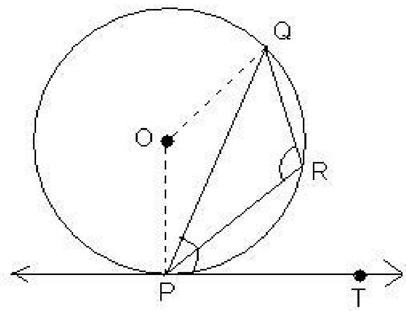
CBSE Board
Class X Mathematics
Board Paper – 2015 Solution
All India Set – 2

1.



$m\angle OPT = 90°$ $(\because$ radius is perpendicular to the tangent$)$

$$\begin{aligned}\text{So,} \angle OPQ &= \angle OPT - \angle QPT \\ &= 90° - 60° \\ &= 30°\end{aligned}$$

$m\angle POQ = 2\angle QPT = 2 \times 60° = 120°$

reflex $m\angle POQ = 360° - 120° = 240°$

$$\begin{aligned}\angle PRQ &= \frac{1}{2} \text{ reflex } \angle POQ \\ &= \frac{1}{2} \times 240 \\ &= 120\end{aligned}$$

$\therefore m\angle PRQ = 120°$

2. The given quadratic equation is,

$px^2 - 2\sqrt{5}px + 15 = 0$

Here, $a = p,\ b = 2\sqrt{5}p,\ c = 15$

For real equal roots, discriminant $= 0$

$\therefore b^2 - 4ac = 0$

$\therefore \left(2\sqrt{5}p\right)^2 - 4p(15) = 0$

$\therefore 20p^2 - 60p = 0$

$\therefore 20p(p-3) = 0$

$\therefore p = 3$ or $p = 0$

But, $p = 0$ is not possible.

$\therefore p = 3$

3. Let AB be the tower and BC be its shadow.

$AB = 20,\ BC = 20\sqrt{3}$

In ΔABC,

$\tan\theta = \frac{AB}{BC}$

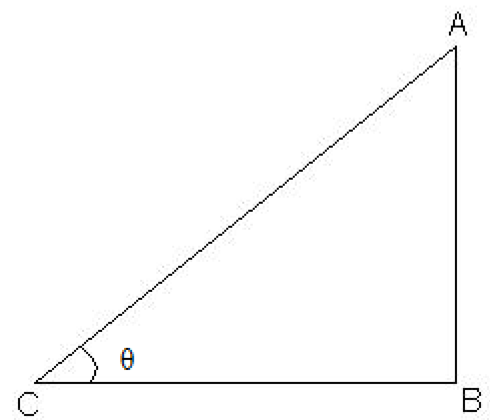
$\tan\theta = \frac{20}{20\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

but, $\tan 30 = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30$

$\therefore$ The Sun is at an altitude of 30°.



4. Two dice are tossed

S = [(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)]

Total number of outcomes when two dice are tossed = 6 x 6= 36

Favourable events of getting the product as 6 are:

$(1\times6=6), (6\times1=6),(2\times3=6),(3\times2=6)$

i.e.(1,6), (6,1), (2,3), (3,2)

Favourable events of getting product as 6 = 4

$\therefore$ P(getting product as 6) = $\frac{4}{36}=\frac{1}{9}$

SECTION B

5. Given that the points $A(x,y)$, B(−5,7) and C(−4,5) are collinear.

So, the area formed by these vertices is 0.

$$\therefore \frac{1}{2}\left[x(7-5)+(-5)(5-y)+(-4)(y-7)\right]=0$$

$$\Rightarrow \frac{1}{2}[2x-25+5y-4y+28]=0$$

$$\Rightarrow \frac{1}{2}[2x+y+3]=0$$

$$\Rightarrow 2x+y+3=0$$

$$\Rightarrow y=-2x-3$$

6. $S_5 + S_7 = 167 \quad \text{and } S_{10} = 235$

$$\text{Now, } S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\therefore S_5 + S_7 = 167$$

$$\Rightarrow \frac{5}{2}\{2a + 4d\} + \frac{7}{2}\{2a + 6d\} = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow 12a + 31d = 167 \qquad ....(1)$$

$$\text{Also, } S_{10} = 235$$

$$\therefore \frac{10}{2}\{2a + 9d\} = 235$$

$$\Rightarrow 10a + 45d = 235$$

$$\Rightarrow 2a + 9d = 47 \qquad ....(2)$$

Multiplying equation (2) by 6, we get

$$12a + 54d = 282 \qquad .....(3)$$

Subtracting (1) from (3), we get

$$\begin{array}{rl} & 12a + 54d = 282 \\ (-) & 12a + 31d = 167 \\ & - \quad\;\; - \qquad - \\ \hline & \qquad 23d = 115 \end{array}$$

$$\therefore d = 5$$

Substituting value of d in (2), we have

$$2a + 9(5) = 47$$

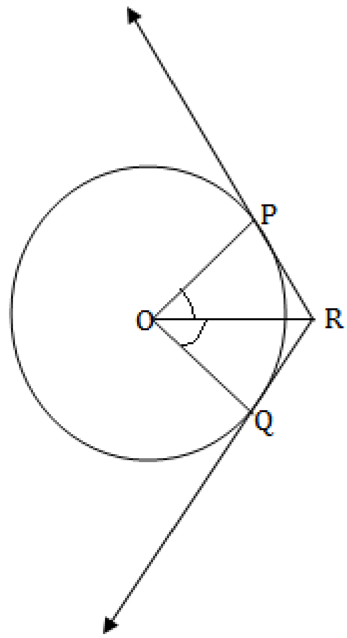
$$\Rightarrow 2a + 45 = 47$$

$$\Rightarrow 2a = 2$$

$$\Rightarrow a = 1$$

Thus, the given A.P. is 1, 6, 11, 16,..........

