CBSE Board
Class X Mathematics
Board Paper – 2015 Solution

1. Given quadratic equation is,

$px^2 - 2\sqrt{5}px + 15 = 0$

Here, $a = p,\ b = 2\sqrt{5}p,\ c = 15$

For real equal roots, discriminant $= 0$

$\therefore b^2 - 4ac = 0$

$\therefore \left(2\sqrt{5}p\right)^2 - 4p(15) = 0$

$\therefore 20p^2 - 60p = 0$

$\therefore 20p(p-30) = 0$

$\therefore p = 30$ or $p = 0$

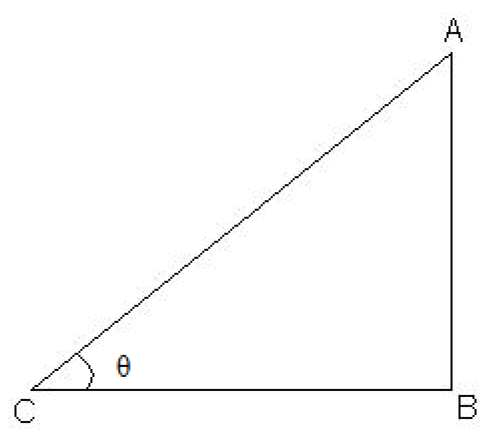
But, $p = 0$ is not possible.

$\therefore p = 30$

2. Let AB be the tower and BC be its shadow.

$AB = 20,\ BC = 20\sqrt{3}$

In ΔABC,



$\tan\theta = \frac{AB}{BC}$

$\tan\theta = \frac{20}{20\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

but, $\tan 30 = \frac{1}{\sqrt{3}}$

$\therefore \theta = 30$

$\therefore$ The Sun is at an altitude of 30° .

3. Two dice are tossed

S = [(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),
(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),
(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),
(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)]

Total number of outcomes when two dice are tossed = 6 x 6= 36

Favourable events of getting product as 6 are:

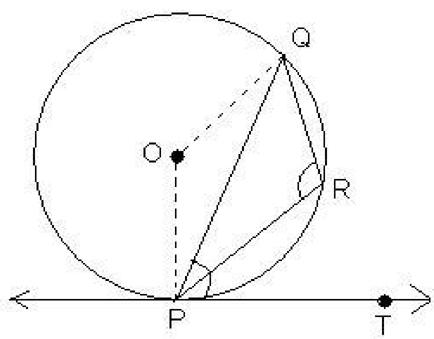
$(1\times 6=6),(6\times 1=6),(2\times 3=6),(3\times 2=6)$

i.e.(1,6), (6,1), (2,3), (3,2)

Favourable events of getting product as 6 = 4

$\therefore$ P(getting product as 6) = $\frac{4}{36}=\frac{1}{9}$

4.



$m\angle OPT=90^\circ$ ($\because$ radius is perpendicular to the tangent)

So, $\angle OPQ=\angle OPT-\angle QPT$

$=90^\circ-60^\circ$

$=30^\circ$

$m\angle POQ=2\angle QPT=2\times 60^\circ=120^\circ$

reflex $m\angle POQ=360^\circ-120^\circ=240^\circ$

$m\angle PRQ=\frac{1}{2}$ reflex $\angle POQ$

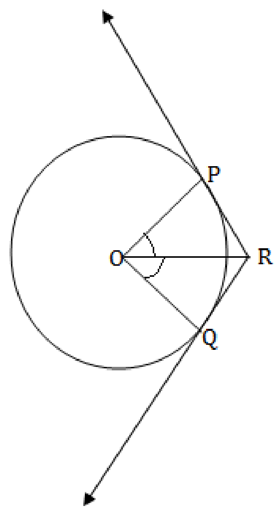
$=\frac{1}{2}\times 240^\circ$

$=120^\circ$

$\therefore m\angle PRQ=120^\circ$

SECTION B

5.



Given that m∠PRQ = 120°

We know that the line joining the centre and the external point is the angle bisector between the tangents.

Thus, m∠PRO = m∠QRO = $\frac{120^\circ}{2} = 60^\circ$

Also we know that lengths of tangents from an external point are equal.

Thus, PR = RQ.

Join OP and OQ.

Since OP and OQ are the radii from the centre O,

$OP \perp PR$ and $OQ \perp RQ$.

Thus, ΔOPR and ΔOQR are right angled congruent triangles.

