Date : 04-11-2012 | Duration : 3 Hours | Max. Marks : 160

STREAM - SB/SX

GENERAL INSTRUCTIONS

- The Test Booklet consists of **120** questions.
- There are Two parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

MARKING SCHEME :

PART-I :

MATHEMATICS
Question No. **1 to 20** consist of **ONE (1)** mark for each correct response.

PHYSICS
Question No. **21 to 40** consist of **ONE (1)** mark for each correct response.

CHEMISTRY
Question No. **41 to 60** consist of **ONE (1)** mark for each correct response.

BIOLOGY
Question No. **61 to 80** consist of **ONE (1)** mark for each correct response.

PART-II :

MATHEMATICS
Question No. **81 to 90** consist of **TWO (2)** marks for each correct response.

PHYSICS
Question No. **91 to 100** consist of **TWO (2)** marks for each correct response.

CHEMISTRY
Question No. **101 to 110** consist of **TWO (2)** marks for each correct response.

BIOLOGY
Question No. **111 to 120** consist of **TWO (2)** marks for each correct response.

PART-I
One Mark Questions

MATHEMATICS

1. Three children, each accompanied by a guardian, seek admission in a school. The principal wants to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this be done?

(A) 60 (B) 90 (C) 120 (D) 180

Sol. Number ways are = $\frac{6!}{2!2!2!} = \frac{720}{8}$ = 90 ways

2. In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has

(A) no solution (B) exactly two distinct solutions
(C) exactly four distinct solutions (D) infinitely many solutions

Sol. $\sqrt{x+3-4\sqrt{x-1}} = x + 8 - 6\sqrt{x-1} + 1 - 2\sqrt{x+8-6\sqrt{x-1}}$

$2\sqrt{x-1} - 6 = -2\sqrt{x+8-6\sqrt{x-1}}$

$\sqrt{x-1} - 3 = -\sqrt{x+8-6\sqrt{x-1}}$

(D) Infinite many solutions.

3. The maximum value M of $3^x + 5^x - 9^x + 15^x - 25^x$, as x varies over reals, satisfies

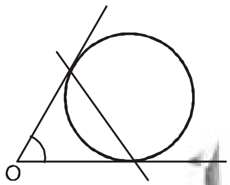
(A) $3 < M < 5$ (B) $0 < M < 2$ (C) $9 < M < 25$ (D) $5 < M < 9$

Sol.

4. Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. Then |p| is equal to

(A) 0 (B) 3 (C) 5 (D) 17

Sol.



Equation of chord of contact T = 0 w.r.t. origin is
$-3(x + 0) - p(y + 0) + 17 = 0$
$3x + py - 17 = 0$
By Homoginization

$x^2 + y^2 - (6x + 2py)\left(\frac{3x+py}{17}\right) + 17\left(\frac{3x+py}{17}\right)^2 = 0$

$\therefore$ For perpendicular coeff of x^2 + coeff of y^2 = 0

$1 - \frac{6.3}{17} + \frac{.9}{14} + 1 - \frac{2p^2}{17} + \frac{p^2}{17} = 0$

$34 - 18 + 9 - p^2 = 0$

$\Rightarrow$ $p^2 = 25$

$\Rightarrow$ $|p| = 5$

5. Let a, b, c be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a - c)^2 + b - d$ is

(A) 0 (B) 5 (C) 30 (D) 36

Sol. $2x^3 + ax + b \neq 2x^3 + cx + d$

$(a - c)x \neq d - b$

If $a - c = 0$ & $d - b \neq 0$

∴ Max. $(a - c)^2 + (b - d)$

$= 0 + 5$ Ans.

6. Consider the conic $ex^2 + \pi y^2 - 2e^2x - 2\pi^2 y + e^3 + \pi^3 = e$. Suppose P is any point on the conic and S_1, S_2 are the foci of the conic, then the maximum value of $(PS_1 + PS_2)$ is

(A) πe (B) $\sqrt{\pi e}$ (C) $2\sqrt{\pi}$ (D) $2\sqrt{e}$

Sol. $e(x^2 - 2ex + e^2) + \pi(y^2 - 2\pi y + \pi^2) = \pi e$

$$\frac{(x-e)^2}{\pi} + \frac{(y-\pi)^2}{e} = 1 \quad (a > b)$$

Ellipse $a = \sqrt{\pi}$, $b = \sqrt{e}$

Req. $PS_1 + PS_2 = 2\sqrt{\pi}$

7. Let $f(x) = \dfrac{\sin(x-a) + \sin(x+a)}{\cos(x-a) - \cos(x+a)}$, then

(A) $f(x + 2\pi) = f(x)$ but $f(x + \alpha) \neq f(x)$ for any $0 < \alpha < 2\pi$
(B) f is a strictly increasing function
(C) f is strictly decreasing function
(D) f is a constant function

Sol. $f(x) = \dfrac{2\sin x.\cos a}{2\sin x \sin a} = \cot a$

(D) = constant function

8. The value of $\tan 81° - \tan 63° - \tan 27° + \tan 9°$ is

(A) 0 (B) 2 (C) 3 (D) 4

Sol. $\tan 81° - \tan 63° - \tan 27° + \tan 9°$

$\cot 9° - \cot 27° - \tan 27° - \tan 9°$

$(\cot 9° + \tan 9°) - (\cot 27° + \tan 27°)$

$2\,\text{cosec}18° - 2\,\text{cosec}54°$

$$2\left(\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right)$$

$$8\left(\frac{2}{4}\right) = 4$$

Ans. (D)

9. The mid–point of the domain of the function $f(x) = \sqrt{4 - \sqrt{2x+5}}$ for real x is

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{5}$

Sol. $4 - \sqrt{2x+5} \geq 0$, $2x + 5 \geq 0$

$0 \leq 2x + 5 \leq 6$

$-5 \leq 2x \leq 11$

$$\frac{-5}{2} \leq x \leq \frac{11}{2}$$

$\therefore$ mid point is $\dfrac{-\frac{5}{2}+\frac{11}{2}}{2} = \dfrac{.6}{2.2} = \dfrac{3}{2}$

Ans. (B)

10. Let n be a natural number and let a be a real number. The number of zeros of $x^{2n+1} - (2n + 1)x + a = 0$ in the interval [–1, 1] is
(A) 2 if a > 0
(B) 2 if a < 0
(C) at most one for every vale of a
(D) at least three for every value of a

Sol.

11. Let $f : R \to R$ be the function $f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - x_3)(x - x_1)$ with $a_1, a_2, a_3 \in R$. Then $f(x) \geq 0$ if and only if
(A) at least two of a_1, a_2, a_3 are equal
(B) $a_1 = a_2 = a_3$
(C) a_1, a_2, a_3 are all distinct
(D) a_1, a_2, a_3 are all positive and distinct

Sol. $f(x) = 3x^2 - 2(a_1 + a_2 + a_3)x + a_1a_2 + a_2a_3 + a_3a_1 \geq 0$

$\therefore$ $D \leq 0$

$4(a_1 + a_2 + a_3)^2 - 4.3.(a_1a_2 + a_2a_3 + a_3a_1) \leq 0$

$a_1^2 + a_2^2 + a_3^2 - a_1a_2 - a_2a_3 - a_3a_1 \leq 0$

$\frac{1}{2}[(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2] \leq 0$

$\therefore$ $a_1 = a_2 = a_3$

Ans. (B)

12. The value $\dfrac{\int_0^{\pi/2}(\sin x)^{\sqrt{2}+1}dx}{\int_0^{\pi/2}(\sin x)^{\sqrt{2}+1}dx}$ is

(A) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$ (B) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ (C) $\dfrac{\sqrt{2}+1}{\sqrt{2}}$ (D) $2-\sqrt{2}$

Sol. $I_1 = \int_0^{\pi/2}(\sin\ x)^{\sqrt{2}}(\sin x)^{-1}\,dx$

$I_1 = \left(-(\cos x)(\sin x)^{\sqrt{2}}\right)_0^{\pi/2} + \sqrt{2}\int_0^{\pi/2}(\cos^2 x)(\sin x)^{\sqrt{2}-1}\,dx$

$I_1 = \sqrt{2}\left(\int_0^{\pi/2}(\sin x)^{\sqrt{2}-1}dx - \int_0^{\pi/2}\sin x^{\sqrt{2}+1}dx\right)$

$I_1 = \sqrt{2}\,(I_2 - I_1)$ Here $I_1 = \int_0^{\pi/2}(\sin x)^{\sqrt{2}-1}\,dx$

$\dfrac{I_1}{I_2} = \dfrac{\sqrt{2}}{(\sqrt{2}+1)} \times \left(\dfrac{\sqrt{2}-1}{(\sqrt{2}-1)}\right)$

$= \left(2-\sqrt{2}\right)$ **Ans.**

13. The value $\int_{-2012}^{2012}(\sin(x^3)+x^5+1)\,dx$ is

(A) 2012 (B) 2013 (C) 0 (D) 4024

Sol. $\int_{-2012}^{2012}(\sin x^3+x^5)\,dx + \int_{-2012}^{2012}1.dx$

= 0 + 2012 – (– 2012)

= 4024 Ans.

14. Let [x] and {x} be the integer part and fractional part of a real number x respectively. The value of the integral $\int_0^5 [x]\{x\}\,dx$ is

(A) 5/2 (B) 5 (C) 34.5 (D) 35.5

Sol. $\int_0^5 [x]\{x\}\,dx$

$= \int_0^1 0.\{x\}dx + \int_1^2 .1.\{x\}\,dx + \int_2^3 2\{x\}\,dx + \int_3^4 3\{x\}\,dx + \int_4^5 4.\{x\}\,dx$

$= 0 + 1\int_0^1 x.dx + 2\int_0^1 x.dx + 3 \times \int_0^1 xdx + 4 \,.\, \int_0^1 xdx$

$= (1 + 2 + 3 + 4)\int_0^1 x.dx$

$= 10 \,.\, \left[\frac{x^2}{2}\right]_0^1$

$= 10 \,.\, \left(\frac{1}{2}-0\right)$

= 5

Ans. (B)

15. Let $S_n = \sum_{k=1}^{n} k$ denote the sum of the first n positive integers. The numbers S_1, S_2, S_3, ...S_{99} are written on 99 cards. The probability of drawing a cards with an even number written on it is

(A) $\frac{1}{2}$ (B) $\frac{49}{100}$ (C) $\frac{49}{99}$ (D) $\frac{48}{99}$

Sol. $S_n \sum_{k=1}^{n} K = \frac{n(n+1)}{2}$ is even (n : 1, 2, ... 49)

n = multi fo a

9, 8, ... 96

Or (+) n is (multi of 4) – 1

3, 7 99

Total favourable cases 24 + 25 = 49

$\because$ P[E] = $\frac{49}{99}$ **Ans.**

16. A purse contains 4 copper coins and 3 silver coins. A second purse contains 6 coins and 4 silver coins. A purse is chosen randomly and a coin is taken out of it. What is the probability that it is a copper coin?

