Answers

PHYSICS

1. (d)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (a)	**10.** (d)
11. (a)	**12.** (b)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (a)	**20.** (a)
21. (d)	**22.** (b)	**23.** (c)	**24.** (d)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (c)
31. (c)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (c)

CHEMISTRY

1. (b)	**2.** (a)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (d)	**9.** (b)	**10.** (c)
11. (d)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (c)	**20.** (b)
21. (d)	**22.** (a)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (c)	**30.** (a)
31. (a)	**32.** (d)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (b)	**40.** (c)

MATHEMATICS

1. (c)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (d)
11. (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (a)	**16.** (d)	**17.** (d)	**18.** (c)	**19.** (b)	**20.** (b)
21. (b)	**22.** (b)	**23.** (b)	**24.** (d)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (a)	**30.** (b)
31. (b)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (c)	**36.** (b)	**37.** (c)	**38.** (a)	**39.** (a)	**40.** (d)

1. When two waves of same frequency, same wavelength and same velocity moves in the same direction. Their superposition results in the interference. The two beams are monochromatic.

2. Width of the central maximum,

$$\beta_0 = \frac{2D\lambda}{a}$$

$$\beta_0 \propto \frac{1}{a}$$

$\therefore$ To increase the width of the central maximum one should decrease a.

3. $n_1\lambda_1 = n_2\lambda_2$

$\therefore \quad n_1 \times 420 = n_2 \times 630$

or $\quad 2n_1 = 3n_2$

If $\quad n_2 = 2$, then $n_1 = 3$

Therefore, thickness of soap solution is given by

$$\mu_1 t = n_1 \frac{\lambda_1}{2}$$

or $$t = \frac{3 \times 420}{1.4 \times 2} = 450 \text{ nm}$$

4. Since frequency remains unchanged

$$\nu = \nu'$$

$$\frac{v}{\lambda} = \frac{v'}{\lambda'}$$

$$\frac{v}{\lambda} = \frac{2v}{\lambda'}$$

$$\lambda' = \frac{2v}{v}\lambda$$

$$\lambda' = 2\lambda$$

Hence, its wavelength will become twice.

5. $KE_{max} = h\nu - \phi$

where $h\nu$ = energy of incident photon,

ϕ = work function

$$KE_{max} = 6.6\times10^{-34} \times 6\times10^{14} - 2\times1.6\times10^{-19}$$

$$= 3.96\times10^{-19} - 3.2\times10^{-19}$$

$$= \frac{0.76\times10^{-19}}{1.6\times10^{-19}} \text{ eV}$$

$$= 0.475 \text{ eV}$$

6. $$\lambda = \frac{h}{\sqrt{2eVm}}$$

$$5\text{Å} = \frac{6.6\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times9.1\times10^{-31}\times V}}$$

$$5\text{Å} = \frac{6.6\times10^{-34}}{5.4\times10^{-25}\sqrt{V}}$$

$$V = \left(\frac{6.6\times10^{-34}}{5.4\times10^{-25}\times5\times10^{-10}}\right)^2$$

$$V = 5.76 \text{ volt}$$

7. Electron diffraction is the diffraction of a beam of electrons by atoms or molecules. The fact that electrons can be diffracted in a similar way to light shows the particles can act as waves.

8. After n half-lives the quantity of a radioactive substance left intact (undecayed) is given by

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$= N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

Here, $\quad N = \frac{1}{16}N_0, t = 2\text{h}$

$$\frac{1}{16}N_0 = N_0\left(\frac{1}{2}\right)^{2/T_{1/2}}$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^{2/T_{1/2}}$$

Equating the powers on both sides

$$4 = \frac{2}{T_{1/2}}$$

$$T_{1/2} = \frac{1}{2}\text{h} = 30 \text{ min}$$

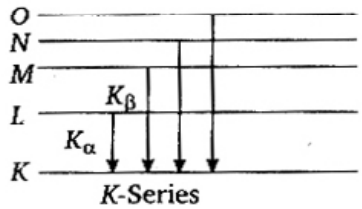
9. Observers in different inertial frames always measure different time intervals between a pair of events.

According to time dilation

$$T_A > T_B$$

10. Alpha particle has mass number 4 and atomic number 2. Therefore, after emission of an alpha particle. The mass number of the ^{234}U will reduce by 4 and after decay it becomes ^{230}Ra.

11. When the colliding electron remove an electron from innermost K-shell (corresponding to $n = 1$) of atom and electron from some higher shell jumps to K-shell to fill up this vacancy, characteristic X-ray of K-series are obtained



K-Series

$\therefore K_\alpha$ and K_β *X*-rays are emitted when there is transition of electron between the levels $n = 2$ to $n = 1$ and $n = 3$ to $n = 1$ respectively.

12. Let there be x α-particles and y β-particles

$$_ZX^A \longrightarrow x\text{He}_2^4 + y\beta_{-1}^0 + Y_{Z-3}^{A-8}$$

then equating the mass numbers

$$A = 4x + A - 8 \qquad \ldots(i)$$

and equating atomic numbers

$$Z = 2x - y + Z - 3 \qquad \ldots(ii)$$

Solving Eqs. (i) and (ii), we get

$$x = 2 \text{ and } y = 1$$

$\therefore$ The number of α and β particles emitted are 2 and 1 respectively.

13. Voltage gain,

$$\frac{V_o}{V_i} = -\frac{R_f}{R_i} = -\frac{10}{1} = -10$$

or $V_o = -10V_i = -10 \times 1 = -10$ volt

14. In *n*-type semiconductor donor energy level lies just below the conduction band also called empty band of minimum energy.

15. Constant potential at the junction

$$V_{\text{constant}} = \frac{kT}{e} \ln\left(\frac{n_a n_d}{n_i^2}\right)$$

$$\therefore V_{\text{constant}} = \frac{kT}{e} \ln\left(\frac{10^{17} \times 10^{16}}{(1.4 \times 10^{10})^2}\right)$$

$$= \frac{kT}{e} \ln (4 \times 10^{12})$$

16. Reverse biased potential for the zener breakdown

$$V_r = Ed$$
$$= 10^6 \times 2.5 \times 10^{-6}$$
$$= 2.5 \text{ volt}$$

17. In Colpitt oscillator two capacitors are placed across a common inductor and the centre of the two capacitors is tapped.

18. The reverse saturation of *p*-*n* diode depends on the doping concentrations, diffusion length and device temperature.