Hence, $\angle POR = 90^\circ - \angle PRO = 90^\circ - 60^\circ = 30^\circ$

$\angle QOR = 90^\circ - \angle QRO = 90^\circ - 60^\circ = 30^\circ$

$\sin\angle QRO = \sin 30^\circ = \frac{1}{2}$

But $\sin 30^\circ = \frac{PR}{OR}$

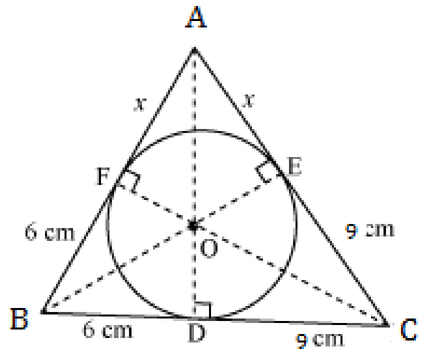
Thus, $\frac{PR}{OR} = \frac{1}{2}$

$\Rightarrow OR = 2PR$

$\Rightarrow OR = PR + PR$

$\Rightarrow OR = PR + QR$

6.



Let the given circle touch the sides AB and AC of the triangle at points F and E respectively and let the length of line segment AF be *x*.

Now, it can be observed that:

BF = BD = 6 cm (tangents from point B)

CE = CD = 9 cm (tangents from point C)

AE = AF = x (tangents from point A)

AB = AF + FB = x + 6

BC = BD + DC = 6 + 9 = 15

CA = CE + EA = 9 + x

2s = AB + BC + CA = x + 6 + 15 + 9 + x = 30 + 2x

s = 15 + x

s – a = 15 + x – 15 = x

s – b = 15 + x – (x + 9) = 6

s – c = 15 + x – (6 + x) = 9

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$54 = \sqrt{(15+x)(x)(6)(9)}$

$54 = 3\sqrt{6(15x+x^2)}$

$18 = \sqrt{6(15x+x^2)}$

$324 = 6(15x+x^2)$

$54 = 15x+x^2$

$x^2+15x-54=0$

$x^2+18x-3x-54=0$

$x(x+18)-3(x+18)$

$(x+18)(x-3)=0$

$x=-18$ and $x=3$

As distance cannot be negative, x = 3

AC = 3 + 9 = 12

AB = AF + FB = 6 + x = 6 + 3 = 9

7. $4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow x^2 + bx - \left(\frac{a^2 - b^2}{4}\right) = 0$

$\Rightarrow x^2 + 2\left(\frac{b}{2}\right)x = \frac{a^2 - b^2}{4}$

$\Rightarrow x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = \frac{a^2 - b^2}{4} + \left(\frac{b}{2}\right)^2$

$\Rightarrow \left(x + \frac{b}{2}\right)^2 = \frac{a^2}{4}$

$\Rightarrow x + \frac{b}{2} = \pm\frac{a}{2}$

$\Rightarrow x = \frac{-b}{2} \pm \frac{a}{2}$

$\Rightarrow x = \frac{-b-a}{2}, \frac{-b+a}{2}$

Hence, the roots are $-\left(\frac{a+b}{2}\right)$ and $\left(\frac{a-b}{2}\right)$.

8. $S_5 + S_7 = 167$ and $S_{10} = 235$

Now, $S_n = \frac{n}{2}\{2a + (n-1)d\}$

$\therefore S_5 + S_7 = 167$

$\Rightarrow \frac{5}{2}\{2a + 4d\} + \frac{7}{2}\{2a + 6d\} = 167$

$\Rightarrow 5a + 10d + 7a + 21d = 167$

$\Rightarrow 12a + 31d = 167$ ....(1)

Also, $S_{10} = 235$

$\therefore \frac{10}{2}\{2a + 9d\} = 235$

$\Rightarrow 10a + 45d = 235$

$\Rightarrow 2a + 9d = 47$ ....(2)

Multiplying equation (2) by 6, we get

$12a + 54d = 282$ .....(3)

Subtracting (1) from (3), we get

$$\begin{array}{rl} & 12a + 54d = 282 \\ (-) & 12a + 31d = 167 \\ & - \quad - \quad - \\ \hline & 23d = 115 \end{array}$$

$\therefore d = 5$

Substituting value of d in (2), we have

$2a+9(5)=47$

$\Rightarrow 2a+45=47$

$\Rightarrow 2a=2$

$\Rightarrow a=1$

Thus, the given A.P. is 1, 6, 11, 16,..........

9. ΔABC is right angled at B.

$\therefore AC^2=AB^2+BC^2$ ....(1)

Also, $A\equiv(4,7)$, $B\equiv(p,3)$ and $C\equiv(7,3)$

Now, $AC^2=(7-4)^2+(3-7)^2=(3)^2+(-4)^2=9+16=25$

$AB^2=(p-4)^2+(3-7)^2=p^2-8p+16+(-4)^2$

$=p^2-8p+16+16$

$=p^2-8p+32$

$BC^2=(7-p)^2+(3-3)^2=49-14p+p^2+0$

$=p^2-14p+49$

From (1), we have

$25=(p^2-8p+32)+(p^2-14p+49)$

$\Rightarrow 25=2p^2-22p+81$

$\Rightarrow 2p^2-22p+56=0$

$\Rightarrow p^2-11p+28=0$

$\Rightarrow p^2-7p-4p+28=0$

$\Rightarrow p(p-7)-4(p-7)=0$

$\Rightarrow (p-7)(p-4)=0$

$\Rightarrow p=7$ and $p=4$

10. Given, the points $A(x,y)$, $B(-5,7)$ and $C(-4,5)$ are collinear.

So, the area formed by these vertices is 0.