(A) $\frac{41}{70}$ (B) $\frac{31}{70}$ (C) $\frac{27}{70}$ (D) $\frac{1}{3}$

Ans. **(A)**

Sol. req. prob = $\frac{1}{2}\left[\frac{4}{7}+\frac{6}{10}\right]$ $=\frac{1}{2}\times\frac{40+42}{70}=\frac{41}{70}$ **Ans.**

17. Let H be the orthocentre of an acute–angled triangle ABC and O be its circumcenter. Then $\overline{HA}+\overline{HB}+\overline{HC}$

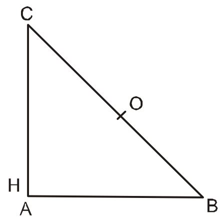
(A) is equal to $\overline{HO}$

(B) is equal to $3\overline{HO}$

(C) is equal to $2\overline{HO}$

(D) is not a scalar multiple of $\overline{HO}$ in general

Sol.



$\therefore$ $\overrightarrow{HA}+\overrightarrow{HB}+\overrightarrow{HC}=2\overrightarrow{HO}$

Ans. (C)

18. The number of ordered pairs (m,n) where m, n $\in$ {1, 2, 3, ..., 50}, such that $6^m + 9^n$ is a multiple of 5 is

(A) 1250 (B) 2500 (C) 625 (D) 500

Sol. $6^m + 9^n$

Unit digit of 6^m is = 6

Unit digit of 9^n will be = 9 or 1

For multiple of 5 unit digit of 9^n must be = 9

It occur when n = odd

Total number of ordered pair = 50 × 25 = 1250

19. Suppose $a_1, a_2, a_3, \ldots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indeced numbers is 3018, what is the sum of all numbers?

(A) 0 (B) 1509 (C) 3018 (D) 6036

Sol. $a_2 = \frac{a_1+a_3}{2}$

$a_3 = \frac{a_2+a_4}{2}$

$a_1 = \frac{a_2+a_{2012}}{2}$

$a_{2012} = \frac{a_{2011}+a_1}{2}$

Now $a_2 + a_4 + \ldots\ldots\ldots + a_{2012} = 3018$ .... (1)

$2a_2 + 2a_4 + \ldots\ldots\ldots + 2a_{2012} = 6036$

$a_1 + a_3 + a_3 + a_5 + \ldots\ldots\ldots + a_{2011} + a_1 = 6036$

$2(a_1 + a_3 + \ldots\ldots\ldots + a_{2011}) = 6036$

$a_1 + a_3 + \ldots\ldots\ldots + a_{2011} = 3018$ .... (2)

By adding (1) and (2) we get

$a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_{2012} = 6036$

20. Let S = 1, 2, 3, ..., n} and A = {(a, b) | $1 \le a, b \le n$} = S × S . A subset B of A is said to be a good subset if (x, x) ∈ B for every x ∈ S. Then the number of good subsets of A is

(A) 1 (B) 2^n (C) $2^{n(n-1)}$ (D) 2^{n^2}

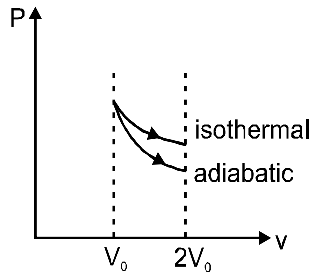
Sol. Number of element in B = n
So number of subsets of B = 2^n

PHYSICS

21. An ideal monatomic gas expands to twice its volume. If the process is isothermal, the magnitude of work done by the gas is W_i . If the process is adiabatic the magnitude of work done by the gas is W_a . Which of the following is true

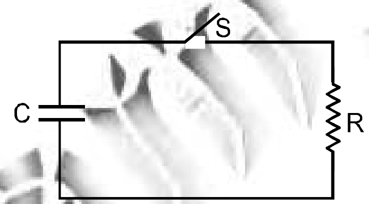
(A) $W_i = W_a > 0$ (B) $W_i > W_a > 0$ (C) $W_i > W_a = 0$ (D) $W_a > W_i = 0$

Sol.



Since area of PV graph under isothermal curve is greater than area under adiabatic curve
So, $w_i > w_a > 0$
Ans. (B)

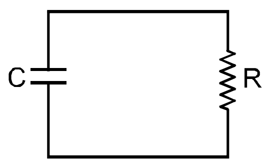
22. The capacitor of capacitance C in the circuit shown is fully charged initially, Resistance is R.



After the switch S is closed, the time taken to reduce the stored energy in the capacitor to half its initial value is :

(A) $\frac{RC}{2}$ (B) $RC\ln 2$ (C) $2RC\ln 2$ (D) $\frac{RC\ln 2}{2}$

Sol. $Q = Q_0 e^{-t/RC}$



$\frac{Q_0}{\sqrt{2}} = Q_0 e^{-t/RC}$

$\frac{1}{\sqrt{2}} = e^{-t/RC}$ **Ans. (D)**

$\ell n\frac{1}{\sqrt{2}} = -\frac{t}{RC}$

$\frac{t}{RC} = \ell n\sqrt{2}$

$t = RC\ \frac{\ell n 2}{2}$

23. A liquid drop placed on a horizontal plane has a near spherical shape (slightly flattened due to gravity). Let R be the radius of its largest horizontal section. A small disturbance causes the drop to vibrate with frequency v about its equilibrium shape. By dimensional analysis the ratio $\frac{c}{\sqrt{\frac{\sigma}{\rho R^3}}}$ can be (Here σ is surface tension, ρ is density, g is acceleration due to gravity, and k is an arbitrary dimensionless constant)

(A) $\frac{k\rho g R^2}{\sigma}$ (B) $\frac{k\rho R^3}{g\sigma}$ (C) $\frac{k\rho R^2}{g\sigma}$ (D) $\frac{k\rho}{g\sigma}$

Sol. $\frac{\upsilon}{\sqrt{\sigma/\rho R^3}} = \upsilon\sqrt{\frac{\rho R^3}{\sigma}}$

$= T^{-1}\left[\frac{ML^{-3}L^3}{MT^{-2}}\right]^{1/2}$ $= T^{-1}\,T$

$= 1$

$\frac{\upsilon}{\sqrt{\sigma/\rho R^3}} = K$

$\upsilon^2\,\frac{\rho R^3}{\sigma} = K^2$

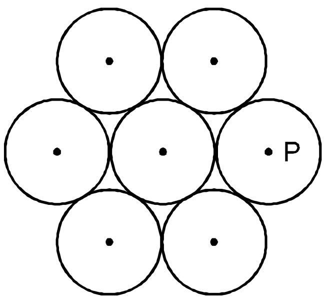
$K^2 = \frac{\rho R^3}{\sigma}T^{-2}$

$= \frac{\rho R^2}{\sigma}RT^{-2}$

$K = \frac{\rho R^2}{\sigma}g$

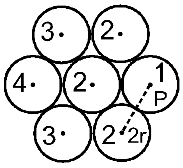
Ans. (A)

24. Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches its neighbours. Each coin is a thin disc of mass m and radius r. Note that the moment of inertia of an individual coin about an axis passing through center and perpendicular to the plane of the coin is $\frac{mr^2}{2}$.



The moment of inertia of the system of seven coins about an axis that passes through the point P (the centre of the coin positioned directly to the right of the central coin) and perpendicular to the plane of the coins is

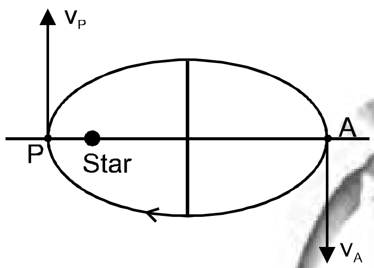
(A) $\frac{55}{2}\,mr^2$ (B) $\frac{127}{2}\,mr^2$ (C) $\frac{111}{2}\,mr^2$ (D) $55\,mr^2$

Sol. 

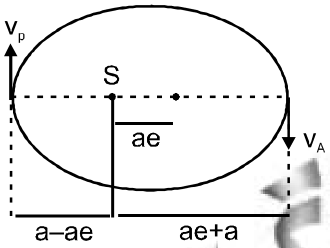
$$I_P = \frac{mr^2}{2} + \left[\frac{mr^2}{2} + m(2r)^2\right] \times 3 + \left[\frac{mr^2}{2} + m(r2\sqrt{3})^2\right] \times 2 + \left[\frac{mr^2}{2} + m(4r)^2\right]$$

$$= \frac{111}{2} mr^2 \ .$$

25. A planet orbits in an elliptical path of eccentricity e around a massive star considered fixed at one of the foci. The point in space where it is closest to the star is denoted by P and the point where it is farthest is denoted by A. Let v_P and v_A be the respective speeds at P and A. Then



(A) $\frac{v_P}{v_A} = \frac{1+e}{1-e}$ (B) $\frac{v_P}{v_A} = 1$ (C) $\frac{v_P}{v_A} = \frac{1+e^2}{1-e}$ (D) $\frac{v_P}{v_A} = \frac{1+e^2}{1-e^2}$

Sol. 

Angular momentum conservation about S

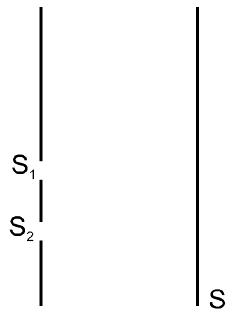
$v_p (a - ae) = v_A(a + ae)$

$$\frac{v_p}{v_A} = \frac{a+ae}{a-ae}$$

$$\frac{v_p}{v_A} = \frac{1+e}{1-e}$$

Ans. (A)

26. In a Young's double slit experiment the intensity of light at each slit is I_0. Interference pattern is observed along a direction parallel to the line $S_1 S_2$ on screen S.



The minimum, maximum, and the intensity averaged over the entire screen are respectively.