19. For efficient radiation and reception, the height of to be the transmitting and receiving antennas should be comparable to a quarter

Therefore FM has shorter antenna and AM has longer antenna.

20. The communication using optical fibres is based on the principle of total internal reflection.

21. If the charge of an electron is taken as elementary unit *ie*, quanta of charge, the charge on any body will be some integral multiple of *e ie*, $q = ne$ with $n = 1, 2, 3\ldots$

22. Energy stored in the capacitor in Fig (a)

$$\frac{1}{2}\frac{Q^2}{C} = 4.5 \times 10^{-6} \text{ J}$$

If battery in Fig. (a) is replaced by capacitor in Fig. (b),

total energy stored

$$= \frac{1}{2}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$
$$= \frac{1}{2} \times 4.5 \times 10^{-6}$$
$$= 2.25 \times 10^{-6} \text{ J}$$

23. Resistance, $R = \rho\frac{l}{A}$

$$R \propto l \propto \frac{1}{A}$$

$\therefore R$ is maximum when length $= 2L$ and area $= \frac{A}{2}$.

24. Force exerted by an electric dipole on a charge is inversely proportional to the cube of distance of the charge from the centre of the dipole

$$F \propto \frac{1}{r^3}$$

$$\therefore \quad \frac{F}{F'} = \left(\frac{r'}{r}\right)^3$$

$$\frac{F}{F'} = \left(\frac{2}{1}\right)^3$$

$$F' = \frac{F}{8}$$

25. For solid sphere of radius R_1

$$q_1 = \int_0^{R_1} 4\pi r^2 dr\rho$$

$$= \int_0^{R_1} 4\pi r^2 dr \frac{\rho_0}{r}$$

$$q_1 = 4\pi\frac{R_1^2}{2}\rho_0$$

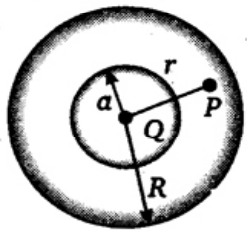
$$q_2 = -4\pi R_2^2\sigma$$

$$q_1 + q_2 = 0$$

$$\left(\frac{R_1}{R_2}\right)^2 = \frac{2\sigma}{\rho_0}$$

$$\frac{R_2}{R_1} = \sqrt{\frac{\rho_0}{2\sigma}}$$

26. The solid conductor with a cavity is shown in figure.



The inner surface of cavity will be $-Q$ due to induction. At outer surface, the charge will be $+Q$ and at a point between P at a position $r(a < r < R)$ will be zero.

27. Energy of a charged capacitor, $E = \frac{1}{2}\frac{Q^2}{C}$

$$C = \frac{2\pi\varepsilon_0 L}{\log_e\left(\frac{b}{a}\right)}$$

$$E = \frac{1}{2}\frac{Q^2}{2\pi\varepsilon_0 L}\log_e\left(\frac{b}{a}\right) \quad \ldots(i)$$

for a cylindrical capacitor.
where L = length of the cylinders
a and b = radii of two concentric cylinders

$$C' = \frac{2\pi\varepsilon_0(2L)}{\log_e\left(\frac{b}{a}\right)}$$

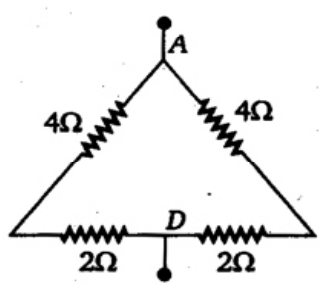
$$E' = \frac{1}{2}\frac{(2Q)^2}{C'}$$

$$= \frac{1}{2}\frac{(2Q)^2}{2\pi\varepsilon_0(2L)}\log_e\left(\frac{b}{a}\right) \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$E' = 2E$$

28. The equivalent circuit is given by



Then 6Ω and 6Ω resistances are in parallel on both sides

$$\frac{1}{R} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

$$= \frac{1}{3}$$

$$R = 3\Omega$$

29. The resistance of a metal increases with increasing temperature this is because, with increase in temperature the ions of the conductor vibrate with greater amplitude and the collision between ions and electrons becomes more frequent.

30. At room temperature, the free electrons in a conductor move randomly with speed of the order of 10^5 m/s. Since, the motion of the electrons is random there is no net charge flow in any direction.

31. In a meter bridge the ratio of two resistances is

$$\frac{R}{R'} = \frac{l}{l'}$$

where l and l' are balancing lengths

Resistance $R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$

If material remains same $\rho = \rho'$

Given $l' = 2l$

$$r' = \frac{r}{2}$$

$$\therefore \quad R' = \frac{\rho l'}{A'}$$

$$= \frac{\rho 2l}{\pi\left(\frac{r}{2}\right)^2}$$

$$= \frac{8\rho l}{\pi r^2}$$

$$R' = 8R$$

Therefore, the new balancing point is expected to be $8\,l$.

32. Supercurrent always flows on the surface of the superconductor.

33. Torque, $\tau = p \times E$

$$\tau_{max} = pE$$

$$= 6 \times 10^{-30} \times 3 \times 10^4$$

$$= 18 \times 10^{-26}\ \text{Nm}$$

34. When a changing magnetic flux is applied to a bulk piece of conducting material then circulating current is called eddy currents are induced in material.

35. The electrical signal is carried by

switching on, an electrical appliance responds almost immediately.

36. At resonance both bulbs will glow with same brightness. At resonance, $X_L = X_C$

or $$2\pi fL = \frac{1}{2\pi fC}$$

or $$f = \frac{1}{2\pi\sqrt{LC}}$$

37. In transformer

$$\frac{n_p}{n_s} = \frac{V_p}{V_s}$$

$$= \frac{5000}{240} = 20.8$$

38. As inductance L_2 was wound using the similar wire but the direction of winding is reversed, so flux through L_2 is zero.

$\therefore$ $L_2 \propto \phi = 0$

Also, $L_1 = L_3$

Therefore, $L_1 = L_3, L_2 = 0$

39. $\mu = \frac{c}{v}$

$\therefore$ $$\frac{\mu}{\mu'} = \frac{v'}{v}$$

$$\frac{1.5}{1.8} = \frac{v'}{2\times10^8}$$

$$v' = \frac{3\times10^8}{1.8}$$

$$= 1.67\times10^8 \text{ m/s}$$

40. Angular fringe width is the ratio of fringe width to distance (D) of screen from the source *ie*,

$$\theta = \frac{\beta}{D}$$

As D is taken large, hence angular fringe width of the central maximum will decrease.

