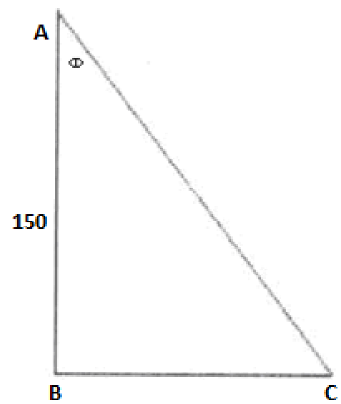
CBSE Board
Class X Summative Assessment – II
Mathematics
Board Question Paper 2014 – Set 2

Time: 3 hrs **Max. Marks: 90**

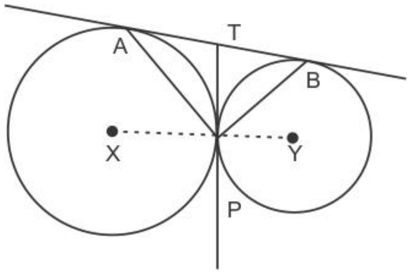
Solution
Section A

1. Correct answer: C
   The multiples of 4 between 1 and 15 are 4, 8 and 12. The probability of getting a multiple of 4 = $\frac{3}{15} = \frac{1}{5}$
2. Correct answer: A



Let AB be the tower and BC be distance between tower and car. Let θ be the angle of depression of the car.
According to the given information,
In ΔABC,
$\tan\theta = \frac{BC}{AB}$ [Using (1)] and tan 30° = $\frac{1}{\sqrt{3}}$
$\therefore$ BC = $\frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$
Hence, distance between the tower and car is $50\sqrt{3}$.

3. Correct answer: D



$TA = TP \Rightarrow \angle TAP = \angle TPA$

$TB = TP \Rightarrow \angle TBP = \angle TPB$

$\therefore \angle TAP + \angle TBP = \angle TPA + \angle TPB = \angle APB$

$\underline{\angle TAP + \angle TBP} + \angle APB = 180^\circ \qquad [\because \text{sum of.....}180^\circ]$

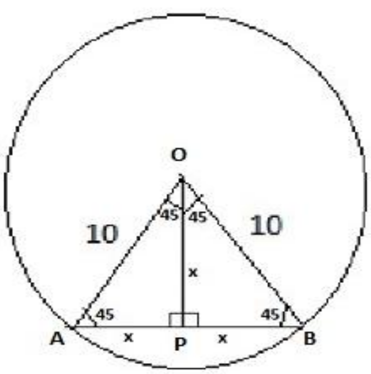
$\therefore \angle APB + \angle APB = 180^\circ$

$\therefore 2\angle APB = 180^\circ$

$\therefore \angle APB = 90^\circ$

4. Correct answer: B
k, 2k - 1, 2k + 1 are in Arithmetic Progression
if a_1, a_2 and a_3 are in A.P. then
$a_2 - a_1 = a_3 - a_2$
$2a_2 = a_3 + a_1$
$2 \times (2k - 1) = (2k + 1) + k$
$4k - 2 = 3k + 1$
$k = 3$

5. Correct answer: B

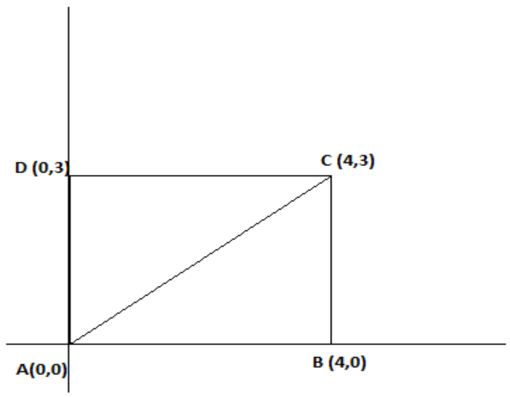


Given $\angle AOB$ is given as 90°
$\triangle AOB$ is an isosceles triangle since OA = OB
Therefore $\angle OAB = \angle OBA = 45^\circ$
Thus $\angle AOP = 45^\circ$ and $\angle BOP = 45^\circ$
Hence $\triangle AOP$ and $\triangle BOP$ also are isosceles triangles
$\therefore AP = OP and OP = PB$

