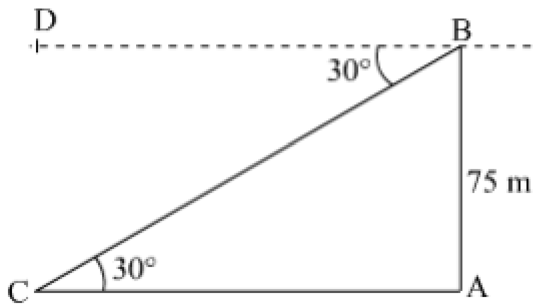
CBSE Board
Class X Mathematics
Board Paper - 2013

Time: 3 hour Total Marks: 90

Solution
Section A

1. Correct answer: C



Let AB be the tower of height 75 m and C be the position of the car.

In $\triangle ABC$,

$$\cot 30^o = \frac{AC}{AB}$$
$$\Rightarrow AC = AB \cot 30^o$$
$$\Rightarrow AC = 75m \times \sqrt{3}$$
$$\Rightarrow AC = 75\sqrt{3}m$$

Thus, the distance of the car from the base of the tower is $75\sqrt{3}$ m.

2. Correct answer: A

S = {1,2,3,4,5,6}

Let event E be defined as 'getting an even number'.

n(E) = {1,4,6}

$$\therefore P\ E = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{3}{6} = \frac{1}{2}$$

3. Correct answer: C

S = {1, 2, 3,..90}

n(S) = 90

The prime number less than 23 are 2, 3, 5, 7, 11, 13, 17, and 19.

Let event E be defined as 'getting a prime number less than 23'.

n(E) = 8

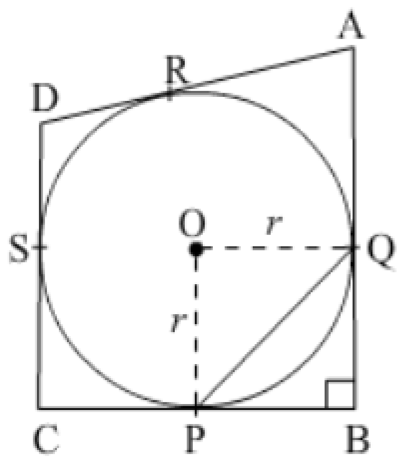
$$\therefore P\ E = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}} = \frac{8}{90} = \frac{4}{45}$$

4. Correct answer: A

Given: AB, BC, CD and AD are tangents to the circle with centre O at Q, P, S and R respectively.

AB = 29 cm, AD = 23, DS = 5 cm and $\angle B = 90°$

Construction: Join PQ.



We know that, the lengths of the tangents drawn from an external point to a circle are equal.

DS = DR = 5 cm

$\therefore$ AR = AD - DR = 23 cm - 5 cm = 18 cm

AQ = AR = 18 cm

$\therefore$ QB = AB - AQ = 29 cm - 18 cm = 11 cm

QB = BP = 11 cm

In Δ PQB,

$PQ^2 = QB^2 + BP^2 = (11\text{ cm})^2 + (11\text{ cm})^2 = 2 \times (11\text{ cm})^2$

$PQ = 11\sqrt{2}$ cm ... (1)

In Δ OPQ,

$PQ^2 = OQ^2 + OP^2 = r^2 + r^2 = 2r^2$

$(11\sqrt{2})^2 = 2r^2$

$121 = r^2$

r = 11

Thus, the radius of the circle is 11 cm.

5. Correct answer: B

AP $\perp$ PB (Given)

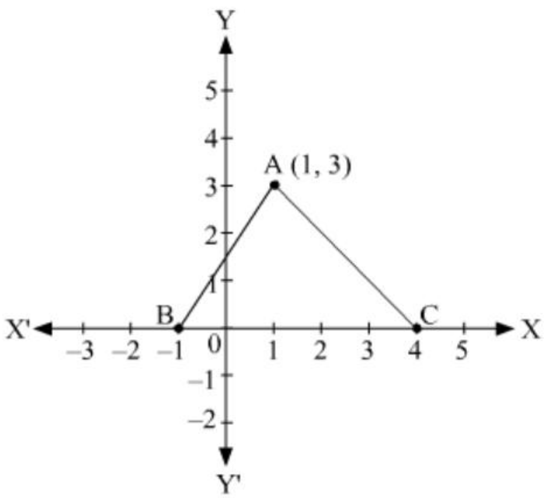
CA $\perp$ AP, CB $\perp$ BP (Since radius is perpendicular to tangent)

AC = CB = radius of the circle

Therefore, APBC is a square having side equal to 4 cm.