Chemistry

1. Under certain conditions, alkanes react with HNO_3, a hydrogen atom being replaced by a nitro group (NO_2). This process is known as nitration. Nitration of alkane may be carried out in the vapour phase between 150° to 475°C. Whereupon a mixture of mono nitro alkanes is obtained

Example Ethane give a mixture of nitroethane and nitromethane

$$\underset{\text{ethane}}{CH_3-CH_3} + HNO_3 \xrightarrow[-H_2O]{673\text{ K}}$$

$$\underset{\substack{\text{nitro ethane}\\\text{(major)}}}{CH_3-CH_2-NO_2} + \underset{\text{(minor)}}{CH_3NO_2}$$

During nitration chain fission of alkanes also takes place, so CH_3NO_2 is also obtained along with $CH_3CH_2NO_2$

2. In presence of dil HCl, acetamide is hydrolysed by boiling, the product obtained is acetic acid (CH_3COOH).

$$CH_3CONH_2 + H_2O \longrightarrow CH_3COOH + NH_3$$

$$CH_3CONH_2 + H_2O + HCl \rightarrow CH_3COOH + NH_4Cl$$

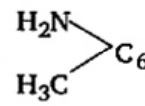
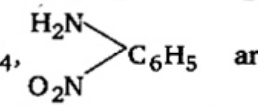
3. Only 1° aromatic amine (primary aromatic amine) form diazonium salts at low temperature (0° – 5°C). A reaction in which $-NH_2$ group is converted into diazo group ($-\overset{+}{N}\equiv N$) is called diazotisation. Diazotised

$$C_6H_5NH_2 + HCl \xrightarrow{0°-5°C} C_6H_5\overset{+}{N}H_3\,Cl^-$$

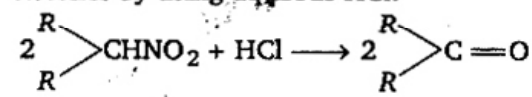
$$NaNO_2 + HCl \xrightarrow{0°-5°C} HNO_2 + NaCl$$

$$C_6H_5\overset{+}{N}H_3\,Cl^- + HNO_2 \xrightarrow{0°-5°C}$$

$$C_6H_5\overset{+}{N}_2\,Cl^- + 2H_2O$$

$C_6H_5NH_2$,   are aromatic primary amines, so undergo diazotisation but $C_6H_5CH_2NH_2$ (aliphatic amine) will not undergo diazotisation.

4. Secondary nitroalkanes can be converted into ketones by using aqueous HCl.



$$+ N_2O + H_2O$$

5. An alkyl cyanide is dissolved in ether or better in ethyl formate or ethyl acetate, and reduced with $SnCl_2$ and HCl, and then steam distilled. This whole process is called Stephen reaction. In this process alkyl cyanide is reduced to aldehyde.

$$CH_3-C\equiv N + 2[H] + HCl \xrightarrow[\text{ether}]{SnCl_2 + HCl}$$

$$CH_3-CH(=O:H_2)=NH\cdot HCl \xrightarrow[\Delta]{H_2O} CH_3CHO + NH_4Cl$$

There is no analogous method for the preparation of ketones.

6. The continuous phase contain the dispersed phase throughout.
   Example is water droplet in mist.

7. Mole of sucrose $= \dfrac{\text{mass of sucrose (in gram)}}{\text{molecular weight of sucrose}}$

$$= \frac{25.6}{342.3} = 0.0747882$$

Formula of sucrose $= C_{12}H_{22}O_{11}$
Number of H atoms in one mole of sucrose = 22
Number of H atoms in 25.6 g of sucrose
$= 22 \times 0.074788 \times 6.023 \times 10^{23}$
$= 9.9 \times 10^{23}$

8. Milk change after digestion into lactose.

9. Essential amino acids(10) are as follows—
   (i) Arginine
   (ii) Histidine
   (iii) Isoleucine
   (iv) Leucine
   (v) Lysine
   (vi) Methionine
   (vii) Phenylalanine
   (viii) Threonine
   (ix) Tryptophane
   (x) Valine

10. The term hexose refers to the presence of six carbon atoms and term keto shows the presence of ketonic group. Thus, the compound which contains 6 C atoms and one $>C=O$ group is called ketohexose. Among the given only glucose and fructose are six C compounds. Out of them, glucose contains an aldehyde group while fructose contains a ketonic group. Hence, the example of ketohexose is fructose.

11. **KO_3**

Suppose O.N. of O = x
$+1 + 3x = 0$
$3x = -1$
$x = -\frac{1}{3}$
$\boxed{x = -0.33}$

Na_2O_2

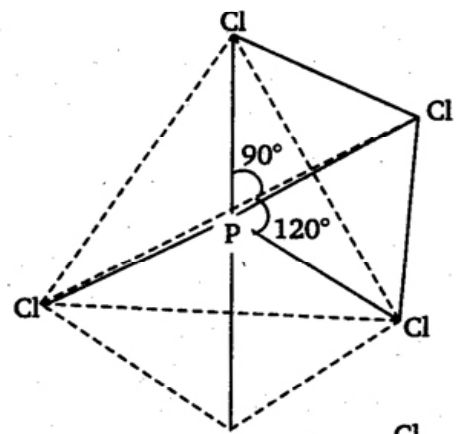
Suppose O.N. of O = x
$2 \times 1 + 2x = 0$
$2 + 2x = 0$
$2x = -2$
$x = -\frac{2}{2}$

12. When phosphorus trichloride reacts with phenyl magnesium bromide (Grignard's reagent), all the three chlorine atoms of PCl_3 are replaced by phenyl group of phenyl magnesium bromide and triphenyl phosphine is obtained.

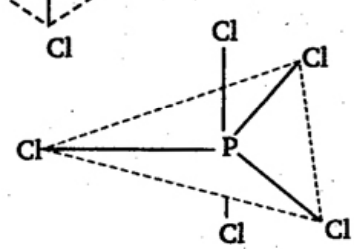
$$PCl_3 + 3PhMgBr \longrightarrow \underset{\text{triphenyl phosphine}}{P(Ph)_3} + 3Mg\begin{matrix}Cl\\Br\end{matrix}$$

13. Natural radioactivity is not a characteristic of transition elements.
   General properties of transition elements are
   (i) Formation of coloured salts
   (ii) Formation of complex salts
   (iii) Magnetic properties
   (iv) Formation of interstitial compounds
   (v) Formation of alloys etc.

