal to $\sqrt{2}$ (b) exists and is equal to $-\sqrt{2}$
(c) does not exist because $x - 1 \to 0$
(d) does not exist because left hand limit $\neq$ right hand limit [IIT 1998]

(70) If $\int_0^x f(t)\,dt = x + \int_1^x t f(t)\,dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$ [IIT 1998]

(71) Let $h(x) = \min[x, x^2]$, for every real number x, then

(a) h is continuous for all x (b) h is differentiable for all x
(c) $h'(x) = 1$ for all $x > 1$ (d) h is not differentiable at two values of x [IIT 1998]

(72) If $h(x) = f(x) - [f(x)]^2$ for every real number x, then

(a) h is increasing whenever f is increasing
(b) h is increasing whenever f is decreasing
(c) h is decreasing whenever f is decreasing
(d) nothing can be said in general [IIT 1998]

(73) If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in this interval

(a) both f(x) and g(x) are increasing functions
(b) both f(x) and g(x) are decreasing functions
(c) f(x) is an increasing function
(d) g(x) is an increasing function [IIT 1997]

(74) $\lim_{n \to p} \frac{1}{n} \sum_{1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals

(a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$ (c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$ [IIT 1997]

(75) If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, (p is a constant), then $\frac{d^3}{dx^3}[f(x)]$ at x = 0 is

(a) p (b) $p + p^2$ (c) $p + p^3$ (d) independent of p [IIT 1997]

(76) The function $f(x) = [x] \cos\left[\frac{2x - 1}{2}\right]\pi$, where [ . ] denote the greatest integer function, is discontinuous at

(a) all x (b) all integer points
(c) no x (d) x which is not an integer [IIT 1995]

(77) If f(x) is defined and continuous for all x > 0 and satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and f(e) = 1, then

(a) f(x) is bounded (b) $\left(\frac{1}{x}\right) \to 0$ as $x \to 0$

(c) $x f(x) \to 1$ as $x \to 0$ (d) f(x) = log x [IIT 1995]

(78) On the interval [0, 1], the function $x^{25}(1 - x)^{75}$ attains maximum value at the point

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ [IIT 1995]

(79) The function $f(x) = |px - q| + r|x|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0, assumes its minimum value only at one point if

(a) $p \neq q$ (b) $r \neq q$ (c) $r \neq p$ (d) p = q = r [IIT 1995]

(80) The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is

(a) increasing on $[0, \infty)$ (b) decreasing on $[0, \infty)$

(c) increasing on $\left[0, \frac{\pi}{e}\right]$ and decreasing on $\left[\frac{\pi}{e}, \infty\right)$

(d) decreasing on $\left[0, \frac{\pi}{e}\right]$ and increasing on $\left[\frac{\pi}{e}, \infty\right)$ [IIT 1995]

# 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

(81) The function $f(x) = \max\{(1 - x), (1 + x), 2\}$, $x \in (-\infty, \infty)$, is

(a) continuous at all points (b) differentiable at all points
(c) differentiable at all points except at $x = 1$ and $x = -1$
(d) continuous at all points except at $x = 1$ and $x = -1$ [ IIT 1995 ]

(82) Let [ . ] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then,

(a) $\lim_{x \to 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$ (d) $f'(0) = 1$ [ IIT 1993 ]

(83) If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$, then

(a) $f(x)$ is increasing on $[-1, 2]$ (b) $f(x)$ is continuous on $[-1, 3]$
(c) $f(x)$ is maximum at $x = 2$ (d) $f'(2)$ does not exist [ IIT 1993 ]

(84) The value of $\lim_{x \to 0} \dfrac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ is

(a) 1 (b) - 1 (c) 0 (d) none of these [ IIT 1991 ]

(85) The following functions are continuous on $(0, \pi)$.

(a) $\tan x$ (b) $\int_0^{\pi} t \sin \frac{1}{t} \, dt$

(c) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2 \sin \frac{2x}{9}, & \frac{3\pi}{4} < x \le \pi \end{cases}$ (d) $\begin{cases} x \sin x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$ [ IIT 1991 ]

(86) If $f(x) = \frac{x}{2} - 1$, then, on the interval $[0, \pi]$, $\tan[f(x)]$ and

(a) $\frac{1}{f(x)}$ are both continuous (b) $\frac{1}{f(x)}$ are both discontinuous
(c) $f^{-1}(x)$ are both continuous (d) $f^{-1}(x)$ are both discontinuous [ IIT 1989 ]

( 87 ) If $y^2 = P(x)$, a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ equals

( a ) $P'''(x) + P'(x)$  ( b ) $P''(x)P'''(x)$
( c ) $P(x)P'''(x)$  ( d ) a constant  [ IIT 1988 ]

( 88 ) The function $f(x) = \begin{cases} x - 3 & x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$ is

( a ) continuous at x = 1  ( b ) differentiable at x = 1
( c ) continuous at x = 3  ( d ) differentiable at x = 3  [ IIT 1988 ]

( 89 ) The set of all points where the function $f(x) = \frac{x}{1 + |x|}$ is differentiable is

( a ) $(-\infty, \infty)$  ( b ) $(0, \infty)$  ( c ) $(-\infty, 0) \cup (0, \infty)$
( d ) $(0, \infty)$  ( e ) none of these  [ IIT 1987 ]

( 90 ) Let f and g be increasing and decreasing functions respectively from $(0, \infty)$ to $(0, \infty)$. Let $h(x) = f[g(x)]$. If $h(0) = 0$, $h(x) - h(1)$ is