Therefore, length of each tangent is 4 cm.

6. Correct answer: C



From the figure, the coordinates of A, B, and C are (1, 3), (-1, 0) and (4, 0) respectively.

Area of ΔABC

$$= \frac{1}{2} | 1(0-0) + (-1)(0-3) + 4(3-0) |$$
$$= \frac{1}{2} | 0 + 3 + 12 |$$
$$= \frac{1}{2} | 15 |$$
$$= 7.5 \text{ sq units}$$

7. Correct answer: B

Let r be the radius of the circle.

From the given information, we have:

$2\pi r - r = 37$ cm

$$\Rightarrow r\ 2\pi - 1\ = 37\text{cm}$$
$$\Rightarrow r\left(2 \times \frac{22}{7} - 1\right) = 37\text{cm}$$
$$\Rightarrow r \times \frac{37}{7} = 37\text{cm}$$
$$\Rightarrow r = 7\text{cm}$$

$\therefore$ Circumference of the circle = $2\pi r = 2 \times \frac{22}{7} \times 7\text{cm} = 44\text{cm}$

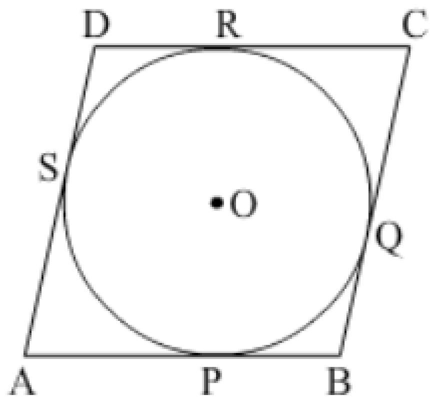
8. Correct answer: C

Common difference =

$$\frac{1-6q}{3q} - \frac{1}{3q} = \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$$

9. Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, AP = AS, BP = BQ, CR = CQ and DR = DS.

Adding the above equations,

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

AB + CD = AD + BC

2AB = 2BC

(Since, ABCD is a parallelogram so AB = DC and AD = BC)

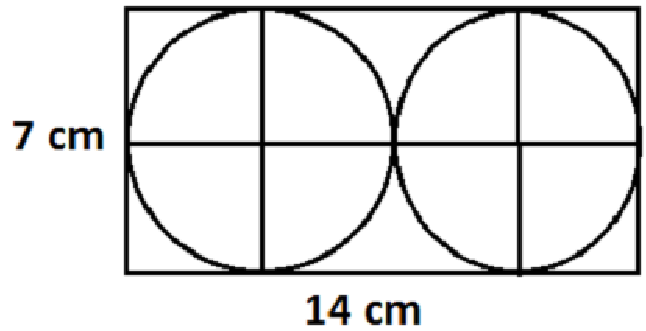
AB = BC

Therefore, AB = BC = DC = AD.

Hence, ABCD is a rhombus.

10. Dimension of the rectangular card board = 14 cm $\times$ 7 cm

Since, two circular pieces of equal radii and maximum area touching each other are cut from the rectangular card board, therefore, the diameter of each of each circular piece is $\frac{14}{2}$ = 7 cm.



Radius of each circular piece = $\frac{7}{2}$ cm.