7.



Given that m$\angle$PRQ = 120°

We know that the line joining the centre and the external point is the angle bisector between the tangents.

Thus, m$\angle$PRO = m$\angle$QRO = $\frac{120°}{2} = 60°$

Also we know that lengths of tangents from an external point are equal.

Thus, PR = RQ.

Join OP and OQ.

Since OP and OQ are the radii from the centre O,

OP $\perp$ PR and OQ $\perp$ RQ.

Thus, ΔOPR and ΔOQR are right angled congruent triangles.

Hence, m$\angle$POR = 90° − m$\angle$PRO = 90° − 60° = 30°

m$\angle$QOR = 90° − m$\angle$QRO = 90° − 60° = 30°

sin$\angle$QRO = sin30° $= \frac{1}{2}$

But $\sin 30° = \frac{PR}{OR}$

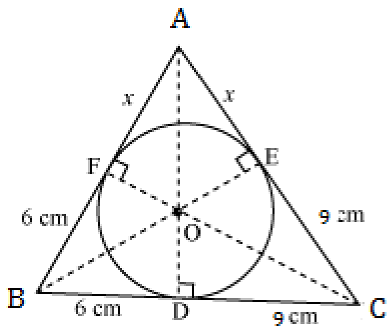
Thus, $\frac{PR}{OR} = \frac{1}{2}$

$\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + QR$

8.



Let the given circle touch the sides AB and AC of the triangle at points F and E respectively and let the length of the line segment AF be *x*.

Now, it can be observed that:

BF = BD = 6 cm (tangents from point B)

CE = CD = 9 cm (tangents from point C)

AE = AF = x (tangents from point A)

AB = AF + FB = x + 6

BC = BD + DC = 6 + 9 = 15

CA = CE + EA = 9 + x

2s = AB + BC + CA = x + 6 + 15 + 9 + x = 30 + 2x

s = 15 + x

s – a = 15 + x – 15 = x

s – b = 15 + x – (x + 9) = 6

s – c = 15 + x – (6 + x) = 9

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$$54=\sqrt{(15+x)(x)(6)(9)}$$

$$54=3\sqrt{6(15x+x^2)}$$

$$18=\sqrt{6(15x+x^2)}$$

$$324=6(15x+x^2)$$

$$54=15x+x^2$$

$$x^2+15x-54=0$$

$$x^2+18x-3x-54=0$$

$$x(x+18)-3(x+18)$$

$$(x+18)(x-3)=0$$

$$x=-18 \text{ and } x=3$$

As distance cannot be negative, x = 3

AC = 3 + 9 = 12

AB = AF + FB = 6 + x = 6 + 3 = 9

9. $4x^2+4bx-(a^2-b^2)=0$

$$\Rightarrow x^2+bx-\left(\frac{a^2-b^2}{4}\right)=0$$

$$\Rightarrow x^2+2\left(\frac{b}{2}\right)x=\frac{a^2-b^2}{4}$$

$$\Rightarrow x^2+2\left(\frac{b}{2}\right)x+\left(\frac{b}{2}\right)^2=\frac{a^2-b^2}{4}+\left(\frac{b}{2}\right)^2$$

$$\Rightarrow \left(x+\frac{b}{2}\right)^2=\frac{a^2}{4}$$

$$\Rightarrow x+\frac{b}{2}=\pm\frac{a}{2}$$

$$\Rightarrow x=\frac{-b}{2}\pm\frac{a}{2}$$

$$\Rightarrow x=\frac{-b-a}{2},\frac{-b+a}{2}$$

Hence, the roots are $-\left(\frac{a+b}{2}\right)$ and $\left(\frac{a-b}{2}\right)$.

10. Given that A(4, 3), B(-1, y) and C(3, 4) are the vertices of the ΔABC.

ΔABC is a right triangle at A.

Hence by applying the Pythagoras Theorem, we have,

$AB^2+AC^2=BC^2$....(1)

Let us find the distances, AB, BC and CA using the distance formula.