$\therefore \frac{1}{2}[x(7-5)+(-5)(5-y)+(-4)(y-7)]=0$

$\Rightarrow \frac{1}{2}[2x-25+5y-4y+28]=0$

$\Rightarrow \frac{1}{2}[2x+y+3]=0$

$\Rightarrow 2x+y+3=0$

$\Rightarrow y=-2x-3$

SECTION C

11. Here it is given that,

$T_{14} = 2(T_8)$

$\Rightarrow a + (14 - 1)d = 2[a + (8 - 1)d]$

$\Rightarrow a + 13d = 2[a + 7d]$

$\Rightarrow a + 13d = 2a + 14d$

$\Rightarrow 13d - 14d = 2a - a$

$\Rightarrow -d = a \quad \ldots (1)$

Now, it is given that its 6^{th} term is –8.

$T_6 = -8$

$\Rightarrow a + (6 - 1)d = -8$

$\Rightarrow a + 5d = -8$

$\Rightarrow -d + 5d = -8 \quad [\because \text{Using } (1)]$

$\Rightarrow 4d = -8$

$\Rightarrow d = -2$

Subs. this in eq. (1), we get a = 2

Now, the sum of 20 terms,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2a + (20-1)d]$$

$$= 10[2(2) + 19(-2)]$$

$$= 10[4 - 38]$$

$$= -340$$

12. For the given equation, $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = \sqrt{3}, b = -2\sqrt{2}, c = -2\sqrt{3}$

Now, $D = \sqrt{b^2 - 4ac}$

$= \sqrt{(-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})}$

$= \sqrt{8 + 24} = \sqrt{32} = 4\sqrt{2}$

Using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-2\sqrt{2}) \pm 4\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{2\sqrt{2}+4\sqrt{2}}{2\sqrt{3}} \text{ or } x = \frac{2\sqrt{2}-4\sqrt{2}}{2\sqrt{3}}$$

$$\Rightarrow x = \frac{\sqrt{2}+2\sqrt{2}}{\sqrt{3}} \text{ or } x = \frac{\sqrt{2}-2\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{3\sqrt{2}}{\sqrt{3}} \text{ or } x = \frac{-\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow x = \sqrt{3}\sqrt{2} \text{ or } x = \frac{-\sqrt{2}}{\sqrt{3}}$$

$$\therefore x = \sqrt{6} \text{ or } x = \frac{-\sqrt{2}}{\sqrt{3}}$$

13. Let BC be the height at which the aeroplane is observed from point A.

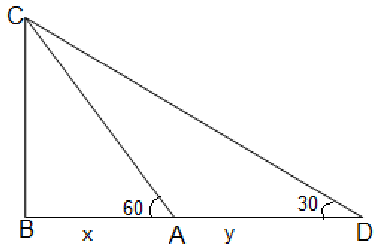
Then, BC = $1500\sqrt{3}$

In 15 seconds, the aeroplane moves from point A to D.

A and D are the points where the angles of elevation 60° and 30° are formed respectively.

Let BA = x metres and AD = y metres

BC = x + y



In ΔCBA,

$$\tan 60° = \frac{BC}{BA}$$

$$\sqrt{3} = \frac{1500\sqrt{3}}{x}$$

$$\therefore x = 1500 \text{ m} \qquad ....(1)$$

In ΔCBD,

$$\tan 30° = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y}$$

$$\therefore x + y = 1500(3) = 4500$$

$$\therefore 1500 + y = 4500$$

$$\therefore y = 3000 \text{ m} \qquad ....(2)$$

We know that, the aeroplane moves from point A to D in 15 seconds and the distance covered is 3000 metres. (by 2)

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$\text{Speed} = \frac{3000}{15}$$

$$\text{Speed} = 200\text{m/s}$$

$$\text{Converting it to km/hr} = 200 \times \frac{18}{5} = 720\text{km/hr}$$

14. Here, P(x,y) divides line segment AB, such that

$$AP = \frac{3}{7}AB$$

$$\frac{AP}{AB} = \frac{3}{7}$$

$$\frac{AB}{AP} = \frac{7}{3}$$

$$\frac{AB}{AP} - 1 = \frac{7}{3} - 1$$

$$\frac{AB - AP}{AP} = \frac{7-3}{3}$$

$$\frac{BP}{AP} = \frac{4}{3}$$

$$\frac{AP}{BP} = \frac{3}{4}$$

$\therefore$ P divides AB in the ratio 3 : 4

$$x = \frac{3\times 2 + 4(-2)}{3+4}; \quad y = \frac{3\times(-4) + 4(-2)}{3+4}$$

$$x = \frac{6-8}{7}; \quad y = \frac{-12-8}{7}$$

$$x = \frac{-2}{7}; \quad y = \frac{-20}{7}$$

$$\therefore \text{The coordinates of P are } \left(\frac{-2}{7}, \frac{-20}{7}\right)$$

15. Here the jar contains red, blue and orange balls.

Let the number of red balls be x.