$\therefore$ Sum of area of two circular pieces = $2 \times \pi \left(\frac{7}{2}\right)^2 = 2 \times \frac{22}{7} \times \frac{49}{4} = 77\text{cm}^2$

Area of the remaining card board

= Area of the card board - Area of two circular pieces

= 14 cm $\times$ 7 cm - 77 cm^2

= 98 cm^2 - 77 cm^2

= 21 cm^2

11. Given: AB = 12 cm, BC = 8 cm and AC = 10 cm.

Let, AD = AF = x cm, BD = BE = y cm and CE = CF = z cm

(Tangents drawn from an external point to the circle are equal in length)

$\Rightarrow$ 2(x + y + z) = AB + BC + AC = AD + DB + BE + EC + AF + FC = 30 cm

$\Rightarrow$ x + y + z = 15 cm

AB = AD + DB = x + y = 12 cm

$\therefore$z = CF = 15 - 12 = 3 cm

AC = AF + FC = x + z = 10 cm

$\therefore$ y = BE = 15 - 10 = 5 cm

$\therefore$ x = AD = x + y + z - z - y = 15 - 3 - 5 = 7 cm

12. Three digit numbers divisible by 7 are

105, 112, 119, … 994

This is an AP with first term (a) = 105 and common difference (d) = 7

Let a_n be the last term.

$a_n = a + (n - 1)d$

994 = 105 + (n - 1)(7)

7(n - 1) = 889

n - 1 = 127

n = 128

Thus, there are 128 three-digit natural numbers that are divisible by 7.

13.

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$
$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$\Rightarrow 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$$
$$\Rightarrow \left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$$
$$\therefore x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}}$$

14.Let E be the event that the drawn card is neither a king nor a queen.

Total number of possible outcomes = 52

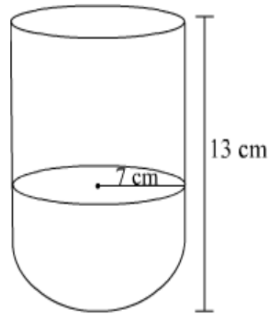
Total number of kings and queens = 4 + 4 = 8

Therefore, there are 52 - 8 = 44 cards that are neither king nor queen.

Total number of favourable outcomes = 44

$\therefore$ Required probability = P(E) = $\dfrac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{44}{52} = \dfrac{11}{13}$

15.



Let the radius and height of cylinder be r cm and h cm respectively.

Diameter of the hemispherical bowl = 14 cm

$\therefore$ Radius of the hemispherical bowl = Radius of the cylinder

$= r = \dfrac{14}{2}\text{cm} = 7\text{cm}$

Total height of the vessel = 13 cm

$\therefore$ Height of the cylinder, h = 13 cm - 7 cm = 6 cm

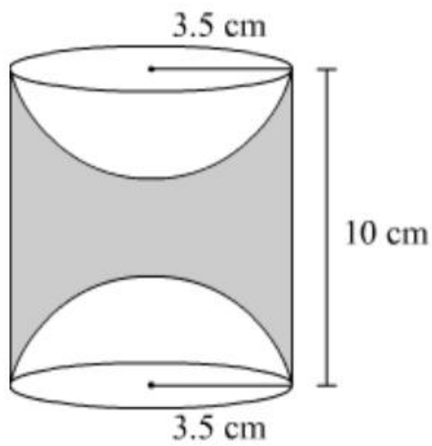
Total surface area of the vessel = 2 (curved surface area of the cylinder + curved surface area of the hemisphere)

(Since, the vessel is hollow)

$= 2\left(2\pi rh + 2\pi r^2\right) = 4\pi r\left(h + r\right) = 4 \times \dfrac{22}{7} \times 7 \times \left(6 + 7\right)\ \text{cm}^2$

= 1144 cm^2

16.



Height of the cylinder, h = 10 cm

Radius of the cylinder = Radius of each hemisphere = r = 3.5 cm

Volume of wood in the toy = Volume of the cylinder - 2 $\times$ Volume of each hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left(h - \frac{4}{3}r\right)$$

$$= \frac{22}{7} \times (3.5)^2 \left(10 - \frac{4}{3} \times 3.5\right)$$

$$= 38.5 \times\ 10 - 4.67$$

$$= 38.5 \times 5.33$$

$$= 205.205\ cm^3$$

Radius = 21 cm

17. The arc subtends an angle of 60^o at the centre.

(i) $l = \dfrac{\theta}{360^o} \times 2\pi r$

$$= \frac{60^o}{360^o} \times 2 \times \frac{22}{7} \times 21cm$$

$$= 22cm$$

(ii) Area of the sector $= \dfrac{\theta}{360^o} \times \pi r^2$

$$= \frac{60^o}{360^o} \times \frac{22}{7} \times 21 \times 21 cm^2$$

$$= 231\ cm^2$$

18. AB and CD are the diameters of a circle with centre O.

$\therefore$ OA = OB = OC = OD = 7 cm (Radius of the circle)