$$AB=\sqrt{(-1-4)^2+(y-3)^2}$$

$$BC=\sqrt{(3+1)^2+(4-y)^2}$$

$$CA=\sqrt{(3-4)^2+(4-3)^2}=\sqrt{2}$$

Squaring both the sides, we have

$$AB^2=25+y^2+9-6y$$

$$BC^2=4+16+y^2-8y$$

$$AC^2=2$$

Therefore, from equation (1), we have,

$$25+y^2+9-6y+2=4+16+y^2-8y$$

$$\Rightarrow 36+y^2-6y=20+y^2-8y$$

$$\Rightarrow 16-6y=-8y$$

$$\Rightarrow 16=-8y+6y$$

$\Rightarrow -2y = 16$

$\Rightarrow -y = \frac{16}{2}$

$\Rightarrow y = -8$

SECTION C

11. Diameter of the tent = 4.2 m

Radius of the tent, r = 2.1 m

Height of the cylindrical part of tent, $h_{cylinder}$ = 4 m

Height of the conical part, h_{cone} = 2.8 m

Slant height of the conical part, ℓ

$= \sqrt{{h_{cone}}^2 + r^2}$

$= \sqrt{2.8^2 + 2.1^2}$

$= \sqrt{2.8^2 + 2.1^2}$

$= \sqrt{12.25} = 3.5$ m

Curved surface area of the cylinder = $2\pi r\, h_{cylinder}$

$= 2 \times \frac{22}{7} \times 2.1 \times 4$

$= 22 \times 0.3 \times 8 = 52.8\ m^2$

Curved surface area of the conical tent = $\pi rl = \frac{22}{7} \times 2.1 \times 3.5 = 23.1\ m^2$

Total area of cloth required for building one tent

= Curved surface area of the cylinder + Curved surface area of the conical tent

= 52.8 + 23.1

= 75.9 m^2

Cost of building one tent = 75.9 × 100 = Rs. 7590

Total cost of 100 tents = 7590 × 100 = Rs. 7,59,000

Cost to be borne by the associations = $\frac{759000}{2}$ = Rs. 3,79,500

It shows the helping nature, unity and cooperativeness of the associations.

12. Let BC be the height at which the aeroplane is observed from point A.

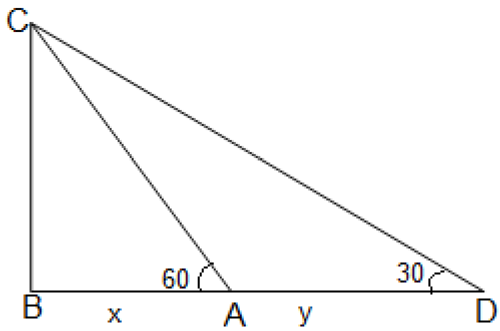
Then, BC = $1500\sqrt{3}$

In 15 seconds, the aeroplane moves from point A to D.

A and D are the points where the angles of elevation 60° and 30° are formed respectively.

Let BA = x metres and AD = y metres

BC = x + y



In ΔCBA,

$\tan 60° = \dfrac{BC}{BA}$

$\sqrt{3} = \dfrac{1500\sqrt{3}}{x}$

$\therefore x = 1500 \text{ m}$ ....(1)

In ΔCBD,

$\tan 30° = \dfrac{BC}{BD}$

$\dfrac{1}{\sqrt{3}} = \dfrac{1500\sqrt{3}}{x+y}$

$\therefore x + y = 1500(3) = 4500$

$\therefore 1500 + y = 4500$

$\therefore y = 3000 \text{ m}$ ....(2)

We know that the aeroplane moves from point A to D in 15 seconds and the distance covered is 3000 metres. (by 2)

Speed $= \dfrac{\text{distance}}{\text{time}}$

Speed $= \dfrac{3000}{15}$

Speed $= 200 \text{m/s}$

Converting it to km/hr = $200 \times \dfrac{18}{5} = 720 \text{ km/hr}$

13. Internal diameter of the bowl = 36 cm

Internal radius of the bowl, r = 18 cm

Volume of the liquid, $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 18^3$

Let the height of the small bottle be 'h'.

Diameter of a small cylindrical bottle = 6 cm

Radius of the small bottle, R = 3 cm

Volume of a single bottle = $\pi R^2 h = \pi \times 3^2 \times h$

No. of small bottles, n = 72

Volume wasted in the transfer = $\frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

Volume of liquid to be transferred in the bottles

$= \frac{2}{3} \times \pi \times 18^3 - \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

$= \frac{2}{3} \times \pi \times 18^3 \left(1 - \frac{10}{100}\right)$

$= \frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}$

Number of the small cylindrical bottles = $\frac{\text{Volume of the liquid to be transferred}}{\text{Volume of a single bottle}}$

$$\Rightarrow 72 = \frac{\frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}}{\pi \times 3^2 \times h}$$

$$\Rightarrow 72 = \frac{\frac{2}{3} \times 18^3 \times \frac{9}{10}}{3^2 \times h}$$

$$\Rightarrow h = \frac{\frac{2}{3} \times 18 \times 18 \times 18 \times \frac{9}{10}}{3^2 \times 72}$$

$\therefore h = 5.4$ cm

Height of the small cylindrical bottle = 5.4 cm

14. Here the jar contains red, blue and orange balls.

Let the number of red balls be x.

Let the number of blue balls be y.