14. Cl — P — Cl bond angles in PCl_5 molecule are 120° and 90°. PCl_5, having sp^3d hybridised P atom (trigonal bipyramidal geometry) has two types of bonds; axial and equatorial. These two types of bond have different bond lengths 1, 2, 3 – equatorial bonds and 4, 5 axial bonds



or



15. Magnetic moment of Zn^{2+} $\mu_{\text{effective}} = \sqrt{n(n+2)}$ BM
   where, n = Number of unpaired electrons
   $_{30}Zn = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6 3d^{10}, 4s^2$

$3d^{10}$

⇅	⇅	⇅	⇅	⇅

$n = 0$

So, magnetic moment of Zn^{2+} = zero

16. $CaF_2 = 146.4$ g

Molecular weight of $CaF_2 = 78.08$ g/mol

$$\text{Moles of } CaF_2 = \frac{\text{wt.}}{\text{mo. wt.}}$$

$$= \frac{146.4}{78.08} = 1.875 \text{ mol}$$

Number of CaF_2 atoms in 146.4 g of CaF_2 = No. of moles $\times 6.022 \times 10^{23}$

$= 1.875 \times 6.022 \times 10^{23}$

$= 11.29 \times 10^{23}$

$= 1.129 \times 10^{24}$ CaF_2

17. The IUPAC name of the compound $[Co(NH_3)_5Cl]Cl_2$ is pentaammine chloro cobalt (III) chloride

18. On adding SCN^- to an aqueous solution of $Fe(NO_3)_3$, a blood red colour, due to the formation of $[Fe(H_2O)_5(SCN)]^{2+}$ complex, is obtained. This test is used for the detection of Fe^{3+} ion.

$$SCN^- + Fe(NO_3)_3 + H_2O \longrightarrow \underset{\text{blood red colour}}{[Fe(OH_2)_5(SCN)]^{2+}} + NO_3^-$$

19. Silver nitrate is used in making hair dyes because it reduced to metallic silver and finely divided silver is black in colour.

20. When equal number of cations or anions are missing from their lattice sites (to maintain electrical neutrality), then the defect is called Schottky defect. The defect is observe in highly ionic compounds which have cations and anions of similar size *eg,* NaCl, KCl etc.

21. In both $[Co(NH_3)_6]^{3+}$ and $[CoF_6]^{3-}$, Co is present as Co^{3+}. Thus, the electronic configuration of Co is

$_{27}Co = [Ar]\ 3d^7, 4s^2$

$_{27}Co^{3+} = [Ar]\ 3d^6, 4s^0$

In case of $[Co(NH_3)_6]^{3+}$, NH_3 is a strong field ligand, so pairing of electrons in 3*d*-orbital takes place.

$_{27}Co^{3+} = [Ar]\ 3d^6, 4s^0$

$3d^6$

⇅	⇅	⇅		

(no unpaired electron)

In $[CoF_6]^{3-}$, F is a weak field ligand, thus does not cause pairing. Hence,

$_{27}Co^{3+} = [Ar]\ 3d^6, 4s^0$

$3d^6$

⇅	1	1	1	1

4 unpaired electrons

22. $\Delta G^\circ = -115 \times 10^3$ J,

$T = 298$ K, $R = 8.314$ $JK^{-1}mol^{-1}$

$-\Delta G^\circ = 2.303RT \log_{10} K_p$

$-(-115 \times 10^3) = 2.303 \times 8.314 \times 298 \log_{10} K_p$

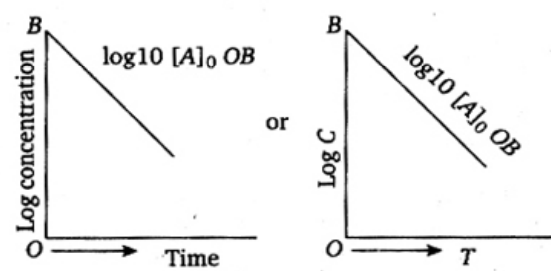
$$\log_{10} K_p = \frac{115000}{2.303 \times 8.314 \times 298}$$

$\log_{10} K_p = 20.16$

23. For endothermic reaction ΔH is positive *ie,* $\Delta H > 0$.

For a spontaneous process ΔG is always Negative and $\Delta G = \Delta H - T\Delta S$ thus, to maintain the value of ΔG negative, ΔS must be positive *ie,* $\Delta S > 0$.

24. A graph between the log concentration (log C) of reactent and time t for the first order, reaction gives a straight line, whose slope is equal to $-\frac{k}{2.303}$



$$\log_{10} C_A = -\frac{kt}{2.303} + \log_{10}(C_A)_0$$

Hence, the order of the above reaction is one.

25. A spontaneous process is one in which the system suffers a lowering of free energy because ΔG = – ve, for a spontaneous reaction

26. $t_{1/2} = 100$ s

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{100}$$

$k = 6.93 \times 10^{-3}$ s^{-1}

27. $NaCl + KNO_3 \longrightarrow NaNO_3 + KCl$

	KCl — NaCl
Molar conductivities	152 128

difference = 24 S cm^2mol^{-1}

Molar conductivities of KNO_3 = 111

Molar conductivities of $NaNO_3$
$= 111 - 24 = 87\ S\ cm^2 mol^{-1}$

28. The electrochemical cell stops working after sometime because electrode potential of both the electrodes become equal.

29. $CuSO_4 \rightleftharpoons Cu^{2+} + SO_4^{2-}$

At cathode : $Cu^{2+}(aq) + 2e^- \longrightarrow Cu(s)$

At anode :

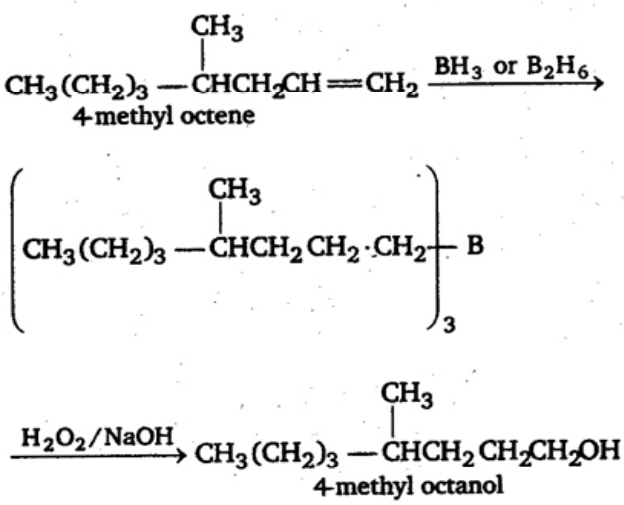
$2H_2O(l) \longrightarrow O_2(g) + 4H^+(aq) + 4e^-$

Thus, for the production of one mole of copper from copper sulphate 2F of electricity is required (because involve 2 electrons).