Area of the shaded region

= Area of the circle with diameter OB + (Area of the semi-circle ACDA - Area of $\triangle$ ACD)

$$= \pi\left(\frac{7}{2}\right)^2 + \left(\frac{1}{2} \times \pi \times 7^2 - \frac{1}{2} \times CD \times OA\right)$$

$$= \frac{22}{7} \times \frac{49}{4} + \frac{1}{2} \times \frac{22}{7} \times 49 - \frac{1}{2} \times 14 \times 7$$

$$= \frac{77}{2} + 77 - 49$$

$$= 66.5\ cm^2$$

19. Let the y-axis divide the line segment joining the points (-4,-6) and (10,12) in the ratio k: 1 and the point of the intersection be (0,y).

Using section formula, we have:

$$\left(\frac{10k + (-4)}{k+1}, \frac{12k + (-6)}{k+1}\right) = (0, y)$$

$$\therefore \frac{10k-4}{k+1} = 0 \Rightarrow 10k - 4 = 0$$

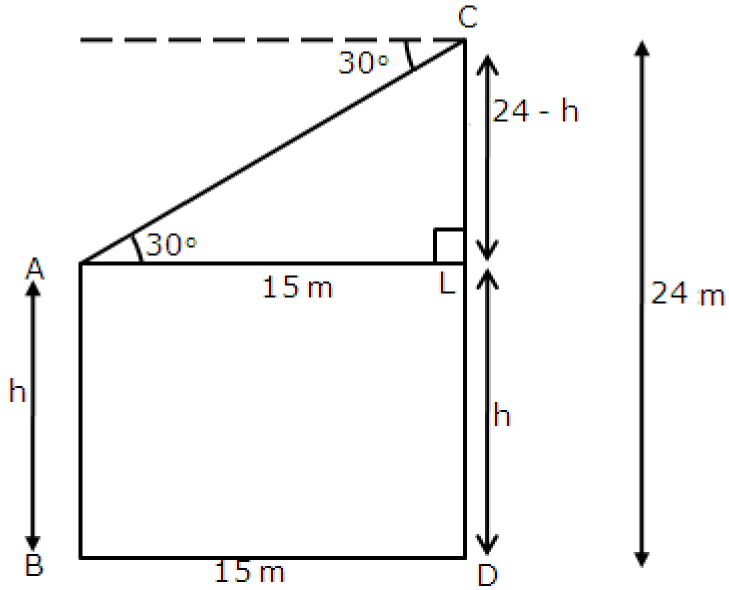
$$\Rightarrow k = \frac{4}{10} = \frac{2}{5}$$

Thus, the y-axis divides the
line segment joining the given points in the ratio 2:5

$$\therefore y = \frac{12k + (-6)}{k+1} = \frac{12 \times \frac{2}{5} - 6}{\frac{2}{5} + 1} = \frac{\left(\frac{24-30}{5}\right)}{\left(\frac{2+5}{5}\right)} = -\frac{6}{7}$$

Thus, the coordinates of the point of division are $\left(0, -\frac{6}{7}\right)$.

20.



Let AB and CD be the two poles, where CD (the second pole) = 24 m.

BD = 15 m

Let the height of pole AB be h m.

AL = BD = 15 m and AB = LD = h

So, CL = CD - LD = 24 – h

In $\triangle$ ACL,

$$\tan 30^{o} = \frac{CL}{AL}$$

$$\Rightarrow \tan 30^{o} = \frac{24 - h}{15}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24 - h}{15}$$

$$\Rightarrow 24 - h = \frac{15}{\sqrt{3}} = 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5\sqrt{3}$$

$$\Rightarrow h = 24 - 5 \times 1.732 \quad \left[\text{Taking} \sqrt{3} = 1.732\right]$$

$$\Rightarrow h = 15.34$$

Thus, height of the first pole is 15.34 m.