Number of orange balls = 10

$\therefore$ Total number of balls = x + y + 10

Now, let P be the probability of drawing a ball from the jar

$$P(\text{a red ball}) = \frac{x}{x+y+10}$$

$$\Rightarrow \frac{1}{4} = \frac{x}{x+y+10}$$

$$\Rightarrow 4x = x + y + 10$$

$$\Rightarrow 3x - y = 10 \qquad \text{----(i)}$$

Next,

$$P(\text{a blue ball}) = \frac{y}{x+y+10}$$

$$\Rightarrow \frac{1}{3} = \frac{y}{x+y+10}$$

$$\Rightarrow 3y = x + y + 10$$

$$\Rightarrow 2y - x = 10 \qquad \text{-----(ii)}$$

Multiplying eq. (i) by 2 and adding to eq. (ii), we get

$$\begin{array}{r} 6x - 2y = 20 \\ -x + 2y = 10 \\ \hline 5x = 30 \end{array}$$

$$\Rightarrow \quad x = 6$$

Subs. x = 6 in eq. (i), we get y = 8

$\therefore$ Total number of balls = x + y + 10 = 6 + 8 + 10 = 24

Hence, total number of balls in the jar is 24.

15. Side of the cubical block, a = 10 cm

Longest diagonal of the cubical block = $a\sqrt{3} = 10\sqrt{3}$ cm

Since the cube is surmounted by a hemisphere, therefore the side of the cube should be equal to the diameter of the hemisphere.

Diameter of the sphere = 10 cm

Radius of the sphere, r = 5 cm

Total surface area of the solid = Total surface area of the cube – Inner cross-section area of the hemisphere + Curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= 6a^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times 5^2$

$= 600 + 78.5 = 678.5 \text{ cm}^2$

Total surface area of the solid = 678.5 cm^2

Cost of painting per sq. cm = Rs. 5

Cost of painting the total surface area of the solid = 678.5 × 5 = Rs. 3392.50

16. Here, P(x,y) divides line segment AB, such that

$$AP = \frac{3}{7} AB$$

$$\frac{AP}{AB} = \frac{3}{7}$$

$$\frac{AB}{AP} = \frac{7}{3}$$

$$\frac{AB}{AP} - 1 = \frac{7}{3} - 1$$

$$\frac{AB - AP}{AP} = \frac{7-3}{3}$$

$$\frac{BP}{AP} = \frac{4}{3}$$

$$\frac{AP}{BP} = \frac{3}{4}$$

$\therefore$ P divides AB in the ratio 3 : 4

$$x = \frac{3\times 2 + 4(-2)}{3+4}; \quad y = \frac{3\times(-4) + 4(-2)}{3+4}$$

$$x = \frac{6-8}{7}; \quad y = \frac{-12-8}{7}$$

$$x = \frac{-2}{7}; \quad y = \frac{-20}{7}$$

$\therefore$ The co-ordinates of P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$

17. No. of cones = 504

Diameter of a cone = 3.5 cm

Radius of the cone, r = 1.75 cm

Height of the cone, h = 3 cm

Volume of a cone

$$=\frac{1}{3}\pi r^2 h$$

$$=\frac{1}{3}\times\pi\times\left(\frac{3.5}{2}\right)^2\times 3$$

$$=\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3\text{ cm}^3$$

Volume of 504 cones

$$=504\times\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3\text{ cm}^3$$

Let the radius of the new sphere be 'R'.

Volume of the sphere $=\frac{4}{3}\pi R^3$

Volume of 504 cones = Volume of the sphere

$$504\times\frac{1}{3}\times\pi\times\frac{3.5}{2}\times\frac{3.5}{2}\times 3=\frac{4}{3}\pi R^3$$

$$\Rightarrow\frac{504\times 1\times\pi\times 3.5\times 3.5\times 3\times 3}{3\times 2\times 2\times 4\times\pi}=R^3$$

$$\Rightarrow R^3=\frac{504\times 3\times 49}{64}$$

$$\Rightarrow R^3=\frac{7\times 8\times 9\times 3\times 7^2}{64}$$

$$\Rightarrow R^3=\frac{8\times 27\times 7^3}{64}$$

$$\Rightarrow R=\frac{2\times 3\times 7}{4}$$

$$\therefore R=\frac{21}{2}=10.5\text{ cm}$$

$\therefore$ Radius of the new sphere = 10.5 cm

Surface area of the new sphere $=4\pi R^2$

$$=4\times\frac{22}{7}\times\frac{21}{2}\times\frac{21}{2}$$

$$=2772\text{ cm}^2$$

18. Given that the area of the circle is 1256 cm^2.

$\pi r^2 = 1256$

$\Rightarrow 3.14 \times r^2 = 1256$

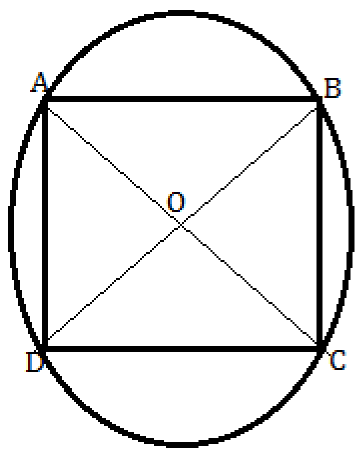
$\Rightarrow r^2 = \frac{1256}{3.14}$

$\Rightarrow r^2 = 400$

$\Rightarrow r = 20 \text{ cm}$

If all the vertices of a rhombus lie on a circle, then the rhombus is square.

Consider the following figure.