30. In electrochemical series, iron is placed below sodium, so it cannot displace sodium from its salt solution. Hence, no reaction takes place.

$Fe + Na_3PO_4 \longrightarrow$ No reaction

31. Terminal alkenes react rapidly with diborane to form primary trialkyl boranes which on oxidation gives primary alcohols.



(In general hydroboration oxidation involve the addition of water according to anti-Markownikoff's rule).

32. When ethyl alcohol is heated with conc H_2SO_4 at 160° – 170°C, the product obtained is ethylene (C_2H_4)

$CH_3 - CH_2OH + H_2SO_4 \longrightarrow CH_3CH_2HSO_4 + H_2O$

ethyl hydrogen sulphate

$CH_3 - CH_2HSO_4 \xrightarrow[\Delta]{160-170°C} CH_2 = CH_2 + H_2SO_4$

33. In presence of NaOH or KOH, phenol reacts with alkyl halide and gives phenolic ether (C_6H_5OR).

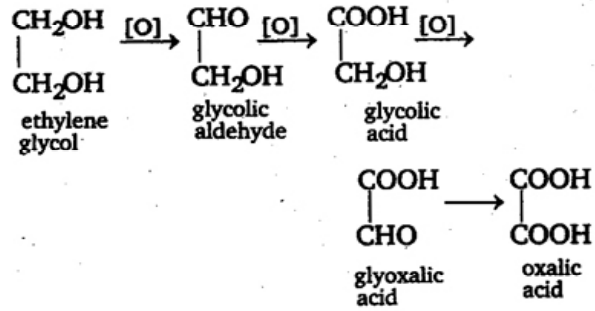
$C_6H_5OH + NaOH \xrightarrow[-H_2O]{} C_6H_5O - Na \xrightarrow[-NaX]{RX} C_6H_5 - O - R$

Vapours of C_6H_5OH and CH_3OH, with red hot ThO_2 (thoria) give anisole (phenolic ether).

$C_6H_5OH + CH_3OH \xrightarrow[\Delta]{ThO_2} C_6H_5OCH_3 + H_2O$

anisole

34. In presence of HNO_3 or alkaline $KMnO_4$



35. In diamond each carbon atom is linked to four other carbon atoms by sigma bond. Each σ C — C bond is formed by the overlapping of sp^3 hybrid orbitals of each carbon atoms. Each carbon atom is present at the centre of a regular tetrahedron. Each carbon atom is surrounded by four other carbon atoms present at the corners of a regular tetrahedron. The octet of each carbon atom is complete.

Structure of diamond is a rigid three dimensional network. This explain high density and hardness of diamond. Diamond is chemically inactive due to rigid three dimensional structure.

High energy is required to break the large number of σ C — C bonds present in diamond. This explain the high melting point of diamond.

tetrahedral structure of diamond

36. This is also a mean of preparing alkene where the position of the double bond is definite. In Wittig reaction, aldehyde (—CHO) and ketone ($>C=O$) react with methylene triphenyl phosphine [$(C_6H_5)_3P=CH_2$] to give alkene.

$$CH_3CHO + (C_6H_5)_3P=CH_2 \longrightarrow \underset{\text{propene}}{CH_3-CH=CH_2} + \underset{\text{triphenyl phosphine oxide}}{(C_6H_5)_3P=O}$$

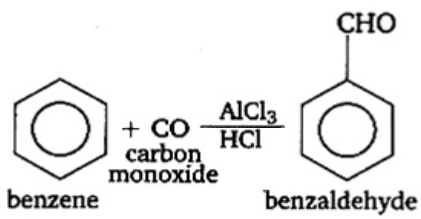
$$\begin{matrix} H_3C \\ H_3C \end{matrix}\!\!>C=O + (C_6H_5)_3P=CH_2 \longrightarrow$$

$$\begin{matrix} H_3C \\ H_3C \end{matrix}\!\!>C=CH_2 + (C_6H_5)_3P=O$$

37. Aldehyde, having no α-hydrogen atom, undergo Cannizaro reaction in which two molecules of the aldehyde are involved, one molecule being converted into the corresponding alcohol, and the other into the acid. The usual reagent for the Cannizaro reaction is aqueous or ethanolic alkali

$$2HCHO + NaOH \longrightarrow HCOONa + CH_3OH$$

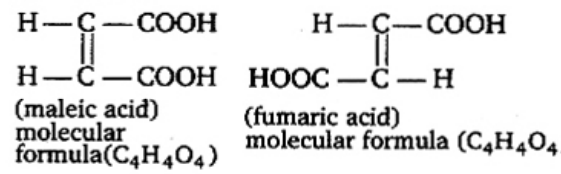
38.



This reaction is known as Gattermann-Koch reaction. Benzaldehyde may be synthesised by bubbling a mixture of CO and HCl through a solution of benzene, and a catalyst consisting of $AlCl_3$ and a small amount of cuprous chloride

$$C_6H_6 + CO + HCl \xrightarrow{AlCl_3} C_6H_5CHO + HCl$$

39. The structures of maleic and fumaric acids are given below



The structures of fumaric and maleic acid suggest that they are geometrical isomers because they have same molecular formula but different spatial arrangement of atoms around a double bond.

When the similar groups are present on same side of double bond, the isomer is called *cis* and when different groups are present on same side of double bond, then the isomer is called *trans*.

Hence, maleic acid is *cis* form and fumaric acid is *trans* form.

40. Generally soda-lime removes CO_2 from an acid but in case of alkali formate it gives alkali carbonate and hydrogen.

$$HCOONa + NaOH \xrightarrow{CaO} Na_2CO_3 + H_2$$

Mathematics

1. Since, $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$

$$\Rightarrow \quad (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{2}\vec{b}$$

On comparing both sides, we get

$\vec{a}\cdot\vec{c} = \frac{1}{2}$ and $\vec{a}\cdot\vec{b} = 0$

Now, $\vec{a}\cdot\vec{c} = \frac{1}{2}$

$$\Rightarrow \quad \vec{a}\cdot\vec{c}\cos\theta_2 = \frac{1}{2}$$

$\Rightarrow \cos\theta_2 = \frac{1}{2}$ ($\because \vec{a}$ and $\vec{c}$ are unit vectors)

$$\Rightarrow \quad \cos\theta_2 = \cos\frac{\pi}{3}$$

$$\Rightarrow \quad \theta_2 = \frac{\pi}{3}$$

and $\vec{a}\cdot\vec{b} = 0$

$$\Rightarrow \quad |\vec{a}||\vec{b}|\cos\theta_1 = 0$$

$\Rightarrow \cos\theta_1 = \cos\frac{\pi}{2}$ ($\because \vec{a}$ and $\vec{b}$ are unit vectors)

$$\Rightarrow \quad \theta_1 = \frac{\pi}{2}$$

Hence, $\theta_1 = \frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{3}$.