Let the number of blue balls be y.

Number of orange balls = 10

$\therefore$ Total number of balls = x + y + 10

Now, let P be the probability of drawing a ball from the jar

$$P(\text{a red ball}) = \frac{x}{x+y+10}$$

$$\Rightarrow \frac{1}{4} = \frac{x}{x+y+10}$$

$$\Rightarrow 4x = x + y + 10$$

$$\Rightarrow 3x - y = 10 \qquad \text{----(i)}$$

Next,

$$P(\text{a blue ball}) = \frac{y}{x+y+10}$$

$$\Rightarrow \frac{1}{3} = \frac{y}{x+y+10}$$

$$\Rightarrow 3y = x + y + 10$$

$$\Rightarrow 2y - x = 10 \qquad \text{-----(ii)}$$

Multiplying eq. (i) by 2 and adding to eq. (ii), we get

$$\begin{array}{r} 6x - 2y = 20 \\ -x + 2y = 10 \\ \hline 5x = 30 \end{array}$$

$$\Rightarrow \quad x = 6$$

Subs. x = 6 in eq. (i), we get y = 8

$\therefore$ Total number of balls = x + y + 10 = 6 + 8 + 10 = 24

Hence, total number of balls in the jar is 24.

16. Radius of the circle =14 cm

Central Angle, $\boldsymbol{\theta}$ = 60°,

Area of the minor segment

$$=\frac{\theta}{360^\circ}\times\pi r^2-\frac{1}{2}r^2\sin\theta$$

$$=\frac{60^\circ}{360^\circ}\times\pi\times14^2-\frac{1}{2}\times14^2\times\sin60^\circ$$

$$=\frac{1}{6}\times\frac{22}{7}\times14\times14-\frac{1}{2}\times14\times14\times\frac{\sqrt{3}}{2}$$

$$=\frac{22\times14}{3}-49\sqrt{3}$$

$$=\frac{22\times14}{3}-\frac{147\sqrt{3}}{3}$$

$$=\frac{308-147\sqrt{3}}{3}\text{cm}^2$$

Area of the minor segment $=\frac{308-147\sqrt{3}}{3}\text{cm}^2$

17. Diameter of the tent = 4.2 m

Radius of the tent, r = 2.1 m

Height of the cylindrical part of tent, $h_{cylinder}$ = 4 m

Height of the conical part, h_{cone} = 2.8 m

Slant height of the conical part, ℓ

$$=\sqrt{h_{cone}{}^2+r^2}$$

$$=\sqrt{2.8^2+2.1^2}$$

$$=\sqrt{2.8^2+2.1^2}$$

$$=\sqrt{12.25}=3.5\text{ m}$$

Curved surface area of the cylinder = $2\pi r\, h_{cylinder}$

= $2\times\frac{22}{7}\times2.1\times4$

= $22\times0.3\times8 = 52.8\text{ m}^2$

Curved surface area of the conical tent = πrl = $\frac{22}{7}\times2.1\times3.5 = 23.1\text{ m}^2$

Total area of cloth required for building one tent

= Curved surface area of the cylinder + Curved surface area of the conical tent

= 52.8 + 23.1

= 75.9 m^2

Cost of building one tent = 75.9 × 100 = Rs 7590

Total cost of 100 tents = 7590 × 100 = Rs 7,59,000

Cost to be borne by the associations = $\frac{759000}{2}$ = Rs. 3,79,500

It shows the helping nature, unity and cooperativeness of the associations.

18. Internal diameter of the bowl = 36 cm

Internal radius of the bowl, r = 18 cm

Volume of the liquid, $V = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \pi \times 18^3$

Let the height of the small bottle be 'h'.

Diameter of a small cylindrical bottle = 6 cm

Radius of a small bottle, R = 3 cm

Volume of a single bottle = $\pi R^2 h = \pi \times 3^2 \times h$

No. of small bottles, n = 72

Volume wasted in the transfer = $\frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

Volume of liquid to be transferred in the bottles

$= \frac{2}{3} \times \pi \times 18^3 - \frac{10}{100} \times \frac{2}{3} \times \pi \times 18^3$

$= \frac{2}{3} \times \pi \times 18^3 \left(1 - \frac{10}{100}\right)$

$= \frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}$

Number of small cylindrical bottles = $\frac{\text{Volume of the liquid to be transferred}}{\text{Volume of a single bottle}}$

$\Rightarrow 72 = \frac{\frac{2}{3} \times \pi \times 18^3 \times \frac{90}{100}}{\pi \times 3^2 \times h}$

$\Rightarrow 72 = \frac{\frac{2}{3} \times 18^3 \times \frac{9}{10}}{3^2 \times h}$

$\Rightarrow h = \frac{\frac{2}{3} \times 18 \times 18 \times 18 \times \frac{9}{10}}{3^2 \times 72}$

$\therefore h = 5.4$ cm

Height of the small cylindrical bottle = 10.8 cm

19. Side of the cubical block, a = 10 cm

Longest diagonal of the cubical block = $a\sqrt{3} = 10\sqrt{3}$ cm

Since the cube is surmounted by a hemisphere, therefore the side of the cube should be equal to the diameter of the hemisphere.