21.k + 4)x^2 + (k + 1)x + 1 = 0

a = k + 4, b = k + 1, c = 1

For equal roots, dicriminant, D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)(k + 3) = 0$

$\Rightarrow k = 5$ or $k = -3$

Thus, for k = 5 or k = -3, the given quadratic equation has equal roots.

22. $S_n = 3n^2 + 4n$

First term $(a_1) = S_1 = 3(1)^2 + 4(1) = 7$

$S_2 = a_1 + a_2 = 3(2)^2 + 4(2) = 20$

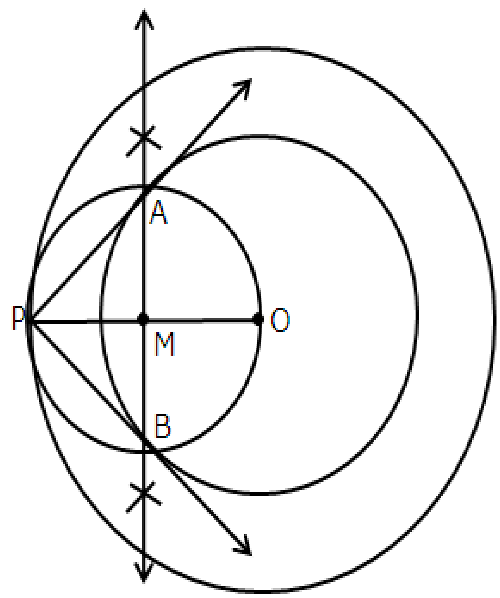
$a_2 = 20 - a_1 = 20 - 7 = 13$

So, common difference $(d) = a_2 - a_1 = 13 - 7 = 6$

Now, $a_n = a + (n - 1)d$

$\therefore a_{25} = 7 + (25 - 1) \times 6 = 7 + 24 \times 6 = 7 + 144 = 151$

23.



Steps of construction:

1. Draw two concentric circle with centre O and radii 4 cm and 6 cm. Take a point P on the outer circle and then join OP.

2. Draw the perpendicular bisector of OP. Let the bisector intersects OP at M.

3. With M as the centre and OM as the radius, draw a circle. Let it intersect the inner circle at A and B.

4. Join PA and PB.

Therefore, $\overline{PA}$ and $\overline{PB}$ are the required tangents.

24. The given points are A(-2,3) B(8,3) and C(6,7).

Using distance formula, we have:

$AB^2 = (8-(-2))^2 + (3-3)^2$

$\Rightarrow AB^2 = 10^2 + 0$

$\Rightarrow AB^2 = 100$

$BC^2 = (6-8)^2 + (7-3)^2$

$\Rightarrow BC^2 = (-2)^2 + 4^2$

$\Rightarrow BC^2 = 4 + 16$

$\Rightarrow BC^2 = 20$

$CA^2 = (-2-6)^2 + (3-7)^2$

$\Rightarrow CA = (-8)^2 + (-4)^2$

$\Rightarrow CA^2 = 64 + 16$

$\Rightarrow CA^2 = 80$

It can be observed that:

$BC^2 + CA^2 = 20 + 80 = 100 = AB^2$

So, by the converse of Pythagoras Theorem,

$\triangle$ ABC is a right triangle right angled at C.

25. Diameter of circular end of pipe = 2 cm

$\therefore$ Radius (r_1) of circular end of pipe = $\frac{2}{200}\text{m} = 0.01$ m

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.01)^2 = 0.0001\pi m^2$

Speed of water = 0.4 m/s = 0.4 $\times$ 60 = 24 metre/min

Volume of water that flows in 1 minute from pipe =
$24 \times 0.0001\pi m^3 = 0.0024\pi m^3$

Volume of water that flows in 30 minutes from pipe =

$30 \times 0.0024\pi m^3 = 0.072\pi m^3$

Radius (r_2) of base of cylindrical tank = 40 cm = 0.4 m

Let the cylindrical tank be filled up to h m in 30 minutes.

Volume of water filled in tank in 30 minutes is equal to the volume of water flowed out in 30 minutes from the pipe.