$Let AP = OP = PB = x$
In $\triangle$ AOP

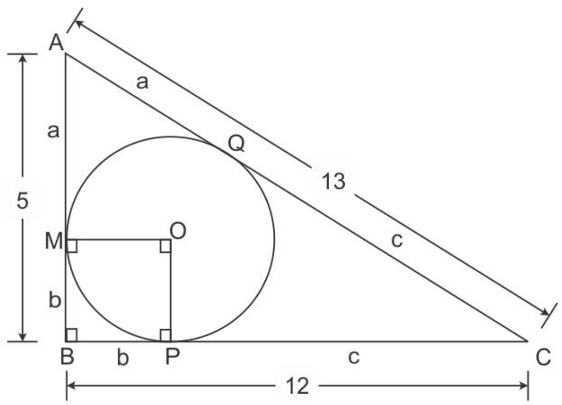
$x^2 + x^2 = 10^2$ [Pythagoras theorem]
Thus $2x^2 = 100$
$x = 5\sqrt{2}$
Hence length of chord AB = $2x = 5\sqrt{2} + 5\sqrt{2} = 10\sqrt{2}$

6. Correct answer: A



We see that AB = 4 units and BC = 3 units
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$= 4^2 + 3^2$
$AC^2 = 25$
Thus AC = 5 units
Hence length of the diagonal of the rectangle is 5 units

7. Correct answer: C



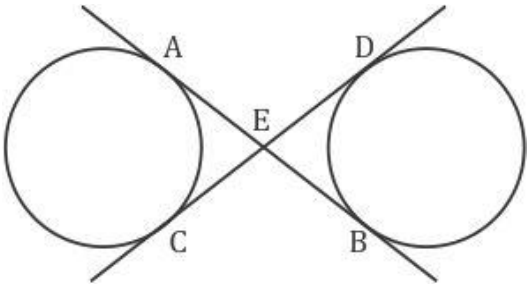
It is given that AB = 5 and BC = 12
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$= 5^2 + 12^2$
$= 169$
Thus AC = 13
We know that two tangents drawn to a circle from the same point that is exterior to the circle are of equal lengths.

Thus AM = AQ = a
Similarly MB = BP = b and PC = CQ = c
We know
AB = a + b = 5
BC = b + c = 12 and
AC = a + c = 13
Solving simultaneously we get a=3, b=2 and c=10
We also know that the tangent is perpendicular to the radius
Thus OMBP is a square with side b
Hence the length of the radius of the circle inscribed in the right angled triangle is 2 cm.

8. Correct answer: A
There are in all 2^3 = 8 combinations or outcomes for the gender of the 3 children
The eight combinations are as follows
BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG
Thus the probability of having at least one boy in a family is $\frac{7}{8}$

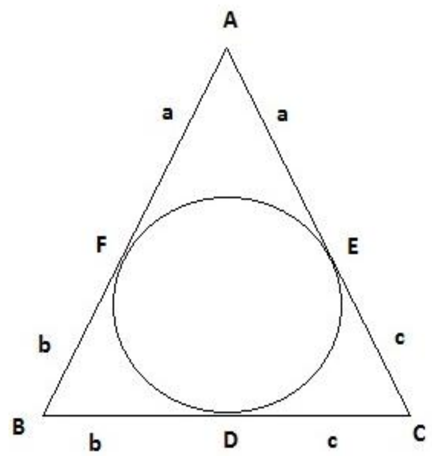
SECTION B

9. Solution:



Given: AB and CD are common tangents to both the circles.
To prove: AB = CD
Proof:
We know that two tangents drawn to a circle for the same exterior point are equal.
Thus we get
AE = EC (i)
Similarly
ED = EB (ii)
AB = AE + EB (iii)
and
CD = CE + ED (iv)
AB = EC + EB from (i) and (iii)
CD = EC + EB from (ii) and (iv)
Therefore AB = CD
Hence proved.