Here A, B, C and D are four points on the circle.
Thus, OA = OB = OC = OD = radius of the circle.
$\Rightarrow$AC and BD are the diameters of the circle.
Consider the Δ ADC.
By Pythagoras theorem, we have,

$AD^2 + CD^2 = AC^2$

$\Rightarrow 2AD^2 = (2 \times 20)^2$ ....[AD = CD, side of the square]

$\Rightarrow 2AD^2 = (40)^2$

$\Rightarrow 2AD^2 = 1600$

$\Rightarrow AD^2 = \frac{1600}{2}$

$\Rightarrow AD^2 = 800 cm^2$

If AD is the side of the square, then AD^2 is the area of the square.

Thus area of the square is $800 cm^2$

19. Consider the given equation:

$$2x^2+6\sqrt{3}-60=0$$

$$\Rightarrow x^2+3\sqrt{3}-30=0$$

Let us the quadratic formula to find x.

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Here, $a = 1, b = 3\sqrt{3}$ and $c = -30$

Thus,

$$x=\frac{-3\sqrt{3}\pm\sqrt{\left(3\sqrt{3}\right)^2-4\times1\times(-30)}}{2\times1}$$

$$\Rightarrow x=\frac{-3\sqrt{3}\pm\sqrt{27+120}}{2}$$

$$\Rightarrow x=\frac{-3\sqrt{3}\pm\sqrt{147}}{2}$$

20. Given that 16th term of an A.P. is five times its third term.

We know that

$$t_n=a+(n-1)d$$

Thus,

$$t_{16}=a+(16-1)d$$

$$t_3=a+(3-1)d$$

Since $t_{16}=5t_3$, we have,

$$a+(16-1)d=5\left[a+(3-1)d\right]$$

$$\Rightarrow a+15d=5[a+2d]$$

$$\Rightarrow a+15d=5a+10d$$

$$\Rightarrow 5d=4a$$

$$\Rightarrow 4a-5d=0 \ldots(1)$$

Also given that $t_{10}=41$

$$\Rightarrow t_{10}=a+(10-1)d$$

$$\Rightarrow 41=a+9d$$

$$\Rightarrow a+9d=41 \ldots(2)$$

Multiplying equation (2) by 4, we have,

$$4a + 36d = 164 \ldots(3)$$

Subtracting equation (1) from equation (3), we have,

$\left[36-(-5)\right]d=164$

$\Rightarrow 41d=164$

$\Rightarrow d=\frac{164}{41}$

$\Rightarrow d=4$

Substituting d = 4 in equation (1) $4a-5d=0$, we have,

$4a-5\times 4=0$

$\Rightarrow 4a-20=0$

$\Rightarrow 4a=20$

$\Rightarrow a=\frac{20}{4}$

$\Rightarrow a=5$

We need to find S_{15}

We know that

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow S_{15}=\frac{15}{2}\left[2\times 5+(15-1)\times 4\right] \quad [\because a=5, n=15, d=4]$

$\Rightarrow S_{15}=\frac{15}{2}[10+14\times 4]$

$\Rightarrow S_{15}=\frac{15}{2}\times 66$

$\Rightarrow S_{15}=495$

SECTION D

21. In the figure, C is the midpoint of the minor arc PQ, O is the centre of the circle and AB is tangent to the circle through point C.

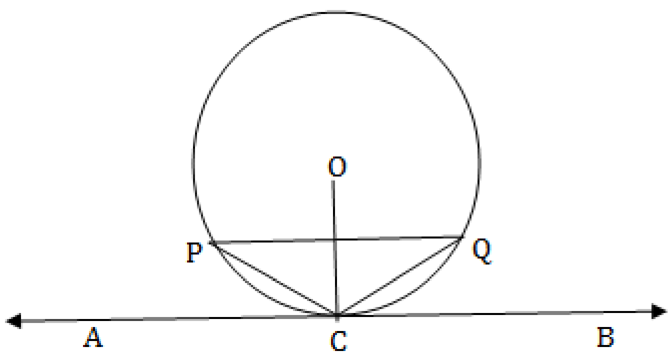
We have to show the tangent drawn at the mid-point of the arc PQ of a circle is parallel to the chord joining the end points of the arc PQ.

We will show PQ || AB.

It is given that C is the midpoint point of the arc PQ.

So, arc PC = arc CQ.

$\Rightarrow$PC = CQ



This shows that ΔPQC is an isosceles triangle.

Thus, the perpendicular bisector of the side PQ of ΔPQC passes through vertex C.

The perpendicular bisector of a chord passes through the centre of the circle.

So the perpendicular bisector of PQ passes through the centre O of the circle.

Thus the perpendicular bisector of PQ passes through the points O and C.

$\Rightarrow PQ \perp OC$

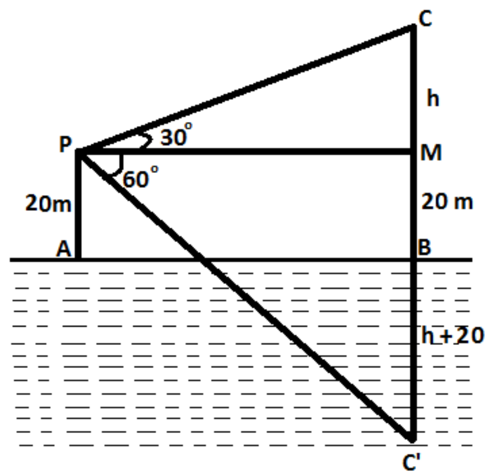
AB is the tangent to the circle through the point C on the circle.