(A) 0, $4I_0$, $2I_0$ (B) I_0, $2I_0$, $3I_0/2$ (C) 0, $4I_0$, I_0 (D) 0, $2I_0$, I_0

Sol. $I_{max} = (\sqrt{I_0} + \sqrt{I_0})^2$

$= 4I_0$

$I_{min} = (\sqrt{I_0} - \sqrt{I_0})^2 = 0$

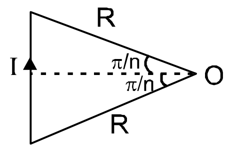
$I_{avg} = I_0 + I_0 = 2I_0$

Ans. (A)

27. A loop carrying current I has the shape of a regular polygon of n sides. If R is the distance from the centre to any vertex, then the magnitude of the magnetic induction vector $\vec{B}$ at the centre of the loop is

(A) $n\frac{\mu_0 I}{2\pi R}\tan\frac{\pi}{n}$ (B) $n\frac{\mu_0 I}{2\pi R}\tan\frac{2\pi}{n}$ (C) $\frac{\mu_0 I}{2R}$ (D) $\frac{\mu_0 I}{\pi R}\tan\frac{\pi}{n}$

Sol.

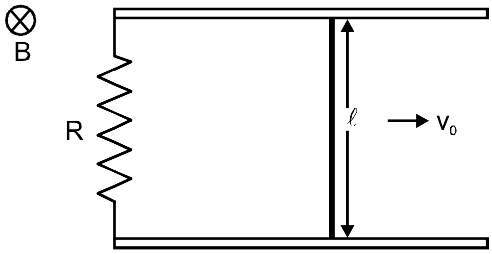


$$B_0 = \frac{\mu_0}{4\pi}\frac{i}{R\cos\frac{\pi}{n}}\left[\sin\frac{\pi}{n} + \sin\frac{\pi}{n}\right]$$

$$B_0 = \frac{\mu_0}{2\pi}\frac{i}{R}\tan\frac{\pi}{n}$$

Ans. (A)

28. A conducting rod of mass m and length ℓ is free to move without friction on two parallel long conducting rails, as shown below. There is a resistance R acorss the the rails. In the entire space around, there is a uniform magnetic filed B normal to the plane of the rod and rails. The rod is given an impulsive velocity v_0.



Finally, the initial energy $\frac{1}{2}mv_0^2$

(A) will be converted fully into heat energy in the resistor
(B) will enable rod to continue to move eith velocity v_0 since the rails are frictionless
(C) will be converted fully into magnetic energy due to induced current
(D) will be converted into the work done against the magnetic field

Sol. Due to the negative work on rod K.E. will decrease and finally become zero.
Ans. (A)

29. A steady current I flows through a wire of radius r, length L and resistivity ρ. The current produces heat in the wire. The rate of heat loss in a wire is proportional its surface area. The steady temperature of the wire is independent of

(A) L (B) r (C) I (D) ρ

Sol. $R = \frac{\rho\ell}{A}$

$R = \frac{\rho L}{\pi r^2}$

$I^2R = K2\pi rL\frac{dT}{dt}$

$\frac{I^2\rho L}{\pi r^2} = K2\pi rL\frac{dT}{dt}$

$\frac{dT}{dt} = \frac{I^2\rho L}{K2\pi^2 r^3}$ **Ans. (A)**

30. The ratio of the speed of sound to the average speed of an air molecule at 300K and 1 atmospheric pressure is close to

(A) 1 (B) $\sqrt{300}$ (C) $\sqrt{\frac{1}{300}}$ (D) 300

Sol. $\frac{\sqrt{\frac{\rho RT}{m}}}{\sqrt{\frac{8RT}{\pi m}}} = \sqrt{\frac{7}{5}\frac{\pi}{8}} = 0.73$

So, closest ans is 1.
Ans. (A)

31. In one model of the election, tho electron of mass m_e is thought to be a uniformly charged shell of radius R and total charge e, whose electrostatic energy E is equivalent to its mass m_e via Einstein's mass energy relation $E = m_e c^2$. In this model, R is approximately ($m_e = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m.s^{-1}, $\frac{1}{4}\pi\varepsilon_0 = 9 \times 10^9$ Farads m^{-1}, magnitude of the electron charge = 1.6×10^{-19} C)

(A) 1.4×10^{-15} m (B) 2×10^{-13} m (C) 5.3×10^{-11} m (D) 2.8×10^{-35} m

Sol. $U = mc^2$

$$\frac{KQ^2}{2R} = mc^2$$

$$\frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2R} = 9.1 \times 10^{-31} \times (3 \times 10^8)^2$$

$$R = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2}$$

$= 1.4 \times 10^{-15}$

Ans. (A)

32. A body is executing simple harmonic motion of amplitude a and period T about the equilibrium position x = 0. Large numbers of snapshots are taken at random of this body in motion. The probability of the body being found in a very small interval x to x + |dx| is highest at

(A) $x = \pm a$ (B) $x = 0$ (C) $x = \pm a/2$ (D) $x = \pm \frac{a}{\sqrt{2}}$

Sol. Probability of being found is maximum where speed is minimum.
So, $x = \pm a$
Ans. (A)

33. Two identical bodies are made of a material for which the heat capacity increases with temperature One of these is held at a temperature of 100°C while the other one is kept at 0°C. If the two are brought into contact, then, assuming no heat loss to the environment, the final temperature that they will reach is

(A) 50°C (B) more than 50°C (C) less than 50°C (D) 0°C

Sol. Since heat capacity at high temperature is high. So for same amount of heat transfer ΔT is more at lower temperature then at higher temperature.
So, final temperature is more than 50ºC .

34. A particle is acted upon by a force given by $F = -\alpha x^3 - \beta x^4$ where α and β are positive constants. At the point x = 0, the particle is

(A) in stable equilibrium (B) in unstable equiibrium
(C) in neutral equilibrium (D) not in equilibrium

Sol. As $F = -\alpha x^3 - \beta x^4$
At x = 0, F = 0
Hence particle is in equilibrium.

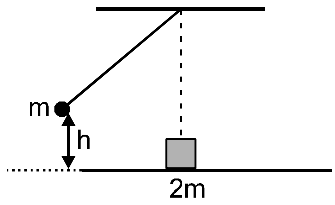
35. The potential energy of a point particle is given by the expression $V(x) = \alpha x + \beta \sin (x / \gamma)$. A dimensionless combination of the constants α, β and γ is

(A) $\frac{\alpha}{\beta\gamma}$ (B) $\frac{\alpha^2}{\beta\gamma}$ (C) $\frac{\gamma}{\alpha\beta}$ (D) $\frac{\alpha\gamma}{\beta}$

Sol. It is clear that
dimension $\alpha x = ML^2T^{-2}$ $\Rightarrow$ $\alpha = MLT^{-2}$
$\beta = ML^2T^{-2}$
and $x = L$

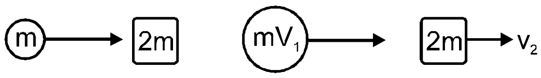
So, $\frac{\alpha\gamma}{\beta}$ will be dimensionless.

36. A ball of mass m suspended from a rigid support by an inextensible massless string is released from a height h above its lowest point. At its lowest point it collides elastically with a block of mass 2m at rest on a frictionless surface. Neglect the dimensions of the ball and the block. After the collision the ball rises to a maximum height of



(A) $\frac{h}{3}$ (B) $\frac{h}{2}$ (C) $\frac{h}{8}$ (D) $\frac{h}{9}$

Sol. $\sqrt{2gh}$



Before collision After collision

From conservation of momentum

$$m\sqrt{2gh} + 0 = mv_1 + 2mv_2$$

$\Rightarrow$ $\sqrt{2gh} = v_1 + 2v_2$ ....(i)

From equation of e

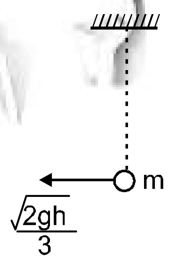
$$1 = \frac{v_2 - v_1}{\sqrt{2gh} - 0} \quad \Rightarrow \quad \sqrt{2gh} = v_2 - v_1 \quad ....(ii)$$

From (i) & (ii)

$$v_1 = -\frac{\sqrt{2gh}}{3}$$

Hence after collision maximum height

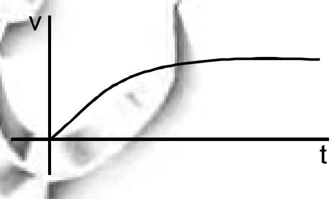
$$h_{max} = \frac{(\sqrt{2gh}/3)^2}{2g}$$

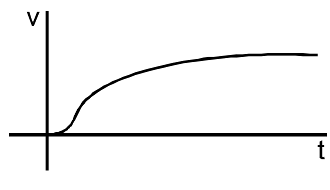


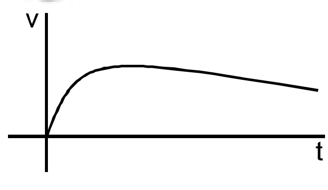
$$h_{max} = \frac{h}{g}$$

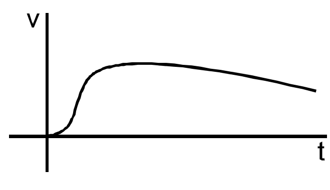
Ans. (D)

37. A particle released from rest is falling through a thick fluid under gravity. The fluid exerts a resistive force on the particle proportional to the square of its speed. Which one of the following graphs best depicts the variation of is speed v with time t?