Diameter of the sphere = 10 cm

Radius of the sphere, r = 5 cm

Total surface area of the solid = Total surface area of the cube – Inner cross-section area of the hemisphere + Curved surface area of the hemisphere

$= 6a^2 - \pi r^2 + 2\pi r^2$

$= 6a^2 + \pi r^2$

$= 6 \times (10)^2 + 3.14 \times 5^2$

$= 600 + 78.5 = 678.5 \text{ cm}^2$

Total surface area of the solid = 678.5 cm^2

20. No. of cones = 504

Diameter of a cone = 3.5 cm

Radius of the cone, r = 1.75 cm

Height of the cone, h = 3 cm

Volume of a cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \left(\frac{3.5}{2}\right)^2 \times 3$

$= \frac{1}{3} \times \pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3\text{cm}^3$

Volume of 504 cones

$= 504 \times \frac{1}{3} \times \pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3\text{cm}^3$

Let the radius of the new sphere be 'R'.

Volume of the sphere $= \frac{4}{3}\pi R^3$

Volume of 504 cones = Volume of the sphere

$504 \times \frac{1}{3} \times \pi \times \frac{3.5}{2} \times \frac{3.5}{2} \times 3 = \frac{4}{3}\pi R^3$

$\Rightarrow \frac{504 \times 1 \times \pi \times 3.5 \times 3.5 \times 3 \times 3}{3 \times 2 \times 2 \times 4 \times \pi} = R^3$

$\Rightarrow R^3 = \frac{504 \times 3 \times 49}{64}$

$$\Rightarrow R^3 = \frac{7\times8\times9\times3\times7^2}{64}$$

$$\Rightarrow R^3 = \frac{8\times27\times7^3}{64}$$

$$\Rightarrow R = \frac{2\times3\times7}{4}$$

$$\therefore R = \frac{21}{2} = 10.5 \text{ cm}$$

$\therefore$ Radius of the new sphere = 10.5 cm

SECTION D

21. Let ℓ be the length of the longer side and b be the length of the shorter side.

Given that the length of the diagonal of the rectangular field is 16 metres more than the shorter side.

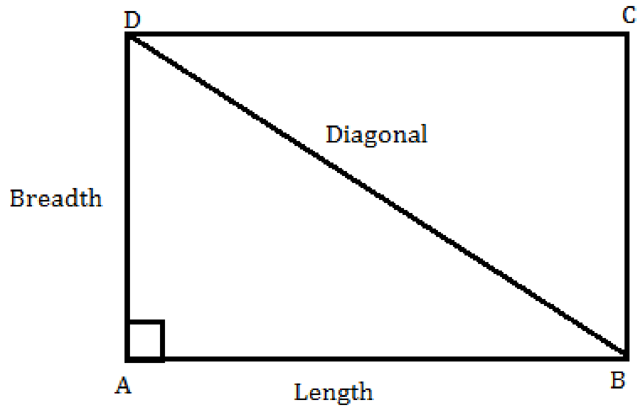
Thus, diagonal = 16 + b

Since longer side is 14 metres more than shorter side, we have,

ℓ = 14 + b

Diagonal is the hypotenuse of the triangle.

Consider the following figure of the rectangular field.



By applying Pythagoras Theorem in $\triangle$ABD, we have,

$$\text{Diagonal}^2 = \text{Length}^2 + \text{Breadth}^2$$

$$\Rightarrow (16+b)^2 = (14+b)^2 + b^2$$

$$\Rightarrow 256 + b^2 + 32b = 196 + b^2 + 28b + b^2$$

$$\Rightarrow 256 + 32b = 196 + 28b + b^2$$

$$\Rightarrow 60 + 32b = 28b + b^2$$

$\Rightarrow b^2 - 4b - 60 = 0$

$\Rightarrow b^2 - 10b + 6b - 60 = 0$

$\Rightarrow b(b-10) + 6(b-10) = 0$

$\Rightarrow (b+6)(b-10) = 0$

$\Rightarrow (b+6) = 0 \text{ or } (b-10) = 0$

$\Rightarrow b = -6 \text{ or } b = 10$

As breadth cannot be negative, breadth = 10 m

Thus, length of the rectangular field = 14 + 10 = 24 m

22. Consider the given A.P. 8, 10, 12, ...

Here the initial term is 8 and the common difference is 10 - 8 = 2 and 12 - 10 = 2

General term of an A.P. is t_n and formula to find out t_n is

$t_n = a + (n-1)d$

$\Rightarrow t_{60} = 8 + (60-1) \times 2$

$\Rightarrow t_{60} = 8 + 59 \times 2$

$\Rightarrow t_{60} = 8 + 118$

$\Rightarrow t_{60} = 126$

We need to find the sum of the last 10 terms.