$\Rightarrow AB \perp OC$

The chord PQ and the tangent PQ of the circle are perpendicular to the same line OC.

$\therefore PQ \parallel AB$.

22.



Let AB be the surface of the lake and P be the point of observation such that AP = 20 metres. Let C be the position of the cloud and C' be its reflection in the lake. Then CB = C'B. Let PM be perpendicular from P on CB.

Then $m\angle CPM = 30°$ and $m\angle C'PM = 60°$

Let CM = h. Then CB = h + 20 and C'B = h + 20.

In ΔCMP we have,

$\tan 30° = \frac{CM}{PM}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$

$\Rightarrow PM = \sqrt{3}h$..................(i)

In $\Delta PMC'$ we have,

$\tan 60° = \frac{C'M}{PM}$

$\Rightarrow \sqrt{3} = \frac{C'B + BM}{PM}$

$\Rightarrow \sqrt{3} = \frac{h + 20 + 20}{PM}$

$\Rightarrow PM = \frac{h + 20 + 20}{\sqrt{3}}$..................(ii)

From equation (i) and (ii), we get

$\sqrt{3}h = \frac{h + 20 + 20}{\sqrt{3}}$

$\Rightarrow 3h = h + 40$

$\Rightarrow 2h = 40$

$\Rightarrow h = 20$ m

Now, $CB = CM + MB = h + 20 = 20 + 20 = 40$.

Hence, the height of the cloud from the surface of the lake is 40 metres.

23. Let S be the sample space of drawing a card from a well-shuffled deck.

$n(S) = {}^{52}C_1 = 52$

(i)There are 13 spade cards and 4 ace's in a deck
As ace of spade is included in 13 spade cards,
so there are 13 spade cards and 3 ace's

A card of spade or an ace can be drawn in ${}^{13}C_1 + {}^{3}C_1 = 13 + 3 = 16$

Probability of drawing a card of spade or an ace $= \frac{16}{52} = \frac{4}{13}$

(ii)There are 2 black king cards in a deck
A card of black king can be drawn in ${}^{2}C_1 = 2$

Probability of drawing a black king $= \frac{2}{52} = \frac{1}{26}$

(iii)There are 4 jack and 4 king cards in a deck.
So there are $52 - 8 = 44$ cards which are neither jack nor king.
a card which is neither a jack nor a king can be drawn in ${}^{44}C_1 = 44$

Probability of drawing a card which is neither a jack nor a king $= \frac{44}{52} = \frac{11}{13}$

(iv)There are 4 king and 4 queen cards in a deck.
So there are $4 + 4 = 8$ cards which are either king or queen.
a card which is either a king or a queen can be drawn in ${}^{8}C_1 = 8$

Probability of drawing a card which is either a king or a queen $= \frac{8}{52} = \frac{2}{13}$

24. PQRS is a square.

So each side is equal and angle between the adjacent sides is a right angle.

Also the diagonals perpendicularly bisect each other.

In ΔPQR using pythagoras theorem,

$$PR^2=PQ^2+QR^2$$

$$PR^2=(42)^2+(42)^2$$

$$PR=\sqrt{2}(42)$$

$$OR=\frac{1}{2}PR=\frac{42}{\sqrt{2}}=OQ$$

From the figure we can see that the radius of the flower bed ORQ is OR.

$$\text{Area of sector ORQ}=\frac{1}{4}\pi r^2$$

$$=\frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2$$

$$\text{Area of the }\Delta ROQ=\frac{1}{2}\times RO\times OQ$$

$$=\frac{1}{2}\times\frac{42}{\sqrt{2}}\times\frac{42}{\sqrt{2}}$$

$$=\left(\frac{42}{2}\right)^2$$

Area of the flower bed ORQ

= Area of sector ORQ – Area of the ΔROQ

$$=\frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2-\left(\frac{42}{2}\right)^2$$

$$=\left(\frac{42}{2}\right)^2\left[\frac{\pi}{2}-1\right]$$

$$=(441)[0.57]$$

$$=251.37cm^2$$

Area of the flower bed ORQ =Area of the flower bed OPS

$$=251.37cm^2$$

Total area of the two flower beds

= Area of the flower bed ORQ+Area of the flower bed OPS

$$=251.37+251.37$$

$$=502.74cm^2$$

25. Height of the cylinder (h) = 10 cm

Radius of the base of the cylinder = 4.2 cm

Volume of original cylinder $=\pi r^2 h$

$$=\frac{22}{7}\times(4.2)^2\times 10$$

$$=554.4 cm^3$$

Volume of hemisphere $=\frac{2}{3}\pi r^3$

$$=\frac{2}{3}\times\frac{22}{7}\times(4.2)^3$$

$$=155.232 cm^3$$

Volume of the remaining cylinder after scooping out the hemisphere from each end

$=$ Volume of original cylinder $-2\times$ Volume of hemisphere

$$=554.4-2\times 155.232$$

$$=243.936 cm^3$$

The remaining cylinder is melted and converted to

a new cylindrical wire of 1.4 cm thickness.