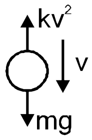
(A)


(B)


(C)


(D)


Sol. Let ball is moving with speed v at anytime t hence



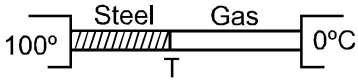
Hence, $m\frac{dv}{dt} = mg - Kv^2$

On the basis of equation we can rays that speed first increase and become constant when $mg = kv^2$
Hence, Ans. (A)

38. A cylindrical steel rod of length 0.10m and thermal conductivity 50 $W.m^{-1}.K^{-1}$ is welded end to end to copper rod of thermal conductivity 400 $W.m^{-1}.K^{-1}$ and of the same area of cross section but 0.20 m long. The free end of the steel rod is maintained at 100°C and that of the copper rod at 0°C. Assuming that the rods are perfectly insulated from the surrounding the temperature at the junction of the two rods is
(A) 20°C (B) 30°C (C) 40°C (D) 50°C

Sol. Resistance of steel = $\frac{0.1}{50 \times A} = \frac{1}{500A}$

Resistance of copper = $\frac{0.2}{400A} = \frac{1}{200A}$



Both are connected in series hence heat current in both will be same. So

$$\frac{100 - T}{\left(\frac{1}{500A}\right)} = \frac{T - 0}{\left(\frac{1}{2000A}\right)} \quad \Rightarrow \quad T = 20^\circ C$$ **Ans. (A)**

39. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. The half lives of X and Y are 4000 years and 20 years respectively. In a certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20}, the number of Y nuclei present in it is
(A) 2×10^{17} (B) 2×10^{20} (C) 4×10^{23} (D) 4×10^{20}

Sol. From

$N_1\lambda_1 = N_2\lambda_2$

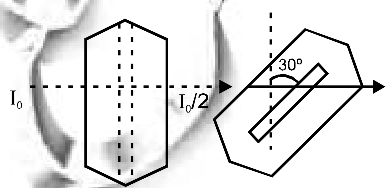
$4 \times 10^{20} = \frac{\ell n(2)}{40000} = N_2\frac{\ell n(2)}{20} \Rightarrow N_2 = 2 \times 10^7$

Ans. (A)

40. An unpolarized beam of light of intensity I_0 passes through two linear polarizers making an angle of 30° with respect to each other. The emergent beam will have an intensity.

(A) $\frac{3I_0}{4}$ (B) $\frac{\sqrt{3}\,I_0}{4}$ (C) $\frac{3I_0}{8}$ (D) $\frac{I_0}{8}$

Sol.



$$\left(\frac{I_0}{2}\right)\cos^2 30^\circ = \frac{I_0}{2}\left(\frac{3}{4}\right)$$

Final intensity will be = $\frac{3I_0}{8}$

Ans. (C)

CHEMISTRY

41. Among the following, the species with the highest bond order is :

(A) O_2 (B) F_2 (C) O_2^+ (D) F_2^-

Ans. **(C)**

Sol. According to MOT bond order of O_2, F_2, O_2^+ and F_2^- is respectively 2, 1, 2.5 and 0.5.

42. The molecule with non-zero dipole moment is :

(A) BCl_3 (B) $BeCl_2$ (C) CCl_4 (D) NCl_3

Ans. **(D)**

Sol. BCl_3, $BeCl_2$ and CCl_4 has zero dipole moment due to symmetric structure where as shape of NCl_3 is pyramidal due to presence of lone pair.

43. For a one-electron atom, the set of allowed quantum numbers is :

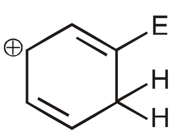
(A) $n = 1, \ell = 0, m_\ell = 0, m_s = +½$ (B) $n = 1, \ell = 1, m_\ell = 0, m_s = +½$

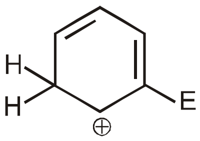
(C) $n = 1, \ell = 0, m_\ell = –1, m_s = –½$ (D) $n = 1, \ell = 1, m_\ell = 1, m_s = –½$

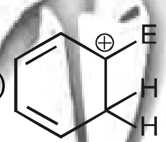
Ans. **(A)**

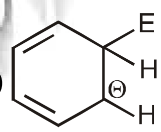
Sol. The value of $n > \ell$ and m should have values from $– \ell$ to $+ \ell$.

44. In the reaction of benzene with an electrophile E^+, the structure of the intermediate σ-complex can be represented as :

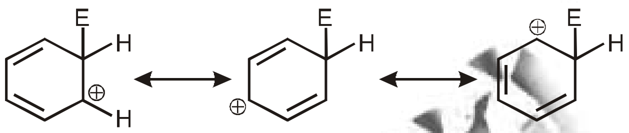
(A) 

(B) 

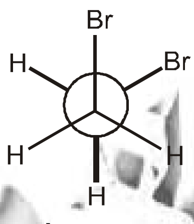
(C) 

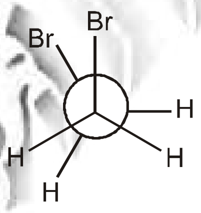
(D) 

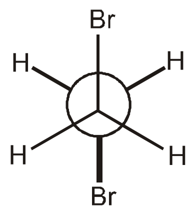
Ans. Option "D" Closest, if (–) is change to **(+).**

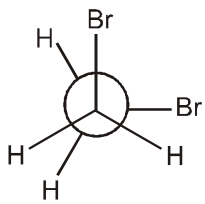
Sol. 

45. The most stable conformation of 2, 3-dibromobutane is :

(A) 

(B) 

(C) 

(D) 

Ans. **(Bonus)**

46. Typical electronic energy gaps in molecules are about 1.0 eV. In terms of temperature, the gap is closest to:

(A) 10^2 K (B) 10^4 K (C) 10^3 K (D) 10^5 K

Ans. **(B)**

Sol. KE = | T.E. |

$\therefore$ $\Delta(KE) = \Delta(T.E.)$

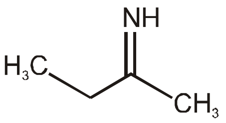
$$\frac{3}{2} \times 8.3 \times (\Delta T) = 1.6 \times 10^{-19} \times 6 \times 10^{23}$$

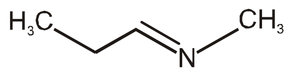
$$(\Delta T) = \frac{9.6 \times 10^4}{8.3} \times \frac{2}{3}$$

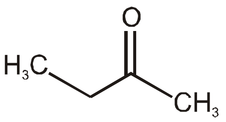
$(\Delta T) = 7.6 \times 10^3$ K i.e. close to 10^4 K.

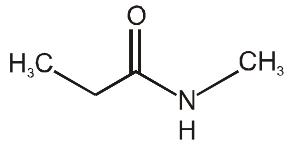
47. The major final product in the following reaction is :

$$CH_3CH_2CN \xrightarrow[2)\ H_3O^+]{1)\ CH_3MgBr}$$

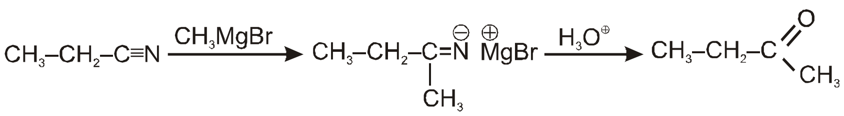
(A) 

(B) 

(C) 

(D) 

Ans. **(C)**

Sol. 

48. A zero-order reaction, A $\rightarrow$ Product, with an initial concentration $[A]_0$ has a half-life of 0.2 s. If one starts with the concentration $2[A]_0$, then the half-life is :

(A) 0.1 s (B) 0.4 s (C) 0.2 s (D) 0.8 s

Ans. **(B)**

Sol. $t_{1/2} \propto (a)^{1-n}$

$$\frac{(t_{1/2})_I}{(t_{1/2})_{II}} = \left(\frac{a_1}{a_2}\right)^{1-0}$$

$$\frac{0.2}{(t_{1/2})_{II}} = \frac{A_0}{2A_0}$$

$(t_{1/2})_{II} = 0.4$ s

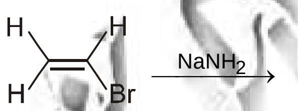
49. The isoelectronic pair of ions is :

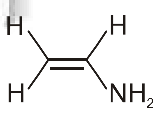
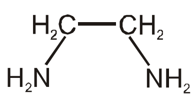
(A) Sc^{2+} and V^{3+} (B) Mn^{3+} and Fe^{2+} (C) Mn^{2+} and Fe^{3+} (D) Ni^{3+} and Fe^{2+}

Ans. **(C)**

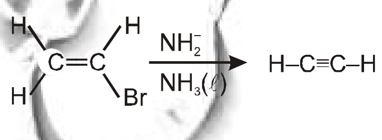
Sol. $_{25}Mn^{2+}$ and $_{26}Fe^{3+}$ both has 23 electrons.

50. The major product in the following reaction is :

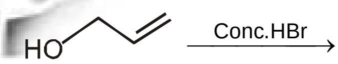


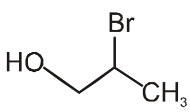
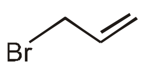
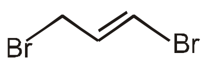
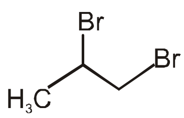
(A)  (B)  (C)  (D) 

Ans. **(A)**

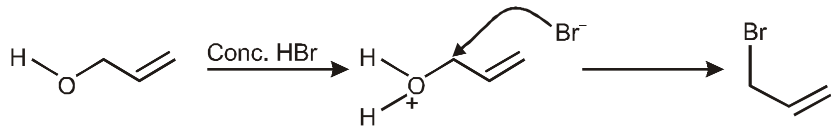
Sol. 

51. The major product of the following reaction is :

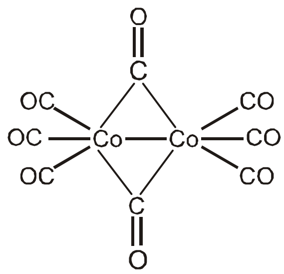


(A)  (B)  (C)  (D) 

Ans. **(B)**

Sol. 

52. The oxidation state of cobalt in the following molecule is :



(A) 3 (B) 1 (C) 2 (D) 0

Ans. **(D)**

Sol. In metal carbonyls oxidation state of metal is equal to zero.

53. The pK_a of a weak acid is 5.85. The concentrations of the acid and its conjugate base are equal at a pH of:

(A) 6.85 (B) 5.85 (C) 4.85 (D) 7.85

Ans. **(B)**

Sol. At pH = pK_a the concenteration of acid and its conjugate base are equal.

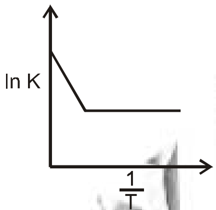
54. For a tetrahedral complex $[MCl_4]^{2-}$, the spin-only magnetic moment is 3.83 BM. The element M is :

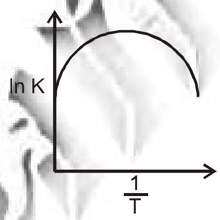
(A) Co (B) Cu (C) Mn (D) Fe

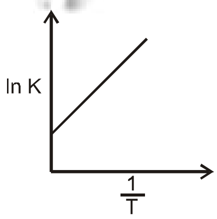
Ans. **(A)**

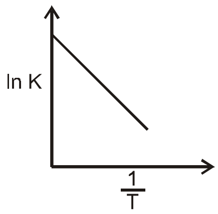
Sol. It is high spin complex as Cl^- is weak field effect ligand. In $[CoCl_4]^{2-}$ oxidation state of Co is +2 in which 3 unpaired electrons are present which gives the spin-only magnetic moment equal to 3.83 BM.