10.



Given: ΔABC is an isosceles triangle in which AB = AC; with a circle inscribed in the triangle.
To prove: BD = DC
Proof:
AF and AE are tangents drawn to the circle from point A.
Since two tangents drawn to a circle from the same exterior point are equal,
AF = AE = a
Similarly BF = BD = b and CD = CE = c
We also know that ΔABC is an isosceles triangle in which AB = AC.
a+b = a+c
Thus b = c
Therefore, BD = DC
Hence proved.

11. Solution:
The total number of outcomes when two dice are tossed together is 36.
The sample space is as follows

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2, 2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

i. Let E be the event 'that number of each die is even'
Favourable outcomes = { (2,2) , (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6) }
Probability that the number on each dice is even

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{9}{36} = \frac{1}{4}$$

ii. Let F be the event
iii. Favourable outcomes = { (1,4) (2,3) (3,2) (4,1) }
Probability that the sum of the numbers appearing on the two dice is 5

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{4}{36} = \frac{1}{9}$$

12. Solution:
Given total surface area of hemisphere = 462 cm^2

$$2\pi r^2 = 462$$

$$r = 8.574\text{cm}$$

$$\text{Volume of a hemisphere} = \frac{2}{3}\pi r^3$$

$$\therefore \quad \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 8.574^3 = 1320.54$$

$$\Rightarrow \quad r^3 = \frac{4851}{2} \times \frac{3}{2} \times \frac{7}{22} = \left(\frac{21}{2}\right)^3$$

13. Solution:
Numbers which are divisible by both 2 and 5 are the numbers which are divisible by 10.
Thus we need to find the number of natural numbers between 101 and 999 which are divisible by 10.
The first number between 101 and 999 which is divisible by 10 is 110
And the last number between 101 and 999 which is divisible by 10 is 990
Using the formula for arithmetic progression where first term (a) = 110, last term (T_n) = 990 and difference (d) =10

$$T_n = a + (n-1)d$$

$$990 = 110 + (n-1)10$$

$$880 = (n-1)10$$

$$88 = n-1$$

$$n = 89$$

Hence there are 89 natural numbers between 101 and 999 which are divisible by both 2 and 5.

14. Solution:

Given: Quadratic equation $9x^2 - 3kx + k = 0$ has equal roots

Let β be the equal roots of the equation

Thus 2β = $\frac{3k}{9} = \frac{k}{3}$ (Sum of the roots is equal to −b/a)

We get $\beta = \frac{k}{6}$ (i)

Or $\beta^2 = \frac{k^2}{36}$

Also, that $\beta^2 = \frac{k}{9}$ (Product of the roots is equal to c/a) (ii)

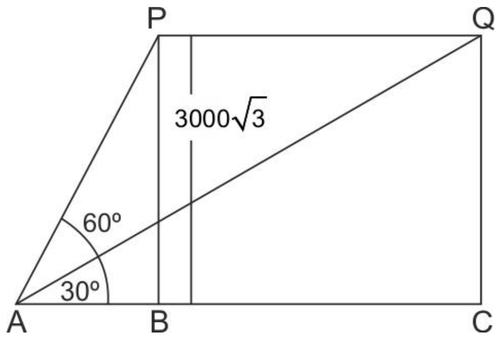
$\frac{k^2}{36} = \frac{k}{9}$

For $k \neq 0$, $\frac{k}{36} = \frac{1}{9}$

Thus k = 4

SECTION – C

15. Solution:



Let P and Q be the two positions of the plane and A be the point of observation. Let ABC be the horizontal line through A.

It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30° respectively.