So they have the same volume and radius of the new cylindrical wire, i.e. 0.7 cm.

Volume of the remaining cylinder = Volume of the new cylindrical wire

$$243.936=\pi r^2 h$$

$$243.936=\frac{22}{7}(0.7)^2 h$$

$$h=158.4 cm$$

$\therefore$ The length of the new cylindrical wire of 1.4 cm thickness is 158.4 cm.

26. Let ℓ be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 metres more than the shorter side.

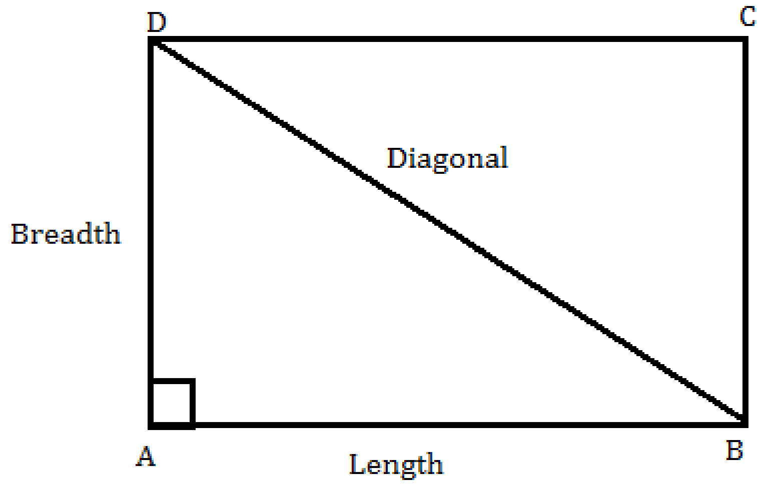
Thus, diagonal = 16 + b

Since longer side is 14 metres more than shorter side, we have,

$\ell = 14 + b$

Diagonal is the hypotenuse of the triangle.

Consider the following figure of the rectangular field.



By applying Pythagoras Theorem in $\triangle ABD$, we have,

$\text{Diagonal}^2 = \text{Length}^2 + \text{Breadth}^2$

$\Rightarrow (16+b)^2 = (14+b)^2 + b^2$

$\Rightarrow 256 + b^2 + 32b = 196 + b^2 + 28b + b^2$

$\Rightarrow 256 + 32b = 196 + 28b + b^2$

$\Rightarrow 60 + 32b = 28b + b^2$

$\Rightarrow b^2 - 4b - 60 = 0$

$\Rightarrow b^2 - 10b + 6b - 60 = 0$

$\Rightarrow b(b-10) + 6(b-10) = 0$

$\Rightarrow (b+6)(b-10) = 0$

$\Rightarrow (b+6) = 0 \text{ or } (b-10) = 0$

$\Rightarrow b = -6 \text{ or } b = 10$

As breadth cannot be negative, breadth =10 m

Thus, length of the rectangular field=14+10=24 m

27. Consider the given A.P. 8, 10, 12, ...

Here the initial term is 8 and the common difference is 10 - 8 = 2 and 12 - 10 = 2

General term of an A.P. is t_n and formula to t_n is

$$t_n = a + (n-1)d$$
$$\Rightarrow t_{60} = 8 + (60-1) \times 2$$
$$\Rightarrow t_{60} = 8 + 59 \times 2$$
$$\Rightarrow t_{60} = 8 + 118$$
$$\Rightarrow t_{60} = 126$$

We need to find the sum of last 10 terms.

Thus,

Sum of last 10 terms = Sum of first 60 terms - Sum of first 50 terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow S_{60} = \frac{60}{2}[2 \times 8 + (60-1) \times 2]$$
$$\Rightarrow S_{60} = 30[16 + 59 \times 2]$$
$$\Rightarrow S_{60} = 30[134]$$
$$\Rightarrow S_{60} = 4020$$

Similarly,

$$\Rightarrow S_{50} = \frac{50}{2}[2 \times 8 + (50-1) \times 2]$$
$$\Rightarrow S_{50} = 25[16 + 49 \times 2]$$
$$\Rightarrow S_{50} = 25[114]$$
$$\Rightarrow S_{50} = 2850$$