2. The given equation is

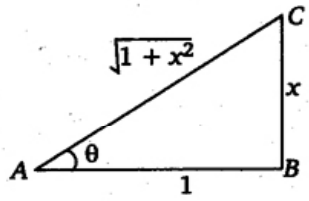
$\vec{r}^2 - 2\vec{r}\cdot\vec{c} + h = 0$, $|\vec{c}| > \sqrt{h}$

This is a equation of sphere in diameter form

ie, $(\vec{r} - \vec{a})\cdot(\vec{r} - \vec{b}) = 0$

3. Let $\tan^{-1} x = \theta$

$\therefore \sin(\tan^{-1} x) = \sin\theta$



$= \frac{BC}{AC} = \frac{x}{\sqrt{1+x^2}}$

4. Let $z = x + iy$

$\therefore \left|\frac{z-25}{z-1}\right| = 5$

$\Rightarrow \quad \left|\frac{(x-25)+iy}{(x-1)+iy}\right| = 5$

$\Rightarrow \quad |(x-25)+iy| = 5|(x-1)+iy|$

$\Rightarrow \quad \sqrt{(x-25)^2+y^2} = 5\sqrt{(x-1)^2+y^2}$

On squaring both sides, we get

$(x-25)^2 + y^2 = 25\{(x-1)^2 + y^2\}$

$\Rightarrow x^2 - 50x + 625 + y^2 = 25x^2 - 50x + 25 + 25y^2$

$\Rightarrow 24x^2 + 24y^2 = 600$

$\Rightarrow x^2 + y^2 = 25$

$\Rightarrow \sqrt{x^2+y^2} = 5 \quad [\because |z| = \sqrt{(x^2+y^2)}]$

$\Rightarrow |z| = 5$

5. $\left(\frac{-1-3i}{2+i}\right) = \frac{-1-3i}{2+i} \times \frac{2-i}{2-i}$

$= \frac{-2+i-6i+3i^2}{4+1}$

$= \frac{-2-5i-3}{5} = \frac{-5-5i}{5} = -1-i$

$\therefore$ Argument of $\left(\frac{-1-3i}{2+i}\right) = \tan^{-1}\left(\frac{-1}{-1}\right)$

$= 225°$

[Since the given number lies on a IIIrd quadrant]

6. The given equation is

$x^2 - 61x + 820 = 0$

$\Rightarrow x^2 - 41x - 20x + 820 = 0$

$\Rightarrow (x-41)(x-20) = 0$

$\Rightarrow x = 41, 20$

Let $b = 41$ and $c = 20$

Also, $A = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore$ By cosine formula,

$a^2 = b^2 + c^2 - 2bc\cos A$

$= 41^2 + 20^2 - 2 \times 41 \times 20 \times \frac{3}{5}$

$= 2081 - 984 = 1097$

7. Here, $\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{a}_2 = -4\hat{i} + 0\hat{j} - \hat{k}$, $\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\therefore$ Shortest distance $= \left[\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right]$

$= \left|\frac{(-10\hat{i} - 2\hat{j} - 3\hat{k})\cdot(8\hat{i} + 8\hat{j} + 4\hat{k})}{\sqrt{64+64+16}}\right|$

$= \left|\frac{-108}{12}\right| = 9$

8. The given equation of sphere is

$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$

On comparing this equation with general equation of sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$,

we get $u = \frac{3}{2}$, $v = 0$, $w = -2$ and $d = 1$

$\therefore$ Coordinates of centre of sphere

$= (-u, -v, -w)$

$= \left(-\frac{3}{2}, 0, 2\right)$

and radius of sphere $= \sqrt{u^2 + v^2 + \omega^2 - d}$

$= \sqrt{\frac{9}{4} + 4 - 1} = \sqrt{\frac{9+12}{4}}$

$= \frac{\sqrt{21}}{2}$

9. If A and B are two fixed points in a plane, then the locus of another point C on the same plane such that $CA + CB$ = constant, $(> AB)$ is an ellipse.

10. The equation of parabola is

$y^2 + 4x + 3 = 0$

$\Rightarrow \quad y^2 = -4\left(x + \frac{3}{4}\right)$

Let $X = x + \frac{3}{4}$ and $Y = y$

$\therefore$ Equation of parabola becomes

$Y^2 = -4X$

The equation of directrix of parabola is

$X = 1 \quad (\because a = 1)$

$\Rightarrow \quad x + \frac{3}{4} = 1$

11. Since, $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$

Now, at $x = 0, y = 2$, we get

$g(0)\,g(2) = g(0) + g(2) + g(0) - 2$

$[\because g(2) = 5]$

$\Rightarrow \quad 5g(0) = 5 + 2g(0) - 2$

$\Rightarrow \quad 3g(0) = 3$

$\Rightarrow \quad g(0) = 1$

$g(x)$ is given in a polynomial and by the relation given $g(x)$ cannot be linear.

Let $g(x) = x^2 + k$

$\Rightarrow \quad g(x) = x^2 + 1 \qquad [\because g(0) = 1]$

$\therefore$ From Eq. (i),

$(x^2 + 1)(y^2 + 1) = x^2 + 1 + y^2 + 1 + x^2y^2 + 1 - 2$

$\therefore \lim_{x \to 3} g(x) = g(3) = 3^2 + 1 = 10$

12. $\because f(x) = \dfrac{-e^x + 2^x}{x}$

and $f(x)$ is continuous at $x = 0$

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad \lim_{x \to 0} \dfrac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x \to 0} \dfrac{-e^{-x} + 2^x \log 2}{1} = f(0)$

(using L'Hospital's rule)

$\Rightarrow \quad -e^0 + 2^0 \log 2 = f(0)$

$\Rightarrow \quad f(0) = -1 + \log 2$

13. Given, $f(x) = [\tan^2 x]$

$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} [\tan^2 x] = 0$

and $\quad f(0) = [\tan^2 0] = 0$

Thus, $f(x)$ is continuous at $x = 0$.