Thus,

Sum of last 10 terms = Sum of first 60 terms - Sum of first 50 terms

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{60} = \frac{60}{2}[2 \times 8 + (60-1) \times 2]$

$\Rightarrow S_{60} = 30[16 + 59 \times 2]$

$\Rightarrow S_{60} = 30[134]$

$\Rightarrow S_{60} = 4020$

Similarly,

$\Rightarrow S_{50} = \frac{50}{2}[2 \times 8 + (50-1) \times 2]$

$\Rightarrow S_{50} = 25[16 + 49 \times 2]$

$\Rightarrow S_{50} = 25[114]$

$\Rightarrow S_{50} = 2850$

Thus the sum of last 10 terms $= S_{60} - S_{50} = 4020 - 2850 = 1170$

Therefore,

Sum of last 10 terms = Sum of first 60 terms - Sum of first 50 terms

23. Let x be the first speed of the train.

We know that $\frac{\text{Distance}}{\text{Speed}}=\text{time}$

Thus, we have,

$\frac{54}{x}+\frac{63}{x+6}=3 \text{ hours}$

$\Rightarrow \frac{54(x+6)+63x}{x(x+6)}=3$

$\Rightarrow 54(x+6)+63x=3x(x+6)$

$\Rightarrow 54x+324+63x=3x^2+18x$

$\Rightarrow 117x+324=3x^2+18x$

$\Rightarrow 3x^2-117x-324+18x=0$

$\Rightarrow 3x^2-99x-324=0$

$\Rightarrow x^2-33x-108=0$

$\Rightarrow x^2-36x+3x-108=0$

$\Rightarrow x(x-36)+3(x-36)=0$

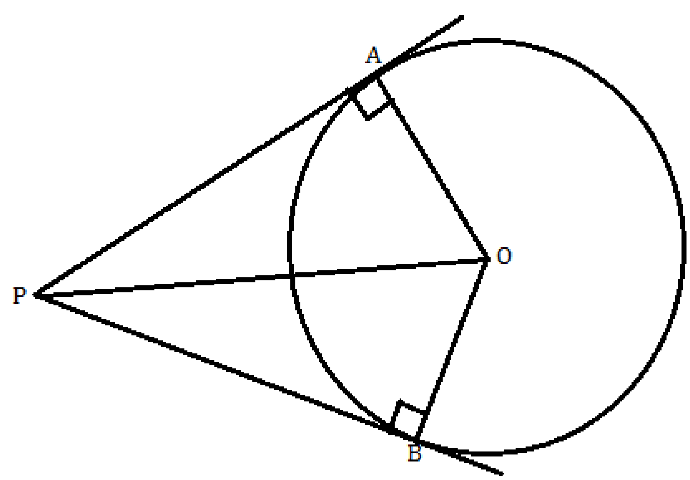
$\Rightarrow (x+3)(x-36)=0$

$\Rightarrow (x+3)=0 \text{ or } (x-36)=0$

$\Rightarrow x=-3 \text{ or } x=36$

Speed cannot be negative and hence intial speed of the train is 36 km/hour.

24. Consider the following diagram.



Let P be an external point and PA and PB be tangents to the circle.

We need to prove that PA = PB

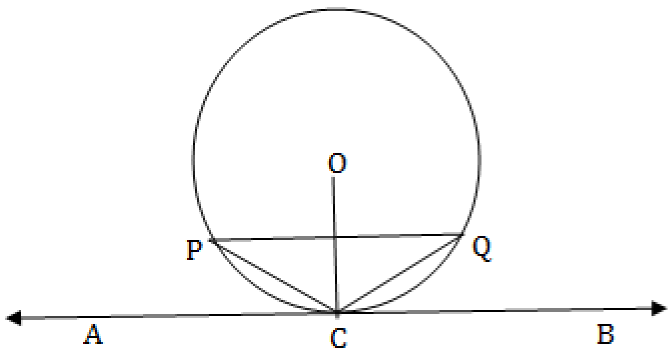
Now consider the triangles ΔOAP and ΔOBP

m∠A = m∠B = 90°

OP = OP [common]

OA = OB = radii of the circle
Thus, by Right Angle-Hypotenuse-Side criterion of congruence we have,
$\Delta OAP \cong \Delta OBP$
The corresponding parts of the congruent triangles are congruent.
Thus,
PA = PB