55. Among the following graphs showing variation of rate (k) with temperature (T) for a reaction, the one that exhibits Arrhenius behavior over the entire temperature range is :

(A) 

(B) 

(C) 

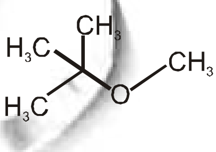
(D) 

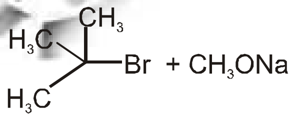
Ans. **(D)**

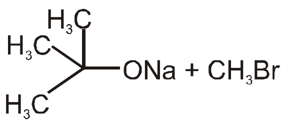
Sol. $K = Ae^{-E_a/RT}$

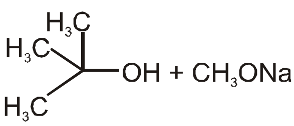
$$\ln K = \frac{-E_a}{RT} + \ln A$$

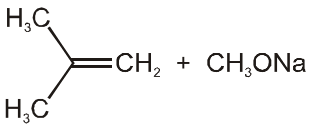
56. The reaction that gives the following molecule as the major product is



(A) 

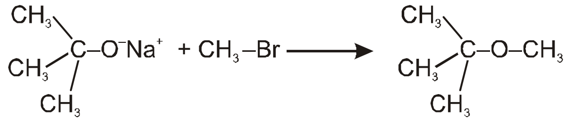
(B) 

(C) 

(D) 

Ans. **(B)**

Sol.



57. The C–O bond length in CO, CO_2 and CO_3^{2-} follows the order :

(A) $CO < CO_2 < CO_3^{2-}$ (B) $CO_2 < CO_3^{2-} < CO$ (C) $CO > CO_2 > CO_3^{2-}$ (D) $CO_3^{2-} < CO_2 < CO$

Ans. **(A)**

Sol. Greater is the bond order shorter will be bond length. In CO_3^{2-} bond order in between 1 and 2, bond order of CO_2 is 2 where as bond order for CO is in between 2 and 3.

58. The equilibrium constant for the following reactions are K_1 and K_2, respectively.

$$2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g)$$

$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g)$$

The the equilibrium constant for the reaction

$$2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g)$$

is :

(A) K_1K_2 (B) $K_1K_2^2$ (C) $K_1^2K_2^2$ (D) $K_1^2K_2$

Ans. **(B)**

Sol.

$$2P(g) + 3Cl_2(g) \rightleftharpoons 2PCl_3(g) \quad ....(i)$$

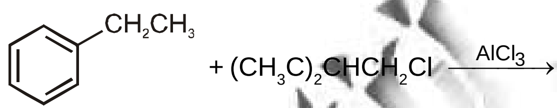
$$PCl_3(g) + Cl_2(g) \rightleftharpoons PCl_5(g) \quad ....(ii)$$

$$2P(g) + 5Cl_2(g) \rightleftharpoons 2PCl_5(g) \quad ....(iii)$$

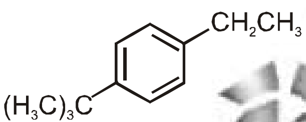
On multipling equation (ii) by 2 and adding in (i) we obtain equation (iii)

Therefore, $K_3 = K_1K_2^2$

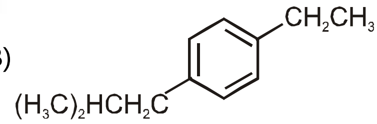
59. The major product of the following reaction is :



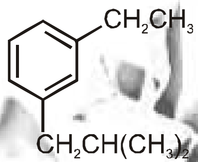
(A)



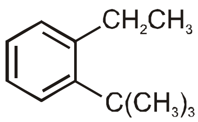
(B)



(C)



(D)

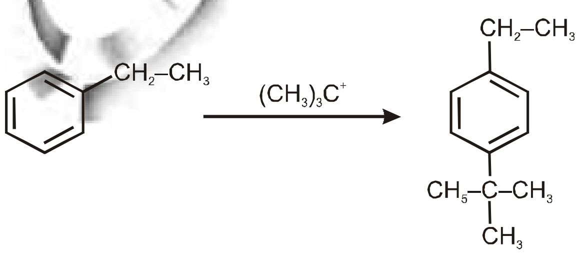


Ans. **(A)**

Sol. The reagent given the question has an addition "C" within bracket.

$(CH_3)_2CH–CH_2–Cl + AlCl_3 \longrightarrow (CH_3)_2CH–CH_2–Cl - - - - AlCl_3 \longrightarrow (CH_3)_2CH–CH_2^+ + AlCl_4^-$

$(CH_3)_2CH–CH_2^+ \xrightarrow{\text{12–H}^- \text{shiffting}} (CH_3)_3C^+$



60. Doping silicon with boron produces a :
(A) n-type semiconductor (B) metallic conductor
(C) p-type semiconductor (D) insulator

Ans. **(C)**

Sol. As boron is trivalent impurity it will proudce p-type semiconductor.

BIOLOGY

61. The disorders that arise when the immune system destroys 'self' cells are called autoimmmune disorders. Which of the following would be classified under this
(A) rheumatoid arthritis (B) asthma (C) rhintis (D) eczema

Ans **(A)**

62. Which of the following class of immunoglobulins can trigger the complement cascade ?
(A) IgA (B) IgM (C) IgD (D) IgE

Ans **(A)**

63. Diabetes insipidus is due to
(A) hypersecretion of vasoperssin
(B) Hyposecretion of insulin
(C) hypersecretion of insulin
(D) hyposecretion of vasopressin

Ans **(D)**

64. Fossils are most often found in which kind of rocks ?
(A) meteorites (B) sedimentary rocks
(C) igneous rocks (D) metamorphic rocks

Ans **(B)**

65. Peptic ulcers are caused by
(A) a fungus, *Candida albicans*
(B) a virus, cytomegalo virus
(C) a parasite, *Trypanosoma brucei*
(D) a bacterium, *Helicobacter pylori*

Ans **(D)**

66. Transfer RNA (tRNA)
(A) is present in the ribosomes and provides structural integrity
(B) usually has clover leaf-like structure
(C) carries genetic information from DNA to ribosomes
(D) codes for proteins

Ans **(B)**

67. Some animals excrete uric acid in urine (uricotelic) as it requires very little water. This is an adaptation to conserve water loss. Which animals among the following are most likely to be uricotelic?
(A) fishes (B) amphibians (C) birds (D) mammals

Ans **(C)**

68. A ripe mango, kept with unripe mangoes causes their ripening. This is due to the release of a gaseous plant hormone
(A) auxin (B) gibberlin (C) cytokinine (D) ethylene

Ans **(D)**

69. Human chromosomes undergo structural changes during the cell cycle. Chromosomal structure can be best visualized if a chromosome is isolated from a cell at
(A) G1 phase (B) S phase (C) G2 phase (D) M phase

Ans **(D)**

70. By which of the following mechanisms is glucose reabsorbed from the glomerular filtrate by the kidney tubule

(A) osmosis (B) diffusion (C) active transport (D) passiver transport

Ans (C)

71. In mammals, the hormones secreted by the pituitary, the master gland, is itself regulated by

(A) Hypothalamus (B) median cortex (C) pineal gland (D) cerebrum

Ans (A)

72. Which of the following is true for TCA cycle in eukaryotes

(A) takes place in mitochondrion

(B) produces no ATP

(C) takes place in Golgi complex

(D) independent of electron transport chain

Ans (A)

73. A hormone molecule binds to a specific protein on the plasma membrane inducing a signal. The protein it binds to it called

(A) ligand (B) antibody (C) receptor (D) histone

Ans (C)

74. DNA mutations that do not cause my functional change in the protein product are known as

(A) nonsense mutations (B) missense mutations (C) deletion mutations (D) silent mutations

Ans (D)

75. Plant roots are usually devoid of chlorophyll and cannot perform photosynthesis. However, three are exceptions. Which of the following plant root can perform photosynthesis

(A) Arabidopsis (B) Tinospora (C) Rice (D) Hibiscus

Ans (B)

76. Vitamin A deficiency leads to night-blindness. Which of the following is the reason for the disease ?

(A) rod cells are not converted to cone cells

(B) rhodopsin pigment of rod cells is defective

(C) melanin pigment is not synthesized in cone cells

(D) cornea of eye gets dried

Ans (B)

77. In Dengue virus infection, patients often develop haemorrhagic fever due to internal bleeding. This happens due to the reduction of

(A) platelets (B) RBCs (C) WBCs (D) lymphocytes

Ans (A)

78. If the sequence of bases in sense strand of DNA is 5'-GTTCATCG-3, then the sequence ofd bases in its RNA transcript would be

(A) 5'-GTTCATCG-3' (B) 5'GUUCAUCG-3 (C) 5'CAAGTAGC-3' (D) 5'CAAGUAGC -3

Ans (B)

79. A refilex aciton is a quick involuntary response to stimulus. Which of the following is an example of BOTH, unconditioned and conditioned reflex

(A) knee jerk reflex

(B) secretion of saliva in response to the aroma of food

(C) sneezing reflex

(D) contration of the pupil in response to bright light

Ans (A)

80. In a food chain such as grass $\rightarrow$ deer $\rightarrow$ lion, the energy cost of respiration as a proportion of total assimilated energy at each level would be

(A) 60%- 30%-20% (B) 20%- 30%-60% (C) 20%- 60%-30% (D) 30%- 30%-30%

Ans (B)

PART-II
Two Mark Questions

MATHEMATICS

81. Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has

(A) two distinct positive real roots
(B) two equal roots
(C) one positive and one negative root
(D) no real roots

Sol. $D_1 = 4a^2 - 4b^2 > 0 \Rightarrow 4(a^2 - b^2) > 0$

$a^2 - b^2 > 0$ .... (1)

$D_2 = 4b^2 - 4c^2 > 0 \Rightarrow b^2 - c^2 > 0$ .... (2)

now $D = 4c^2 - 4a^2 = 4(c^2 - b^2 + b^2 - a^2)$

$= -4(b^2 - c^2 + a^2 - b^2)$

$= -4(D_1 + D_2)$

= Negative

$\therefore$ Equation have no real root.