( a ) always zero  ( b ) always negative  ( c ) always positive
( d ) strictly increasing  ( e ) none of these  [ IIT 1987 ]

( 91 ) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. The function $P(x)$ has

( a ) neither a maximum nor a minimum ( b ) only one maximum
( c ) only one minimum ( d ) only one maximum and only one minimum
( e ) none of these  [ IIT 1986 ]

( 92 ) The function $f(x) = 1 + |\sin x|$ is

( a ) continuous nowhere  ( b ) continuous everywhere  ( c ) differentiable
( d ) not differentiable at x = 0  ( e ) not differentiable at infinite number of points
[ IIT 1986 ]

( 93 ) Let [ x ] denote the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then f ( x ) is
( a ) continuous at x = 0  ( b ) continuous in ( -1, 0 )  ( c ) differentiable at x = 1
( d ) differentiable in ( - 1, 1 )  ( e ) none of these  [ IIT 1986 ]

( 94 ) If $f(x) = \dfrac{\sin[x]}{[x]}$, $[x] \neq 0$

$= 0$, $[x] = 0$,

where x ] denotes the greatest integer less than or equal to x, then $\lim\limits_{x \to 0} f(x)$ equals

( a ) 1 ( b ) 0 ( c ) - 1 ( d ) none of these [ IIT 1985 ]

( 95 ) If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

( a ) f ( x ) is continuous but not differentiable at x = 0
( b ) f ( x ) is differentiable at x = 0
( c ) f ( x ) is not differentiable at x = 0 ( d ) none of these [ IIT 1985 ]

( 96 ) $\lim\limits_{n \to \infty} \left\{ \dfrac{1}{1 - n^2} + \dfrac{2}{1 - n^2} + \ldots + \dfrac{n}{1 - n^2} \right\}$ is equal to

( a ) 0 ( b ) $-\dfrac{1}{2}$ ( c ) $\dfrac{1}{2}$ ( d ) none of these [ IIT 1984 ]

( 97 ) If $x + |y| = 2y$, then y as a function of x is

( a ) defined for all real x ( b ) continuous at x = 0
( c ) differentiable for all real x ( d ) such that $\dfrac{dy}{dx} = \dfrac{1}{3}$ for $x < 0$ [ IIT 1984 ]

( 98 ) If $G(x) = -\sqrt{25 - x^2}$, then $\lim\limits_{x \to 1} \dfrac{G(x) - G(1)}{x - 1}$ has the value

( a ) $\dfrac{1}{24}$ ( b ) $\dfrac{1}{5}$ ( c ) $-\sqrt{24}$ ( d ) none of these [ IIT 1983 ]

( 99 ) If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$, $g'(a) = 2$, then the value of

$\lim\limits_{x \to a} \dfrac{g(x)f(a) - g(a)f(x)}{x - a}$ is

( a ) - 5 ( b ) $\dfrac{1}{5}$ ( c ) 5 ( d ) none of these [ IIT 1983 ]

( 100 ) The function $f(x) = \dfrac{\ln(1 + ax) - \ln(1 - bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0, so that it is continuous at x = 0, is

( a ) a - b ( b ) a + b ( c ) ln a + ln b ( d ) none of these [ IIT 1983 ]

## 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

( 101 ) The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point '$\theta$' is such that

( a ) it makes a constant angle with the X-axis ( b ) it passes through the origin
( c ) it is at a constant distance from the origin ( d ) none of these

[ IIT 1983 ]

( 102 ) If $y = a\ln x + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$ then

( a ) $a = 2, b = -1$ ( b ) $a = 2, b = -\frac{1}{2}$

( c ) $a = -2, b = \frac{1}{2}$ ( d ) none of these

[ IIT 1983 ]

( 103 ) There exists a function $f(x)$ satisfying $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ for all x and

( a ) $f''(x) > 0$ for all x ( b ) $-1 < f''(x) < 0$ for all x
( c ) $-2 \le f''(x) \le -1$ for all x ( d ) $f''(x) < -2$ for all x

[ IIT 1982 ]

( 104 ) For a real number y, let [ y ] denote the greatest integer less than or equal to y. Then the function $f(x) = \dfrac{\tan[\pi(x - \pi)]}{1 + [x]^2}$ is

( a ) discontinuous at some x
( b ) continuous at all x, but the derivative $f''(x)$ does not exist for some x
( c ) $f'(x)$ exists for all x, but the derivative $f''(x)$ does not exist for some x
( d ) $f''(x)$ exists for all x

[ IIT 1981 ]

( 105 ) If $f(x) = \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$, then $\lim_{x \to \infty} f(x)$ is

( a ) 0 ( b ) $\infty$ ( c ) 1 ( d ) none of these

[ IIT 1979 ]

# 08 - DIFFERENTIAL CALCULUS

( Answers at the end of all questions )

## Answers

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| c | d | c | a | c | a | b | b | d | b | b | c | c | d | b | a | a | a | c | c |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| b | d | c | d | c | c | d | b | d | a | a | d | d | d | d | a | a | c | a | b |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 5 | 5 | 57 | 58 | 59 | 60 |
| b | d | b | a | c | d | a | d | c | c | d | d | a | a | b | a | d | b | c | d |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| d | d | b | b | b | d | c | b,d | d | a | a,c,d | a c | c | | d | b | d | b | c | b |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 |
| a,c | b | a,b,c,d | d | b,c | b | c | a,b,c | a | a | c | b,d,e | a,d | d | a | b |

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 97 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 |
| a,b,d | d | c | b | c | b | a | d | c | | | | | | | | |