25. In the figure, C is the midpoint of the minor arc PQ, O is the centre of the circle and AB is tangent to the circle through point C.
We have to show the tangent drawn at the midpoint of the arc PQ of a circle is parallel to the chord joining the end points of the arc PQ.
We will show PQ || AB.
It is given that C is the midpoint point of the arc PQ.
So, arc PC = arc CQ.
$\Rightarrow PC = CQ$



This shows that ΔPQC is an isosceles triangle.
Thus, the perpendicular bisector of the side PQ of ΔPQC passes through vertex C.
The perpendicular bisector of a chord passes through the centre of the circle.
So the perpendicular bisector of PQ passes through the centre O of the circle.
Thus perpendicular bisector of PQ passes through the points O and C.
$\Rightarrow PQ \perp OC$
AB is the tangent to the circle through the point C on the circle.
$\Rightarrow AB \perp OC$
The chord PQ and the tangent PQ of the circle are perpendicular to the same line OC.
$\therefore PQ \parallel AB$.

26. Construct the ΔABC as per given measurements.

In the half plane of $\overline{AB}$ which does not contain C, draw $\overrightarrow{AX}$ such that $\angle BAX$ is an acute angle.

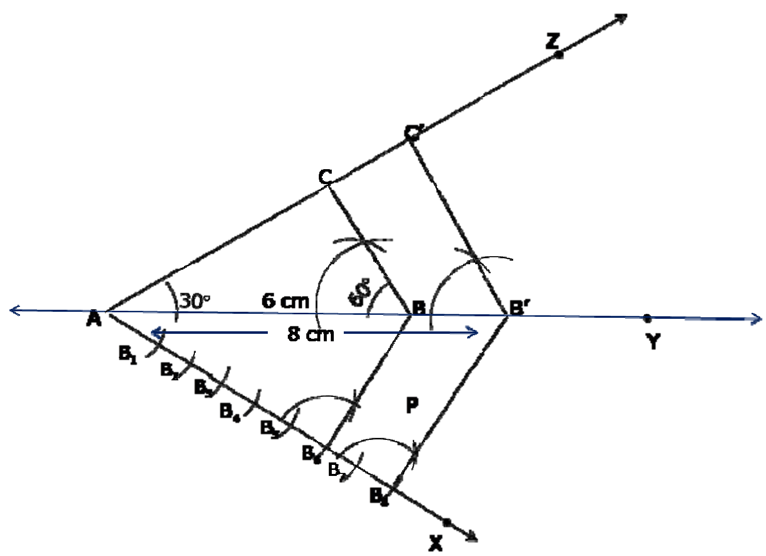
3) With some approprriate radius and centre A, Draw an arc to intersect $\overrightarrow{AX}$ at B_1. Similarly, with center B_1 and the same radius, draw an arc to intersect $\overrightarrow{BX}$ at B_2 such that $B_1B_2 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7 = B_7B_8$

4) Draw $\overline{B_6B}$.

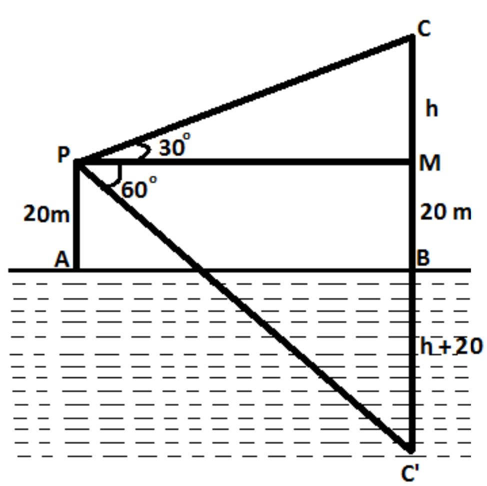
5) Through B_8 draw a ray parallel to $\overline{B_6B}$. to intersect $\overline{AY}$ at B'.

6) Through B' draw a ray parallel to $\overline{BC}$ to intersect $\overline{AZ}$ at C'.

Thus, $\Delta AB'C'$ is the required triangle.



27.



Let AB be the surface of the lake and P be the point of observation such that AP = 20 metres. Let C be the position of the cloud and C' be its reflection in the lake. Then CB = C'B. Let PM be perpendicular from P on CB.

Then $m\angle CPM = 30°$ and $m\angle C'PM = 60°$

Let CM = h. Then CB = h + 20 and C'B = h + 20.

In ΔCMP we have,

$$\tan 30^\circ = \frac{CM}{PM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{PM}$$

$$\Rightarrow PM = \sqrt{3}h..................(i)$$

In $\Delta PMC'$ we have,

$$\tan 60^\circ = \frac{C'M}{PM}$$

$$\Rightarrow \sqrt{3} = \frac{C'B + BM}{PM}$$

$$\Rightarrow \sqrt{3} = \frac{h + 20 + 20}{PM}$$

$$\Rightarrow PM = \frac{h + 20 + 20}{\sqrt{3}}..................(ii)$$

From equation (i) and (ii), we get

$$\sqrt{3}h = \frac{h + 20 + 20}{\sqrt{3}}$$

$$\Rightarrow 3h = h + 40$$

$$\Rightarrow 2h = 40$$

$$\Rightarrow h = 20 \text{ m}$$

Now, $CB = CM + MB = h + 20 = 20 + 20 = 40$.