82. The coefficient of x^{2012} in $\frac{1+x}{(1+x^2)(1-x)}$ is

(A) 2010
(B) 2011
(C) 2012
(D) 2013

Sol. $(1+x)(1+x^2)^{-1}(1-x)^{-1}$

$$(1+x)\sum_{r=0}^{\infty} x^{2r}(-1)^r \sum_{r=0}^{\infty} x^r = \sum_{r=0}^{\infty} x^{2r}(-1)^r \sum_{r=0}^{\infty} x^r + \sum_{r=0}^{\infty} x^{2r}(-1)^r \sum_{r=0}^{\infty}(x^{r+1})$$

for Coffi. of $x^{2012} = (-1)^0 + (-1)^1 + (-1)^2 + ...(-1)^{1006}) + ((-1)^0 + (-1)^1 + ... + (-1)^{1005})$

= 1 + 0

= 1

No answer

83. Let (x, y) be a variable point on the curve $4x^2 + 9y^2 - 8x - 36y + 15 = 0$. Then $\min(x^2 - 2x + y^2 - 4y + 5) + \max(x^2 - 2x + y^2 - 4y + 5)$ is

(A) $\frac{325}{36}$
(B) $\frac{36}{325}$
(C) $\frac{13}{25}$
(D) $\frac{25}{13}$

Sol. $4x^2 + 9y^2 - 8x - 36y + 15 = 0$

$4(x^2 - 2x + 1) + 9(y^2 - 4y + 4) - 25 = 0$

$4(x-1)^2 + 9(y-2)^2 = 25$

$$\frac{(x-1)^2}{\left(\frac{5}{2}\right)^2} + \frac{(y-2)^2}{\left(\frac{5}{3}\right)^2} = 1$$

$$\min\left((x-1)^2 + (y-2)^2\right) + \max.\left((x-1)^2 + (y-2)^2\right)$$

$$= \left(\frac{5}{3}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{100+225}{36} = \frac{325}{36}$$

Ans. **(A)**

84. The sum of all $x \in [0, \pi]$ which satisfy the equation $\sin x + \frac{1}{2}\cos x = \sin^2(x + \frac{\pi}{4})$ is

(A) $\frac{\pi}{6}$
(B) $\frac{5\pi}{6}$
(C) π
(D) 2π

Sol. $\sin x + \frac{1}{2}\cos x = \sin^2\left(x+\frac{\pi}{4}\right)$

$\sin x + \frac{1}{2}\cos x = \frac{1-\cos(2x+\pi/2)}{2}$

$\sin x + \frac{1}{2}\cos x = \frac{1}{2}+\frac{1}{2}\sin 2x$

$2\sin x + \cos x = 1 + \sin 2x$

$2\sin x + \cos x = 1 + 2\sin x\cos x$

$2\sin x(1 - \cos x) - 1(1 - \cos x) = 0$

$(1 - \cos x)(2\sin x - 1) = 0$

$\cos x = 1$ or $\sin x = \frac{1}{2}$

$x = 0$ $\qquad x = \frac{\pi}{6}, \frac{5\pi}{6}$

sum of roots $= 0 + \frac{\pi}{6} + \frac{5\pi}{6} = \pi$

Ans. (C)

85. A polynomila P(x) with real coefficients has the property that $P''(x) \neq 0$ for all x. Suppose $P(0) = 1$ and $P'(0) = -1$. What can you say about P(1)?

(A) $P(1) \geq 0$ (B) $P(1) \neq 0$ (C) $P(1) \leq 0$ (D) $\frac{-1}{2} < P(1) < \frac{1}{2}$

Sol. $P(x) = ax^2 + bx + c \qquad a \neq 0$

$P'(x) = 2ax + b$

$P''(x) = 2a$

$P(0) = c = 1$

$P'(0) = b = -1$

$\therefore\ P(x) = ax^2 - x + 1$

$P(1) = a - 1 + 1 = a \neq 0$

Ans. (C)

86. Define a sequence (a_n) by $a_1 = 5$, $a_n = a_1a_2 \ldots a_{n-1} + 4$ for $n > 1$. Then $\lim_{n\to\infty}\frac{\sqrt{a_n}}{a_{n-1}}$

(A) equals $\frac{1}{2}$ (B) equal 1 (C) equals $\frac{2}{5}$ (D) does not exists

87. The value of the integral $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, where $a > 0$, is

(A) π (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) 2π

Sol. $I = \int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx$, $(a > 0)$ ...(1)

$I = \int_{-\pi}^{\pi}\frac{\cos^2(-x)}{1+a^{-x}}dx$

$I = \int_{-\pi}^{\pi}\frac{a^x\cos^2 x}{1+a^x}dx$ ...(2)

(1) + (2)

$$2I = \int_{-\pi}^{\pi} \frac{\cos^2 x(1+a^x)}{(1+a^x)} dx$$

$$2I = 2\int_0^{\pi} \cos^2 x \, dx$$

$$I = \left(\frac{1+\cos 2x}{2}\right)_0^{\pi}$$

$$I = \frac{1}{2}\left((x+\cos 2\pi) - (0+\cos 0^0)\right)$$

$$= \frac{1}{2}\left((\pi+1)-1\right)$$

$$= \frac{\pi}{2}.$$

88. Consider

$L = \sqrt[3]{2012} + \sqrt[3]{2013} + ... + \sqrt[3]{3011}$

$R = \sqrt[3]{2013} + \sqrt[3]{2014} + ... + \sqrt[3]{3012}$

and $I = \int_{2012}^{3012} 3\sqrt{x}\, dx$.

(A) L + R < 2I (B) L + R = 2I (C) L + R > 2I (D) $\sqrt{LR}$ = I

89. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let P(K) be the probability of scroing at least K points. The largest value of K such that P(K) > $\frac{1}{2}$ is

(A) 14 (B) 15 (C) 16 (D) 17

Sol. P(K) = P (at least K points)

$x_1 + x_2 + x_3 + ... x_{10} = K$

Coeff x^K in $(x^1 + x^2)^{10}$

$= x^{10}(1 + x)^{10}$

coeff x^k in $(1 + x)^{10}$

$= {}^{10}C_{K-10}$ $K \geq 10$

Now $P(K) > \frac{1}{2}$

$$\frac{{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{K-10}}{2^{10}} \geq \frac{1}{2}$$

${}^{10}C_0 + {}^{10}C_1 + ... + {}^{10}C_{K-10} > 2^9$

1 + 10 + 45 + 120 + 210 + 252 + ... > 512

${}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 > 512$

K – 10 = 5

K = 15

Ans. (B)

90. Let $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$. Let $f^1(x) = f(x)$, $f^2(x) = f(f(x))$ and generally $f^n(x) = f(f^{n-1}(x))$ for n > 1. Let P = $f^1(2)\, f^2(3)\, f^3(4)\, f^4(5)$ which of the following is a multiple of P

(A) 125 (B) 375 (C) 250 (D) 147

Sol. P = f(2) . f(f(3)) f(f(f(4)) f(f(f(5)

$= \left(\frac{3}{1}\right) \left(f\left(\frac{4}{2}\right)\right) f \left(f\left(\frac{5}{3}\right)\right) f\left(f\left(f\left(f\left(\frac{6}{4}\right)\right)\right)\right)$

$= (3) \left(\frac{3}{1}\right) \left(f\left(\frac{\frac{5}{3}+1}{\frac{5}{3}-1}\right)\right) f\left(f\left(\frac{\frac{3}{2}+1}{\frac{3}{2}-1}\right)\right)$

= (3) (3) (f(4)) f(f(5))

$= 9 \left(\frac{5}{3}\right) \left(f\left(\frac{6}{4}\right)\right)$

$= (15) \left(\frac{\frac{3}{2}+1}{\frac{3}{2}-1}\right)$

= (15) (5) = 75

375 is multiple of 75.

Ans. (B)

PHYSICS

91. The total energy of a black body radiation source is collected for five minutes and used to heat water. The temperature of the water increases from 10.0°C to 11.0°C. The absolute temperature of the black body is doubled and its surface area halved and the experiment repeated for the same time. Which of the following statements would be most nearly correct?

(A) The temperature of the water would increase from 10.0°C to a final temperature of 12°C
(B) The temperature of the water would increase from 10.0°C to a final temperature of 18°C
(C) The temperature of the water would increase from 10.0°C to a final temperature of 14°C
(D) The temperature of the water would increase from 10.0°C to a final temperature of 11°C

Sol. $H_1 = \sigma AT^4 \times 5 = Ms\Delta\theta_1$

$H_2 = \sigma \frac{A}{2} (2T)^4 \times 5 = ms\Delta q_2$

$\Delta\theta_2 = 8\Delta\theta_1$

$\Delta\theta_2 = 8ºC$

The temperature of water would increase from 10ºC to a final temperature of 18ºC

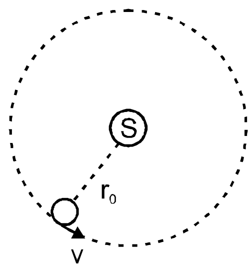
92. A small asteroid is orbiting around the sun in a circular orbit of radius r_0 with speed V_0. A rocket is launched from the asteroid with speed $V = \alpha V_0$ where V is the speed relative to the sun. The highest value of α for which the rocket will remain bound to the solar system is (ignoring gravity due to the asteroid and effect of other planets)

(A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) 1

Sol. Mechanical energy of asteroid

$= \frac{1}{2} mv^2 - \frac{GMm}{r_0}$

Rocket will remain band to the solar system its B moving energies negative.



$-\frac{GMm}{r_0} + \frac{1}{2}mv^2 = 0$

$\frac{GMm}{r_0} = \frac{1}{2}mv^2$

$\frac{GMm}{r_0} = \frac{1}{2}m\alpha^2 v_0^2$

$mv_0^2 = \frac{1}{2} m\alpha^2 v_0^2$

$\alpha = \sqrt{2}$.

93. A radioactive nucleus A has a single decay mode with half life τ_A. Another radioactive nucleus B has two decay modes 1 and 2. If decay mode 2 were absent, the half life of B would have been $\tau_A/2$. If decay mode 1 were absent, the half life of B would have been 3 τ_A. If the actual half life of B is τ_B , then the ratio $\frac{\tau_B}{\tau_A}$ is

(A) $\frac{3}{7}$ (B) $\frac{7}{2}$ (C) $\frac{7}{3}$ (D) 1

Sol. $\lambda_B = \lambda_1 + \lambda_2$

$$\frac{\ell n2}{\tau_B} = \frac{2\ell n2}{\tau_A} + \frac{\ell n2}{3\tau_A} = \frac{7\ell n2}{3\tau_A}$$

$= \frac{\tau_B}{\tau_A} = \frac{3}{7}$.