Therefore,

Sum of last 10 terms = Sum of first 60 terms $-$ Sum of first 50 terms

Thus the sum of last 10 terms $= S_{60} - S_{50} = 4020 - 2850 = 1170$

28. Let x be the initial speed of the bus.

We know that $\frac{\text{Distance}}{\text{Speed}} = \text{time}$

Thus, we have,

$$\frac{75}{x} + \frac{90}{x+10} = 3 \text{ hours}$$

$$\Rightarrow \frac{75(x+10) + 90x}{x(x+10)} = 3$$

$$\Rightarrow 75(x+10) + 90x = 3x(x+10)$$

$$\Rightarrow 75x + 750 + 90x = 3x^2 + 30x$$

$$\Rightarrow 165x + 750 = 3x^2 + 30x$$

$$\Rightarrow 3x^2 - 165x - 750 + 30x = 0$$

$$\Rightarrow 3x^2 - 135x - 750 = 0$$

$$\Rightarrow x^2 - 45x - 250 = 0$$

$$\Rightarrow x^2 - 50x + 5x - 250 = 0$$

$$\Rightarrow x(x-50) + 5(x-50) = 0$$

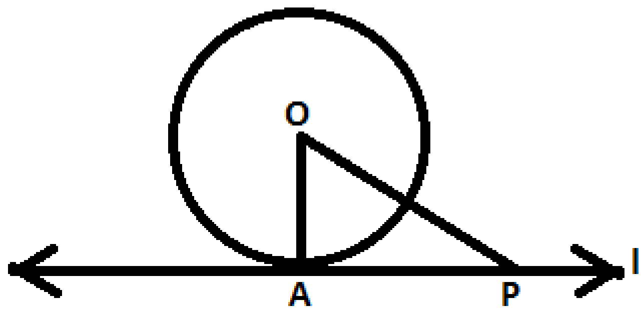
$$\Rightarrow (x+5)(x-50) = 0$$

$$\Rightarrow (x+5) = 0 \text{ or } (x-50) = 0$$

$$\Rightarrow x = -5 \text{ or } x = 50$$

Speed cannot be negative and hence first speed of the train is 50 km/hour.

29.



Given: Line l is tangent to the $\odot(O, r)$ at point A.

To prove: $\overline{OA} \perp l$

Proof: Let $P \in l$, $P \neq A$.

If P is in the interior of $\odot(O, r)$, then the line l will be a secant of the circle and not a tangent.

But l is a tangent of the circle, so P is not in the interior of the circle.

Also $P \neq A$.

$\therefore$ P is the point in the exterior of the circle.

$\therefore OP > OA$. $(\overline{OA}$ is the radius of the circle$)$

Therefore each point $P \in l$ except A satisfies the inequality $OP > OA$.

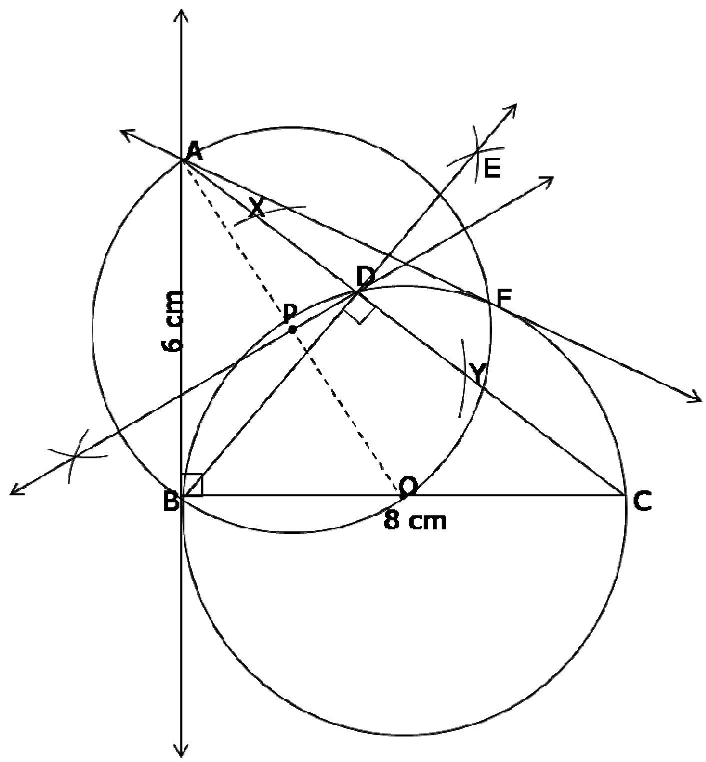
Therefore OA is the shortest distance of line l from O.

$\therefore \overline{OA} \perp l$.

30. Steps of construction:

1) Construct the triangle as per given measurements.
2) Take any arbitrary radius and draw two arcs of circle from point B on AC, intersecting AC at X and Y.
3) Taking X and Y as centres, draw two arcs of circles to intersect each other at point E. Join B and E. BE is the perpendicular from B on AC.
4) ΔBDC is a right angled. Hence, BC the hypotenuse will form the diameter of the circle passing through the vertices of ΔBDC.
5) BC = 8 cm $\therefore$ OC = 4 cm. draw a circle of radius equal 4 cm, passing through B, D and C.
6) Join O and A. Obtain the mid-point P of segment OA by drawing perpendicular bisector to OA.
7) Draw a circle with centre P and radius AP.
8) Let B and F be the points of intersection of these two circles.

Hence, AB and AF are the required tangents.



31. Let A(k + 1, 1), B(4, -3) and C(7, - k) are the vertices of the triangle.
Given that the area of the triangle is 6 sq. units.
Area of the triangle is given by

$$A=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\Rightarrow 6=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\Rightarrow 12=(k+1)(-3+k)+4(-k-1)+7(1+3)$$

$$\Rightarrow 12=-3k+k^2-3+k-4k-4+28$$

$$\Rightarrow 12=k^2-6k+21$$

$$\Rightarrow k^2-6k+21-12=0$$

$$\Rightarrow k^2-6k+9=0$$

$$\Rightarrow (k-3)^2=0$$

$$\Rightarrow k=3,3$$