∴ ∠PAB = 60°, ∠QAB = 30°. It is also given that PB = $3000\sqrt{3}$ meters

In ΔABP, we have

$$\tan 60 = \frac{BP}{AB}$$

$$\frac{\sqrt{3}}{1} = \frac{3000\sqrt{3}}{AB}$$

AB = 3000 m

In ΔACQ, we have

$$\tan 30 = \frac{QC}{AC}$$

$$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AC}$$

AC = 9000 m

∴ Distance = BC = AC – AB = 9000m – 3000m = 6000m

Thus, the plane travels 6km in 30 seconds

Hence speed of plane = 6000/30 = 200 m/sec = 720km/h

16. Solution:

Diameter of sphere curved out = side of cube = 7cm or Radius =3.5cm

Volume of cube = a^3

= 7^3

= 343 cm^3

Volume of sphere curved out = 4/3 πr^3

= 4/3 x 22/7 x 7/2 x 7/2 x 7/2 = 179.66 cm^3

17. Solution:

Given:-

Speed of flowing water = 4km/h = 200/3 metres per minute

Width of canal = 6 m and height of canal = 1.5 m

Standing water requirement for irrigation = 8 cm = 0.08 m

Cross section area of canal = width of canal * height of canal

= (6 * 1.5) m^2

Volume of water needed to irrigate in 10 min = 10 x 6 x 1.5 x 200/3 = 6000m^3

Volume of water irrigated = base area (of irrigated land) x height

= base area x 0.08m

6000 = base area x 0.08

Base area = 6000/0.08 =75000 m^2

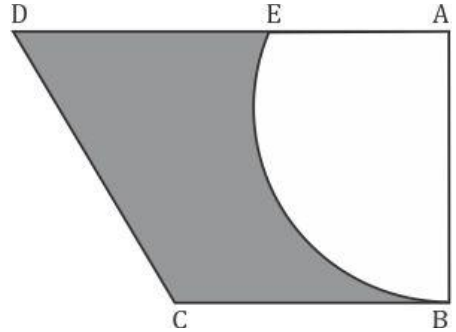
= 7.5 hectare [1 hectare = 10000 m^2]

$$\therefore \quad \text{Sum} = \frac{n}{2}\{\text{first term} + \text{last term}\}$$

$$= \frac{128}{2}\{a_1 + a_{128}\}$$

$$= 64\{105 + 994\} = (64)(1099) = 70336$$

18. Solution:



Given :-
AD = 10 cm
BC = 4 cm
Area of trapezium = 24.5 cm^2

Area of trapezium = $\frac{a+b}{2}$ height [where 'a' and 'b' are parallel sides]

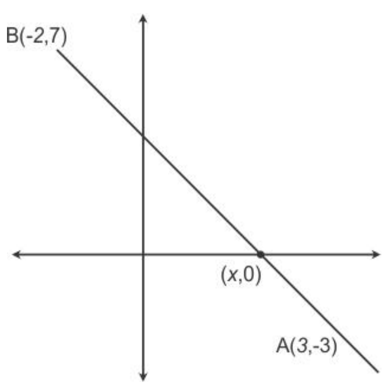
$$= \frac{AD+BC}{2} AB$$

$$24.5 = \frac{10+4}{2} AB$$

$$\frac{24.5}{7} = AB$$

AB = 3.5 cm
Thus, radius = 3.5 cm
Area of quadrant = Area of a circle / 4
Area of quadrant = ¼ * pi * r^2 = 0.25 x 22/7 x3.5 x 3.5 = 9.625 cm^2
The area of shaded region = Area of trapezium – Area of the given quadrant
The area of shaded region = 24.5 - 9.625 = 14.875 cm^2

19. Solution:
Point p lies on x axis so it's ordinate is 0 (Using section formula)



Let the ratio be k: 1 Let the coordinate of the point be P(x , 0)
As given A(3,-3) and B(-2,7)
The co-ordinates of the point P(x,y), which divide the line segment joining the points A(x_1,y_1) and B(x_2,y_2), internally in the ratio m_1:m_2 are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$P_y = \frac{my_2+ny_1}{m+n}$$

Hence, A(3,-3) be the co-ordinates (x_1,y_1) and
B(-2,7) be the co-ordinates (x_2,y_2)
m = k
n = 1