94. A stream of photons having energy 3 eV each impinges on a potassium surface. The work function of potassium is 2.3 eV. The emerging photo-electrons are slowed down by a copper plate placed 5 mm away. If the potential difference between the two metal plates is 1V, the maximum distance the electrons can move away from the potassium surface before being turned back is

(A) 3.5 mm (B) 1.5 mm (C) 2.5 mm (D) 5.0 mm

Sol. 3eV

ϕ = 2.3 eV

K_{max} = 3 – 2.3 = 0.7 eV

5 mm ≡ 1V

1V ≡ 5mm

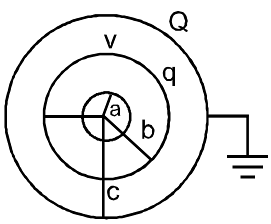
0.7 V ≡ 5 × 0.7 mm

= 3.5 mm.

95. Consider three concentric metallic spheres A, B and C of radii a, b, c respectively where a<b<c. A and B are connected whereas C is grounded. The potential of the middle sphere B is raised to V then the charge on the sphere C is

(A) $-4\pi\varepsilon_0 V \frac{bc}{c-b}$ (B) $+4\pi\varepsilon_0 V \frac{bc}{c-b}$ (C) $-4\pi\varepsilon_0 V \frac{ac}{c-a}$ (D) zero

Sol. $V = \frac{Kq}{b} + \frac{KQ}{c}$



$\frac{k(q+Q)}{c} = 0$

q + Q = 0

q = –Q

$\frac{K}{b}(-Q) + \frac{KQ}{c} = V$

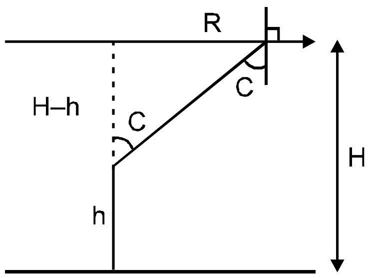
$KQ\left(\frac{1}{c} - \frac{1}{b}\right) = V$

$Q = \frac{bcV}{(b-c)} 4\pi \in_0$.

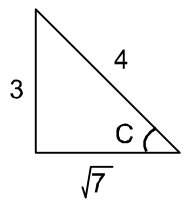
96. On a bright sunny day a diver of height h stands at the bottom of a lake of depth H. Looking upward, he can see objects outside the lake in a circular region of radius R. Beyond this circle he sees the images of objects lying on the floor of the lake. If refractive index of waler is 4/3, then the value of R is

(A) $\frac{3(H-h)}{\sqrt{7}}$ (B) $3h\sqrt{7}$ (C) $\frac{(H-h)}{\sqrt{\frac{7}{3}}}$ (D) $\frac{(H-h)}{\sqrt{\frac{5}{3}}}$

Sol.



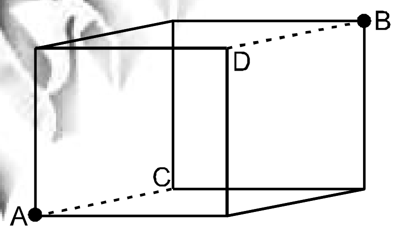
$\frac{4}{3}(\sin C) = 1 \quad \Rightarrow \quad \sin C = \frac{3}{4}$



$\tan C = \frac{3}{\sqrt{17}} = \frac{R}{H-h}$

$R = \frac{3}{\sqrt{7}}(H-h)$.

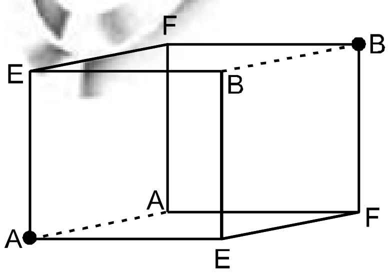
97. As shown in the figure below, a cube is formed with ten indentical resistance R (thick lines) and two shorting wires (dotted lines) along the arms AC and BD.

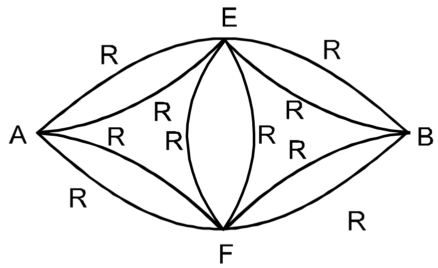


Resistance between point A and B is

(A) $\frac{R}{2}$ (B) $\frac{5R}{6}$ (C) $\frac{3R}{4}$ (D) R

Sol.





W.S. Bridge
R_{AB} = R/2.

98. A standing wave in a pipe with a length L = 1.2 m is described y

$$y(x, t) = y_0 \sin\left[\left(\frac{2\pi}{L}\right)x\right] \sin\left[\left(\frac{2\pi}{L}\right)x + \frac{\pi}{4}\right]$$

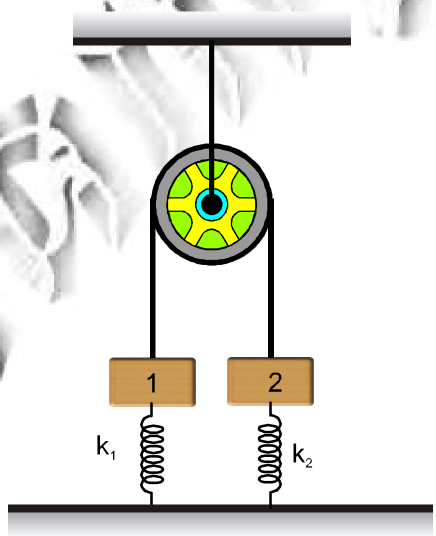
Based on above information, which one of the following statement is incorrect.
(Speed of sound in air is 300 ms^{-1})
(A) A pipe is closed at both ends
(B) The wavelength of the wave could be 1.2 m
(C) There could be a nods at x = 0 and antinode at x = L/2
(D) The frequency of the fundamental mode of vibrations is 137.5 Hz

Sol. From equation

$$\frac{2\pi}{\lambda} = \frac{4\pi}{L} \quad \Rightarrow \quad \lambda = \frac{L}{2} = 0.6 \text{ m.}$$

Ans. (B)

99. Two block (1 and 2) of equal mass m are connected by an ideal string (see figure below) over a frictionless pulley. The blocks are attached to the ground by springs having spring constants k_1 and k_2 such that $k_1 > k_2$.



Initially, both springs are unstretched. The block 1 is slowly pulled down a distance x and released. Just after the release the possible value of the magnitudes of the acceleration of the blocks a_1 and a_2 can be

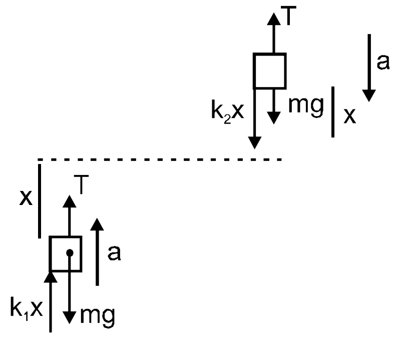
(A) either $\left(a_1 = a_2 = \frac{(k_1 + k_2)x}{2m}\right)$ or $\left(a_1 = \frac{k_1 x}{m} - g \text{ and } a_2 = \frac{k_2 x}{m} + g\right)$

(B) $\left(a_1 = a_2 = \frac{(k_1 + k_2)x}{2m}\right)$ only

(C) $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m}\right)$ only

(D) either $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m}\right)$ or $\left(a_1 = a_2 = \frac{(k_1 k_2)x}{(k_1 + k_2)m} - g\right)$

Sol.



$T + k_1x - mg = ma$
$k_2x + mg - T = ma$
$T + k_1x - mg = k_2x + mg - T$
$2T = (k_2 - k_1)x + 2mg$

$T = \frac{(k_2 - k_1)x}{2} + mg$

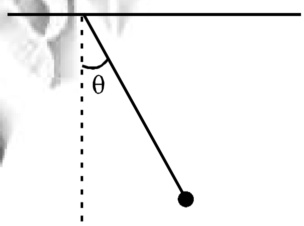
$a = \frac{(k_1 + k_2)x}{2m}$ If T is not zero

If T is 0 then

$a_1 = \frac{k_1x - mg}{m} = \frac{k_1x}{m} - g$

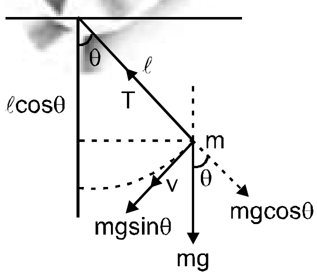
$a_2 = \frac{k_2x + mg}{m} = \frac{k_2x}{m} + g$

100. A simple pendulum is released from rest at the horizontally stretched position. When the string makes an angle θ with the vertical, the angle ϕ which the acceleration vector of the bob makes with the string is given by



(A) $\phi = 0$ (B) $\phi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$ (C) $\phi = \tan^{-1}(2\tan\theta)$ (D) $\phi = \frac{\pi}{2}$

Sol.



$a_t = g \sin\theta$ .... (i)

$a_c = v^2/\ell$ .... (ii)

$0 + 0 = \frac{1}{2}mv^2 - mg\,\ell\cos\theta$ .... (iii)

$a_C = v^2/\ell = 2g\cos\theta$

$a_t = g\sin\theta$

$\tan\phi = a_t/a_c$

$= g\sin\theta/2g\cos\theta$

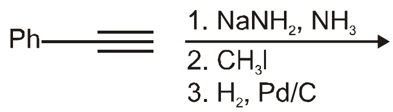
$= \frac{\tan\theta}{2}$

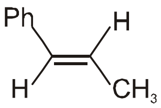
$\phi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$

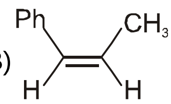
Ans. (B)

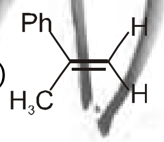
CHEMISTRY

101. The final major product obtained in the following sequence of reactions is :



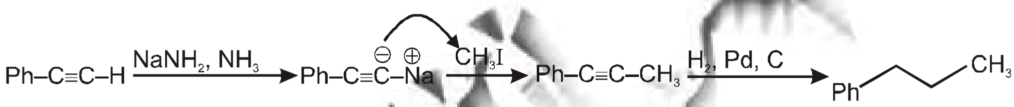
(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. 