14. Let r be the radius of spherical balloon.

$\therefore$ Volume, $V = \dfrac{4}{3}\pi r^3$

$\dfrac{dV}{dt} = \dfrac{4}{3}\pi 3r^2 \dfrac{dr}{dt}$

$= 4\pi r^2 \dfrac{dr}{dt}$

$= 4\pi r^2 \cdot (2) \qquad \left(\because \dfrac{dr}{dt} = 2\right)$

$\Rightarrow \quad \dfrac{dV}{dt} = 8\,\pi r^2\ \text{cm}^3/\text{min}$

Now, when $r = 5$ cm

$\therefore \dfrac{dV}{dt} = 8\,\pi(5)^2 = 200\,\pi\ \text{cm}^3/\text{min}$

and equation of latusrectum is

$x = 3 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$y^2 = 36$

$\Rightarrow \quad y = \pm 6$

$\therefore$ Coordinates of end points of a latusrectum are (3, 6) and (3, – 6).

$\therefore$ Required length $= 2\int_0^3 \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\,dx$

$= 2\int_0^3 \sqrt{1 + \left(\dfrac{6}{y}\right)^2} = 2\int_0^3 \sqrt{\dfrac{12x + 36}{12x}}\,dx$

$= 2\int_0^3 \dfrac{x + 3}{\sqrt{x^2 + 3x}}\,dx$

$= 2\left[\sqrt{x^2 + 3x} + \dfrac{3}{2}\log\left|\left(x + \dfrac{3}{2}\right) + \sqrt{x^2 + 3x}\right|\right]_0^3$

$= 2\left[3\sqrt{2} + \dfrac{3}{2}\log\left(\dfrac{9}{2} + 3\sqrt{2}\right) - \dfrac{3}{2}\log\left(\dfrac{3}{2}\right)\right]$

$= 2[3\sqrt{2} + 3\log(\sqrt{2} + 1)]$

$= 6[\sqrt{2} + \log(1 + \sqrt{2})]$

16. Given, $\quad I = \int \dfrac{x^5}{\sqrt{1 + x^3}}\,dx$

Let $\quad 1 + x^3 = t$

$\Rightarrow \quad 3x^2\,dx = dt$

$\Rightarrow \quad x^2 dx = \dfrac{dt}{3}$

$\therefore \quad I = \int \dfrac{(t - 1)}{\sqrt{t}} \cdot \dfrac{dt}{3} = \dfrac{1}{3}\int (\sqrt{t} - t^{-1/2})\,dt$

$= \dfrac{1}{3}\left[\dfrac{2t^{3/2}}{3} - 2t^{1/2}\right] + c$

$= \dfrac{2}{9}(1 + x^3)^{3/2} - \dfrac{2}{3}(1 + x^3)^{1/2} + c$

17. The given equation can be rewritten as

$\dfrac{(x - \sqrt{2})^2}{2/\pi} + \dfrac{y^2}{8/\pi} = 1$

Which represents an ellipse.

Here, $a = \sqrt{\dfrac{2}{\pi}}$ and $b = \sqrt{\dfrac{8}{\pi}}$

Area enclosed in an ellipse $= \pi ab$

$= \pi\sqrt{\dfrac{2}{\pi}}\sqrt{\dfrac{8}{\pi}} = \sqrt{16}$

$= 4$ sq unit

18. Let $I = \int_0^a \sqrt{\dfrac{a - x}{x}}\,dx$

Put $x = a \sin^2\theta$ and $dx = 2a \sin\theta \cos\theta\,d\theta$

$$= 2a\int_0^{\pi/2} \cos^2\theta\, d\theta$$

$$= 2a \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi a}{2}$$

19. Given that, $\frac{dy}{dt} \propto y$

$$\Rightarrow \quad \frac{dy}{dt} = ky$$

$$\Rightarrow \quad \frac{1}{y}\,dy = k\,dt$$

On integrating, we get

$$\log y = \log c + kt$$

$$\Rightarrow \quad \log y - \log c = kt$$

$$\Rightarrow \quad \log \frac{y}{c} = kt$$

$$\Rightarrow \quad \frac{y}{c} = e^{kt}$$

$$\Rightarrow \quad y = ce^{kt}$$

20. The equation of straight line touching the circle $x^2 + y^2 = a^2$ is

$$x\cos\theta + y\sin\theta = a \qquad \ldots(i)$$

On differentiating w.r.t. x, regarding θ as a constant

$$\cos\theta + y'\sin\theta = 0 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\cos\theta = \frac{ay'}{xy' - y} \text{ and } \sin\theta = -\frac{a}{xy' - y}$$

$\because \cos^2\theta + \sin^2\theta = 1$

$$\therefore \quad \frac{a^2y'^2 + a^2}{(xy' - y)^2} = 1$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$$

21. The differential equation $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ admits no solution, since three positive quantities cannot add to give zero.

22. Given equation can be rewritten as

$$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} \qquad \ldots(i)$$

Which is a homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From Eq. (i),

$$v + x\frac{dv}{dx} = \frac{\sqrt{x^2 + v^2x^2} + vx}{x}$$

$$\Rightarrow \quad \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

On integrating, we get

$$\log(v + \sqrt{1 + v^2}) = \log x + \log c$$

$$\Rightarrow \quad \log\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) = \log cx$$

$$\Rightarrow \quad y + \sqrt{x^2 + y^2} = cx^2$$

23. $\because p \to q \to r \to p$ and $\sim s \to r$

s	q	r	$\sim s$	$\sim q$	r	$\sim s \to r$	$\sim s \to s$
T	T	T	T	F	T	T	T
T	F	T	T	T	T	T	T
F	T	T	F	F	T	F	F
F	F	T	F	T	T	T	T
T	T	F	T	F	F	T	T
T	F	F	T	T	F	T	T
F	T	F	F	F	F	F	F
F	F	F	F	T	F	T	T

$\Rightarrow$ Options (c) and (d) are not true also $\sim s \to r$

$\therefore$ Option (a) is not true.

Hence, option (b) is correct.

24. Required number of ways $= \frac{10!}{4! \times 3! \times 3! \times 2!}$

$$= 2100.$$

25. Since, R is defined as aRb iff $|a - b| > 0$.

For reflexive aRa iff $|a - a| > 0$

Which is not true. So R is not reflexive.