Hence, the height of the cloud from the surface of the lake is 40 metres.

28. Let S be the sample space of drawing a card from a well-shuffled deck.

$$n(S) = {}^{52}C_1 = 52$$

(i)There are 13 spade cards and 4 ace's in a deck
As ace of spade is included in 13 spade cards,
so there are 13 spade cards and 3 ace's

a card of spade or an ace can be drawn in ${}^{13}C_1 + {}^{3}C_1 = 13 + 3 = 16$

Probability of drawing a card of spade or an ace $= \frac{16}{52} = \frac{4}{13}$

(ii)There are 2 black King cards in a deck

a card of black King can be drawn in $^{2}C_{1}=2$

Probability of drawing a black king $=\frac{2}{52}=\frac{1}{26}$

(iii)There are 4 Jack and 4 King cards in a deck.

So there are $52-8=44$ cards which are neither Jacks nor Kings.

a card which is neither a Jack nor a King can be drawn in $^{44}C_{1}=44$

Probability of drawing a card which is neither a Jack nor a King $=\frac{44}{52}=\frac{11}{13}$

(iv)There are 4 King and 4 Queen cards in a deck.

So there are $4+4=8$ cards which are either King or Queen.

a card which is either a King or a Queen can be drawn in $^{8}C_{1}=8$

Probability of drawing a card which is either a King or a Queen $=\frac{8}{52}=\frac{2}{13}$

29. Take $(x_1,y_1)=(1,-1),(-4,2k)$ and $(-k,-5)$

It is given that the area of the triangle is 24 sq. unit

Area of the triangle having vertices $(x_1,y_1),(x_2,y_2)$ and (x_3,y_3)

is given by

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\therefore 24=\frac{1}{2}\left[1(2k-(-5))+(-4)((-5)-(-1))+(-k)((-1)-2k)\right]$$

$$48=\left[(2k+5)+16+(k+2k^2)\right]$$

$$\therefore 2k^2+3k-27=0$$

$$\therefore (2k+9)(k-3)=0$$

$$\therefore k=-\frac{9}{2} \text{ or } k=3$$

$\therefore$ The values of k are $-\frac{9}{2}$ and 3.

30. PQRS is a square.

So each side is equal and angle between the adjacent sides is a right angle.

Also the diagonals perpendicularly bisect each other.

In ΔPQR using pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$PR^2 = (42)^2 + (42)^2$

$PR = \sqrt{2}(42)$

$OR = \frac{1}{2}PR = \frac{42}{\sqrt{2}} = OQ$

From the figure we can see that the radius of flower bed ORQ is OR.

Area of sector ORQ $= \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2$

Area of the ΔROQ $= \frac{1}{2} \times RO \times OQ$

$= \frac{1}{2} \times \frac{42}{\sqrt{2}} \times \frac{42}{\sqrt{2}}$

$= \left(\frac{42}{2}\right)^2$

Area of the flower bed ORQ

=Area of sector ORQ – Area of the ΔROQ

$= \frac{1}{4}\pi\left(\frac{42}{\sqrt{2}}\right)^2 - \left(\frac{42}{2}\right)^2$

$= \left(\frac{42}{2}\right)^2\left[\frac{\pi}{2} - 1\right]$

$= (441)[0.57]$

$= 251.37 cm^2$

Area of the flower bed ORQ = Area of the flower bed OPS

$= 251.37 cm^2$

Total area of the two flower beds

= Area of the flower bed ORQ + Area of the flower bed OPS

$= 251.37 + 251.37$

$= 502.74\ cm^2$

31. Height of the cylinder (h) = 10 cm

Radius of the base of the cylinder = 4.2 cm

Volume of original cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (4.2)^2 \times 10$

$= 554.4 cm^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times (4.2)^3$

$= 155.232\ cm^3$

Volume of the remaining cylinder after scooping out hemisphere from each end

= Volume of original cylinder $- 2 \times$ Volume of hemisphere

$= 554.4 - 2 \times 155.232$

$= 243.936\ cm^3$

The remaining cylinder is melted and converted to
a new cylindrical wire of 1.4 cm thickness.
So they have same volume and radius of new cylindrical wire is 0.7 cm.
Volume of the remaining cylinder = Volume of the new cylindrical wire

$243.936 = \pi r^2 h$

$243.936 = \frac{22}{7}(0.7)^2 h$

$h = 158.4\ cm$

$\therefore$ The length of the new cylindrical wire of 1.4 cm thickness is 158.4 cm.