102. In the DNA of E. Coli the mole ratio of adenine to cytosine is 0.7. If the number of moles of adenine in the DNA is 350000, the number of moles of guanine is equal to :

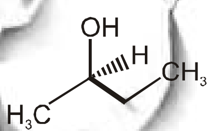
(A) 350000 (B) 500000 (C) 225000 (D) 700000

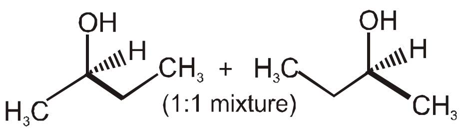
Ans. (B)

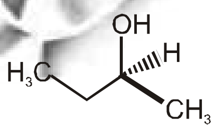
Sol. $\frac{T}{G} = \frac{A}{C} = 0.7 = \frac{350000}{G}$

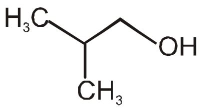
$G = \frac{350000}{0.7} = 5{,}00{,}000$ **Ans.**

103. (R)-2-bromobutane upon treatment with aq. NaOH gives

(A) 

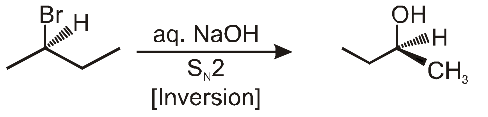
(B) 

(C) 

(D) 

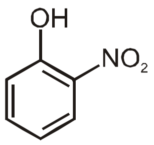
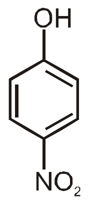
Ans. (C)

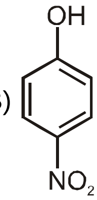
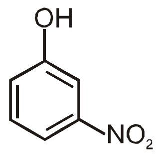
Sol.

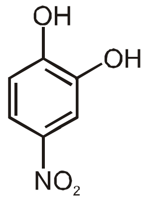
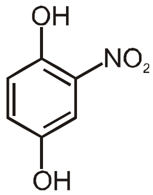


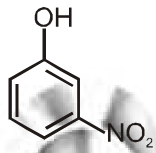
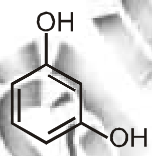
(R)–2–bromobutane (S)Butane–2–ol

104. Phenol on treatment with dil. HNO_3 gives two products P and Q. P is steam volatile but Q is not. P and Q are respectively

(A)  and 

(B)  and 

(C)  and 

(D)  and 

Ans. **(A)**

Sol. Phenol on treatment with dil HNO_3 gives o–nitrophenol and p-nitrophenol.
o–nitrophenol has intra molecular H–bonding hence steam volatile ie P, where as "Q" is p-nitrophenol, Q has inter molecular H-bonding.

105. A metal is irradiated with light of wavelength 660 nm. Given that the work function of the metal is 1.0 eV, the de-Broglie wavelength of the ejected electron is close to :

(A) 6.6×10^{-7} m (B) 8.9×10^{-11} m (C) 1.3×10^{-9} m (D) 6.6×10^{-13} m

Ans. **(C)**

Sol. Kinetic energy = $h\nu$ – work function

$$KE = \frac{1240}{660} - 1$$

KE = 0.878 eV

$$\lambda = \sqrt{\frac{150}{V}}\ \text{Å}$$

$$\lambda = \sqrt{\frac{150}{0.878}}$$

$\lambda = 13.07$ Å $= 1.3 \times 10^{-9}$ m

106. The inter-planar spacing between the (2 2 1) planes of a cubic lattice of length 450 pm is :

(A) 50 pm (B) 150 pm (C) 300 pm (D) 450 pm

Ans. **(B)**

Sol. $$\frac{a}{\sqrt{h^2+K^2+\ell^2}} = \frac{450}{\sqrt{(2)^2+(2)^2+(1)^2}} = \frac{450}{3} = 150 \text{ pm}$$

107. The ΔH for vaporization of a liquid is 20 kJ/mol. Assuming ideal behaviour, the change in internal energy for the vaporization of 1 mole of the liquid at 60ºC and 1 bar is close to :

(A) 13.2 kJ/mol (B) 17.2 kJ/mol (C) 19.5 kJ/mol (D) 20.0 kJ/mol

Ans. **(B)**

Sol. $\Delta H = 20$ kJ/mol
$\Delta U = \Delta H - \Delta n_g RT$
$\Delta U = 20 \times 10^3 - 1 \times 8.314 \times (273 + 60)$
$\Delta U = 20 - 2.768$
$\Delta U = 17.2$ kJ/mol

108. Among the following, the species that is both tetrahedral and diamagnetic is :
(A) $[NiCl_4]^{2-}$ (B) $[Ni(CN)_4]^{2-}$ (C) $Ni(CO)_4$ (D) $[Ni(H_2O)_6]^{2+}$

Ans. **(C)**

Sol. As in $Ni(CO)_4$ hybridisation of Ni is sp^3 and CO is strong field effect ligand therefore, it is diamangetic.

109. Three moles of an ideal gas expands reversibly under isothermal condition from 2 L to 20 L at 300 K. The amount of heat-change (in kJ/mol) in the process is :
(A) 0 (B) 7.2 (C) 10.2 (D) 17.2

Ans. **(D)**

Sol. $W = -nRT \ln \frac{V_2}{V_1}$
$W = -3R \times 300 \ln 10$
$= \frac{-900 \times 8.314}{1000} \times 2.3 = -17.2$ kJ/mol.
$q = -W = 17.2$ kJ/mol. (For isothermal process $\Delta E = 0$).

110. The following data are obtained for a reaction, $X + Y \rightarrow$ Products.

Expt.	$[X_0]$/mol	$[Y_0]$/mol	rate/mol L^{-1} s^{-1}
1	0.25	0.25	1.0×10^{-6}
2	0.50	0.25	4.0×10^{-6}
3	0.25	0.50	8.0×10^{-6}

The overall order of the reaction is :
(A) 2 (B) 4 (C) 3 (D) 5

Ans. **(D)**

Sol. Rate = K . $[X]^p [Y]^q$
From expt. 1 and 2, p = 2 and from expt. 1 and 3, q = 3. Therefore, over all order = 5.

BIOLOGY

111. When hydrogen peroxide is applied on the wound as a disinfectant, there is frothing at the site of injury, which is due to the presence of an enzyme in th skin that uses hydrogen peroxide as a substrate to produce
(A) hydrogen (B) carbon dioxide (C) water (D) oxygen

Ans **(D)**

112. Persons suffering from hypertension (high blood pressure) are advised a low-salt diet because
(A) more salt is absorbed in the body of a patient with hypertension
(B) high salt leads to water retention in the blood that further increases the blood pressure
(C) high salt increases nerve conduction and increases blood pressure
(D) high salt causes adrenaline release that increases blood pressure

Ans **(B)**

113. Insectivorous plants that mostly grow on swampy soil use insects as a source of
(A) carbon (B) nitrogen (C) phosphorous (D) magnesium

Ans **(B)**

114. In cattle, the coat colour red and white are two dominant traits, which express equally in F1 to produce roan (red and white clour in equal proportion). If F1 progeny are self-bred, the resulting progeny in F2 will have phenotypic ratio (red:roan:white) is -
(A) 1 : 1 : 1 (B) 3 : 9 : 3 (C) 1 : 2 : 1 (D) 3 : 9 : 4

Ans **(C)**

115. The restriction endonuclease EcoR-I recognises and cleaves DNA sequence as shown below -

5' – G A A T T C – 3'
3' – C T T A A G – 5'

What is the probable number of cleavage sites that can occur in a 10 kb long random DNA sequence?

(A) 10 (B) 2 (C) 100 (D) 50

Ans (B)

116. Which one of the following is true about enzyme catalysis ?

(A) the enzyme changes at the end of the reaction
(B) the activation barrier of the process is lower in the presence of an enzyme
(C) the rate of the reaction is retarded in the presence of an enzyme
(D) the rate of the reaction is independent of substrate concentration

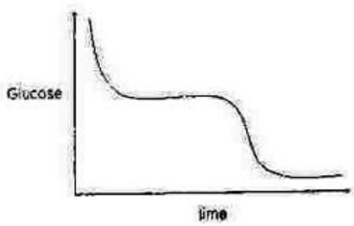
Ans (B)

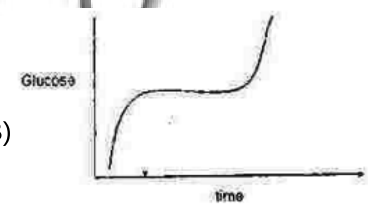
117. Vibrio cholerae causes cholera in humans. Ganga water was once used successfully to combat the infection. The possible reason could be -

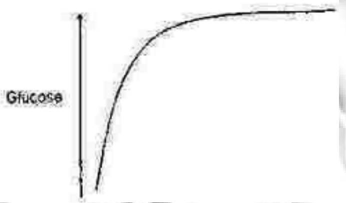
(A) high salt content of Ganga water
(B) low salt content of Ganga water
(C) presence of bacteriophages in Ganga water
(D) presence of antibiotics in Ganga water

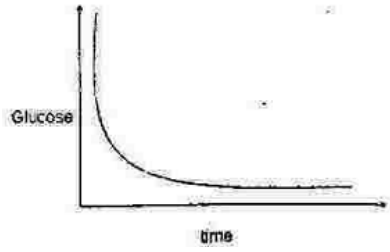
Ans (C)

118. When a person begins to fast, after some time glycogen stored in the liver is mobilized as a source of glucose. Which of the following graphs best represents the change of glucose level (y-axis) in his blood, starting from the time (x-axis) when he begins to fast ?

(A) 

(B) 

(C) 

(D) 

Ans (C)

119. The following sequence contains the open reading frame of a polypeptide. How many amino acids will the polypeptide consist of ?

5'–AGCATATGATCGTTTCTCTGCTTTGAACT–3

(A) 4 (B) 2 (C) 10 (D) 7

Ans (D)

120. Insects constitute the largest animal group on earth. About 25-30% of the insect species are known to be herbivores. In spite of such huge herbiore perssure, globally, green plants have persisted. One possible reason for this persistence is :

(A) food preference of insects has tended to change with time
(B) herbivore insects have become inefficient feeders of green plants
(C) herbivore population has been kept in control by predators
(D) decline in reproduction of herbivores with time

Ans (C)