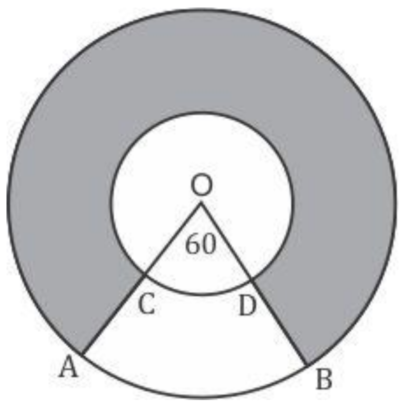
0 = k x 7 + 1 x -3 / (k+1)
0(k+1) =7k -3
0 =7k - 3
3=7k
k = 3 /7
k:1 = 3 : 7

$$P_x = \frac{mx_2 + nx_1}{m+n}$$

$$Px = \frac{\left[\left(\frac{3}{7}\times -2\right)+(1\times 3)\right]}{\left(\frac{3}{7}+1\right)} = 2.41$$

20. Solution:
Given :
Radii of inner circle = 21 cm = r
Radii of outer circle = 42 cm = R
$\angle AOB = \theta = 60^{\circ}$

Also,
Area of ring = pi (R^2 – r^2)
Area of a sector = $\frac{\theta}{360} * \pi r^2$



The area of shaded region = Area of ring – Area of ABCD
= Area of ring – Area of sector of Outer Circle - Area of sector of Inner Circle

$$= \pi(R^2 - r^2) - \frac{\pi(R^2 - r^2)}{1} \times \frac{\theta}{360}$$

$$= \pi(R^2 - r^2) - \frac{\pi(R^2 - r^2)}{1} \times \frac{\theta}{360}$$

$$= \pi(R^2 - r^2)[1 - \frac{\theta}{360}]$$

$$= \frac{22}{7}(42^2 - 21^2)[1 - \frac{60}{360}]$$

= 3465 cm^2

21. Solution:

Given: $\frac{16}{x} - 1 = \frac{15}{x+1}$; $x \# 0, -1$

$\therefore \frac{16-x}{x} = \frac{15}{x+1}$

$\frac{16-x}{x} = \frac{15}{x+1}$

$\frac{(16-x)(x+1)}{1} = \frac{15x}{1}$

$16x + 16 - x^2 - x = 15x$

$16 = x^2$

$x = 4$

22. Solution:

The sum of 2^{nd} and the 7^{th} terms of an AP is 30

(a + d) + (a + 6 d) = 30

2a + 7d = 30 (i)

Now,

15^{th} term is 1 less than twice the 8^{th} term

(a + 14d) = 2(a + 7d) -1

a + 14d = 2a + 14d -1

a = 1 (ii)

Substituting the values in (i)

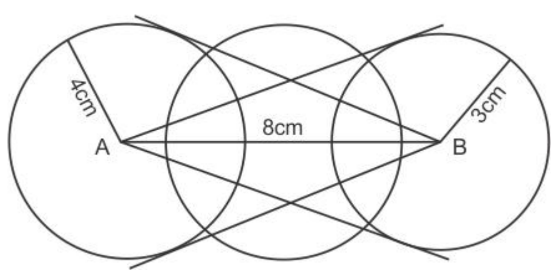
2x1 + 7d = 30

d = 4 (iii)

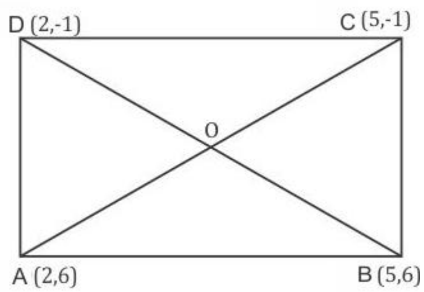
Hence, the terms in AP are …. a, a+d, a+2d, a+3d….