$\therefore \pi \times r_2^2 \times h = 0.072\pi$

$\Rightarrow 0.4^2 \times h = 0.072$

$\Rightarrow 0.16\ h = 0.072$

$\Rightarrow h = \dfrac{0.072}{0.16}$

$\Rightarrow h = 0.45\ m = 45\ cm$

Therefore, the rise in level of water in the tank in half an hour is 45 cm.

26. The group consists of 12 persons.

$\therefore$ Total number of possible outcomes = 12

Let A denote event of selecting persons who are extremely patient

$\therefore$ Number of outcomes favourable to A is 3.

Let B denote event of selecting persons who are extremely kind or honest.

Number of persons who are extremely honest is 6.

Number of persons who are extremely kind is 12 - (6 + 3) = 3

$\therefore$ Number of outcomes favourable to B = 6 + 3 = 9.

(i)

$$P(A) = \frac{\text{Number of outcomes favrouableto A}}{\text{Total number of possible outcomes}} = \frac{3}{12} = \frac{1}{4}$$

(ii)

$$P(B) = \frac{\text{Number of outcomes favorableto B}}{\text{Total number of possible outcomes}} = \frac{9}{12} = \frac{3}{4}$$

Each of the three values, patience, honesty and kindness is important in one's life.

27. Diameter of upper end of bucket = 30 cm

$\therefore$ Radius (r_1) of upper end of bucket = 15 cm

Diameter of lower end of bucket = 10 cm

$\therefore$ Radius (r_2) of lower end of bucket = 5 cm

Slant height (*l*) of frustum

$= \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(15-5)^2 + (24)^2} = \sqrt{(10)^2 + (24)^2} = \sqrt{100+576}$

$= \sqrt{676} = 26\text{cm}$

Area of metal sheet used to make the bucket

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \pi(15+5)26 + \pi(5)^2$

$= 520\pi + 25\pi = 545\pi\text{cm}^2$

Cost of 100 cm^2 metal sheet = Rs 10

Cost of 545 π cm^2 metal sheet

$= \text{Rs.} \dfrac{545 \times 3.14 \times 10}{100} = \text{Rs.}171.13$

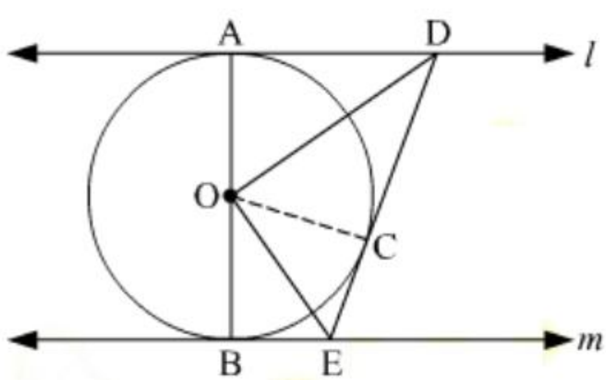
Therefore, cost of metal sheet used to make the bucket is Rs 171.13.

28. Given: *l* and m are two parallel tangents to the circle with centre O touching the circle at A and B respectively. DE is a tangent at the point C, which intersects *l* at D and m at E.

To prove: $\angle DOE = 90^o$

Construction: Join OC.

Proof:



In $\triangle ODA$ and $\triangle ODC$,

OA = OC (Radii of the same circle)

AD = DC (Length of tangents drawn from an external point to a circle are equal)

DO = OD (Common side)

$\Delta ODA \cong \Delta ODC$ (SSS congruence criterion)

$\therefore \angle DOA = \angle COD$ ... (1)

Similarly, $\Delta OEB \cong \Delta OEC$

$\therefore \angle EOB = \angle COE$ ... (2)

Now, AOB is a diameter of the circle. Hence, it is a straight line.

$\angle DOA + \angle COD + \angle COE + \angle EOB = 180^o$

From (1) and (2), we have:

$2\angle COD + 2\angle COE = 180^o$

$\Rightarrow \angle COD + \angle COE = 90^o$

$\Rightarrow \angle DOE = 90^o$

Hence, proved.

29. Let the sides of the two squares be x cm and y cm where x > y.

Then, their areas are x^2 and y^2 and their perimeters are 4x and 4y.