For symmetric aRb iff $|a - b| > 0$

Now, bRa iff $|b - a| > 0$

$$\Rightarrow \quad |a - b| > 0 \Rightarrow aRb$$

Thus, R is symmetric.

For transitive aRb iff $|a - b| > 0$,

bRc iff $|b - c| > 0$

$$\Rightarrow \quad |a - b + b - c| > 0$$

$$\Rightarrow \quad |a - c| > 0$$

$$\Rightarrow \quad |c - a| > 0 \Rightarrow aRc$$

Thus, R is also transitive.

26. $\because S$ be a finite set containing n elements. Then total number of commutative binary operation on S is $n^{\left[\frac{n(n+1)}{2}\right]}$.

27. Required probability distribution is poisson distribution.

28. Required probability $= {}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2$

$$= \frac{4!}{2!2!} \times \frac{9}{16} \times \frac{1}{16}$$

$$= \frac{24}{4} \times \frac{9}{16} \times \frac{1}{16} = \frac{27}{128}$$

29. $\bar{x} = \dfrac{8+12+13+15+22}{5} = \dfrac{70}{5} = 14$

x	$(x-\bar{x})$	$(x-\bar{x})^2$
8	−6	36
12	−2	4
13	−1	1
15	+1	1
22	+8	64
$\Sigma x = 70$	$\Sigma(x-\bar{x}) = 0$	$\Sigma x^2 = 106$

$$\therefore \sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{N}}$$
$$= \sqrt{\frac{106}{5}} = \sqrt{21.2}$$
$$= 4.604$$

30. For binomial distribution
$0 <$ variance $<$ mean
$\Rightarrow \quad 0 < \beta < \alpha$

31. The given system of equations are
$$x + y + z = 0,$$
$$2x + 3y + z = 0,$$
and $$x + 2y = 0$$
Here, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 1(0-2) - 1(0-1) + 1(4-3)$
$$= -2 + 1 + 1 = 0$$
$\therefore$ This system has infinite solutions.

32. $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = I$

Now, $$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$$
$$= \begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$$

and $$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^2$$
$$= \begin{bmatrix} ab & a \\ 0 & ab \end{bmatrix}\begin{bmatrix} ab & 0 \\ 0 & ab \end{bmatrix}$$
$$= \begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix}$$

But $$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(given)}$$

$\Rightarrow$ $$\begin{bmatrix} a^2b^2 & 0 \\ 0 & a^2b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$a^2b^2 = 1$$

33. If $D = \text{diag}(d_1, d_2, \ldots, d_n)$
$\therefore D^{-1} = \text{diag}(d_1^{-1}, d_2^{-1}, \ldots, d_n^{-1})$

34. $\begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$

$\Rightarrow$
$$a\begin{vmatrix} b & c-z \\ b-y & c \end{vmatrix} - (b-y)\begin{vmatrix} a-x & c-z \\ a-x & c \end{vmatrix} + (c-z)\begin{vmatrix} a-x & b \\ a-x & b-y \end{vmatrix} = 0$$
$\Rightarrow a(bc - bc + bz + cy - yz) - (b-y)$
$(ac - cx - ac + az + cx - xz) + (c-z)$
$(ab - ay - bx + xy - ab + bx) = 0$
$\Rightarrow a(bz + cy - yz) - (b-y)(az - xz) + (c-z)$
$(xy - ay) = 0$
$\Rightarrow abz + acy - ayz - abz + bxz + ayz - xyz$
$+ cxy - acy - xyz + ayz = 0$
$\Rightarrow ayz + bxz - 2xyz + cxy = 0$
$\Rightarrow ayz + bxz + cxy = 2xyz$
$\Rightarrow \dfrac{ayz}{xyz} + \dfrac{bxz}{xyz} + \dfrac{cxy}{xyz} = 2$
$\Rightarrow \dfrac{a}{x} + \dfrac{b}{y} + \dfrac{c}{z} = 2$

35. Given, $x^2 - n = 0$
$\Rightarrow \quad x = \pm\sqrt{n}$
$\therefore n = 1, 4, 9, 16, 25, 36$
$\therefore$ Required probability $= \dfrac{6}{40} = \dfrac{3}{20}$

36. $x^4 + \sqrt{x^4 + 20} = 22$
$\Rightarrow x^4 + 20 + \sqrt{x^4 + 20} = 22 + 20$
$\Rightarrow (x^4 + 20) + \sqrt{x^4 + 20} = 44$
Let $\sqrt{x^4 + 20} = y$
$\therefore y^2 + y - 44 = 0$
Hence, the number of real roots of the equation is 2.

37. $\because \alpha$ and β be the roots of $x^2 - ax + b = 0$.
$\therefore \alpha^2 - a\alpha + b = 0$ and $\beta^2 - a\beta + b = 0$
Now, $A_{n+1} - aA_n + bA_{n-1}$
$$= \alpha^{n+1} + \beta^{n+1} - a(\alpha^n + \beta^n) + b(\alpha^{n-1} + \beta^{n-1})$$
$$= \alpha^{n-1}(\alpha^2 - a\alpha + b) + \beta^{n-1}(\beta^2 - a\beta + b)$$
$$= 0$$

38. $\because b, c$ and a are in AP.
$\therefore \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\Rightarrow \quad a = \frac{b}{\sin B} = \frac{c}{\sin C} \quad (\because \angle A = 90°)$

$\Rightarrow \quad \sin B = \frac{b}{a}, \sin C = \frac{c}{a}$

Hence, option (a) satisfies this equation.

39. The cartesian form of an equation of plane is

$$x + 3y - z = 0$$

and $\quad y + 2z = 0$

The line of intersection of two planes is

$(x + 3y - z) + \lambda(y + 2z) = 0$

Since, it is passing through $(-1, -1, -1)$

$\therefore \quad (-1, -3 + 1) + \lambda(-1 - 2) = 0$

$\Rightarrow \quad -3 - 3\lambda = 0$

$\Rightarrow \quad \lambda = -1$

$\therefore (x + 3y - z) - 1(y + 2z) = 0$

$\Rightarrow \quad x + 2y - 3z = 0$

Hence, equation of plane is

$\vec{r} \cdot (\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

40. Area of triangle

$$= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 4 & 3 \end{matrix} \right\|$$

$= |-7\hat{i} - \hat{j} + 6\hat{k}| = \sqrt{49 + 1 + 36}$

$= \sqrt{86}$