AP : 1,5,9 ……

23. Solution:



24. Solution:



ΔADC and ΔBDC are right angled triangles with AD and BC as hypotaneus

$AC^2 = AB^2 + DC^2$

$AC^2 = (5-2)^2 + (6+1)^2 = 9 + 49 = 58$ sq. unit

$BD^2 = DC^2 + CB^2$

$BD^2 = (5-2)^2 + (-1-6)^2 = 9 + 49 = 58$ sq. unit

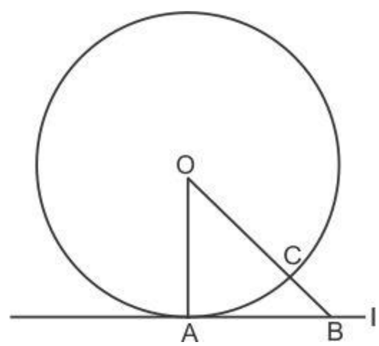
Hence, both the diagonals are equal in length.

SECTION D

25. Solution:

Given: A circle C (0, r) and a tangent l at point A.

To prove: OA ⊥ l



Construction: Take a point B, other than A, on the tangent l. Join OB. Suppose OB meets the circle in C.

Proof: We know that, among all line segments joining the point O to a point on l, the perpendicular is shortest to l.

OA = OC (Radius of the same circle)

But, OB = OC + BC.

∴ OB > OC

⇒ OB > OA

⇒ OA < OB

This is true for all positions of B on I.

Thus, OA is the shortest distance between point A and line segment I.

Hence, OA ⊥ l

26. Solution:

Given:-

Diameter of cylindrical vessel = 7 cm

Diameter of spherical marbles = 1.4 cm

Volume of a sphere =

Volume of 150 spherical marbles, each of diameters 1.4 cm = volume of cylindrical vessel of diameter 7 cm displaced

Volume of a Sphere = $\frac{4}{3}\pi r^3$

$150 \times \frac{4}{3}\pi(\frac{1.4}{2})^3 = \pi(\frac{7}{2})^2 \times h$

h = 5.6 cm

27. Solution:

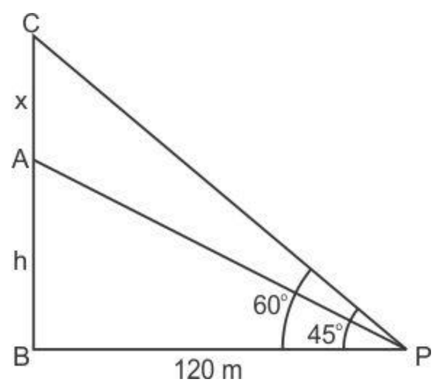
Volume of a frustum of a cone = $\frac{1}{3}\pi h(r1^2 + r2^2 + r1 \times r2)$

Volume of container = 1/3 pi* h (R^2 + r^2 +Rr)
= 1/3 x 22/7 x 24[20x 20 + 8 x 8 + 20 x 8]
= 15689.14 cm^3
= 15.69 litre

The cost of milk which can completely fill the container at the rate of Rs.21 per liter = Rs(21 x 15.69) = 329.49

28. Solution:

Let AB is the tower of height h meter and AC is flagstaff of height x meter.



$\angle APB = 45^o$ and $\angle BPC = 60^o$

Tan 60 = (x+h)/120)

$\sqrt{3} = \frac{x+h}{120}$

$x = 120\sqrt{3} - h$

Tan 45 = h/120

1 = h/120

h = 120

Therefore height of the flagstaff =

$= 120\sqrt{3} - 120$

$= 120(\sqrt{3} - 1)$

$= 120 * .73$

$= 87.6m$

29. Solution:

Let speed of stream = x km/h

Speed f boat in steel water = 18 km/h

Speed f boat in upstream = (18 −x)km/h
Speed f boat in downstream = (18 + x) km/h
Distance = 24 km
As per question, it takes 1 hour more to go upstream 24 km, than downstream
24 km /(18 −x) = 24 km/(18 + x) + 1
$x^2 + 48x - 324 = 0$
x = 6 or x = – 54
But, as speed can not be negative
Hence, the speed of stream = 6 km/h

30. Solution:
Class 1 plant trees = 2 x class 1 x 2 section = 2x1x2 = 4 x classs= 4x1 = 4 trees
Class 2 plant trees =4 x classs= 4x2 = 8 trees
a = 4
d = 8
n = 12
S12 = 12/2[2x4 + 11x4] = 312 trees

31. Solution:

$$\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}$$

$$\frac{\left[(x-3)(x-6)+(x-4)(x-5)\right]}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{x^2-9x+18+x^2-9x+20}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{2x^2-18x+38}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{2\left(x^2-9x+19\right)}{1}=\frac{10}{3}\left(x^2-10x+24\right)$$

$$\frac{x-9x+19}{1}=\frac{5}{3}\left[x^2-10x+24\right]$$

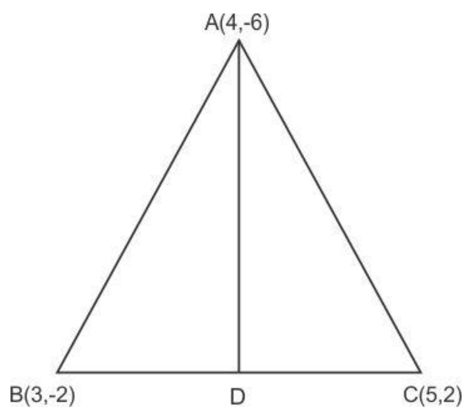
$$3x^2-27x+57=5x^2-50x+120$$

$$2x^2-23x+63=0$$

$$\therefore x=7 \text{ or } x=\frac{9}{2}$$

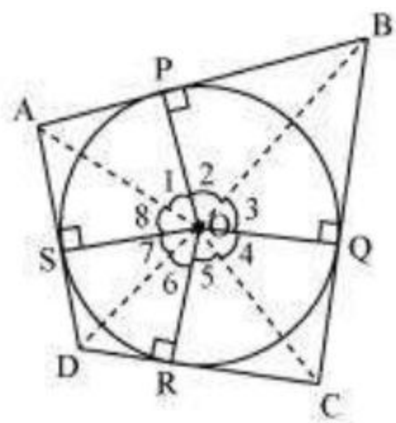
32. Solution:
(i) face card are removed from a pack of 52 playing card = 6
Total favorable outcomes = 52 – 6 = 46
Number of red color cards in the remaining pack = 26 -6= 20
P[E] = 20/46 = 10/23
(ii) Number of queen cards in the remaining pack = 2
P[E] = 2/46= 1/23
(iii) Number of aces in the remaining pack = 2
P[E] = 2/46= 1/23
(iv) Number of face cards in the remaining pack = 6
P[E] = 6/46= 3/23

33. Solution:
Let the co-ordinates of D be D (x,y) and D is midpoint of BC
x = (3+5)/2 = 4; y = (2-2)/2 = 0



Now Area of triangle ABD = ½ [4(-2-0) +3(0+6)+4(-6+2)] = 3 sq unit
and Area of triangle ACD = ½ [5(-6-0) + 4(0 - 2) + 4(2+6)] = 3 sq unit
Hence, the median AD divides triangle ABC into two triangle of equal area.

34. Solution:



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us
join the vertices of the quadrilateral ABCD to the center of the circle.
Consider ΔOAP and ΔOAS,
AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)
OA = OA (Common side)
Thus, by Side-Side-Side criterion of congruence, we have
$\Delta OAP \cong \Delta OAS$ (SSS congruence criterion)
Therefore, $A \leftrightarrow A$, $P \leftrightarrow S$, $O \leftrightarrow O$
The corresponding parts of the congruent triangles are congruent.
And thus, $\angle POA = \angle AOS$
$\angle 1 = \angle 8$
Similarly,
$\angle 2 = \angle 3$
$\angle 4 = \angle 5$
$\angle 6 = \angle 7$
$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^{\circ}$
$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^{\circ}$
$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^{\circ}$
$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^{\circ}$
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^{\circ}$
$\angle AOB + \angle COD = 180^{\circ}$
Similarly, we can prove that $\angle BOC + \angle DOA = 180^{\circ}$
Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.