By the given condition:

$x^2 + y^2 = 400$ ... (1)

and $4x - 4y = 16$

$\Rightarrow 4(x - y) = 16 \Rightarrow x - y = 4$

$\Rightarrow x = y + 4$ ... (2)

Substituting the value of x from (2) in (1), we get:

$(y + 4)^2 + y^2 = 400$

$\Rightarrow y^2 + 16 + 8y + y^2 = 400$

$\Rightarrow 2y^2 + 16 + 8y = 400$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow$ y(y + 16) - 12 (y + 16) = 0

$\Rightarrow$ (y + 16) (y - 12) = 0

$\Rightarrow$ y = -16 or y = 12

Since, y cannot be negative, y = 12.

So, x = y + 4 = 12 + 4 = 16

Thus, the sides of the two squares are 16 cm and 12 cm.

30.

$$\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

$$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$$

$$\Rightarrow \frac{2x-2a-b-2x}{2x\ \ 2a+b+2x}=\frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-\ \ 2a+b}{2x\ \ 2a+b+2x}=\frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-1}{x\ \ 2a+b+2x}=\frac{1}{ab}$$

$$\Rightarrow 2x^2+2ax+bx+ab=0$$

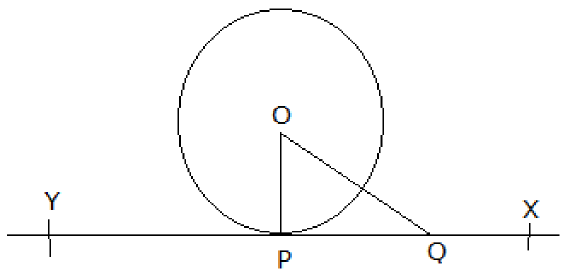
$$\Rightarrow 2x\ \ x+a\ \ +b\ \ x+a\ \ =0$$

$$\Rightarrow\ \ x+a\ \ 2x+b\ \ =0$$

$$\Rightarrow x+a=0 \text{ or } 2x+b=0$$

$$\Rightarrow x=-a, \text{ or } x=\frac{-b}{2}$$

31.



Given: A circle with centre O and a tangent XY to the circle at a point P

To Prove: OP is perpendicular to XY.

Construction: Take a point Q on XY other than P and join OQ.

Proof: Here the point Q must lie outside the circle as if it lies inside the tangent XY will become secant to the circle.

Therefore, OQ is longer than the radius OP of the circle, That is, OQ > OP. This happens for every point on the line XY except the point P.

So OP is the shortest of all the distances of the point O to the points on XY.

And hence OP is perpendicular to XY.

Hence, proved.

32. Given AP is -12, -9, -6, ..., 21

First term, a = -12

Common difference, d = 3

Let 21 be the n^{th} term of the A.P.

21 = a + (n - 1)d

$\Rightarrow$ 21 = -12 + (n - 1) $\times$ 3

$\Rightarrow$ 33 = (n - 1) $\times$ 3

$\Rightarrow$ n = 12

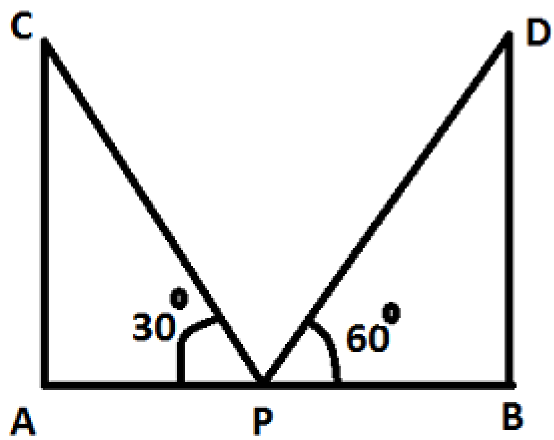
Sum of the terms of the AP = S_{12}

$$= \frac{n}{2}\left[2a + (n-1)d\right] = \frac{12}{2}\left[-24 + 11 \times 3\right] = 54$$

If 1 is added to each term of the AP, the sum of all the terms of the new AP will increase by n, i.e., 12.

$\therefore$ Sum of all the terms of the new AP = 54 + 12 = 66

33. Let AC and BD be the two poles of the same height h m.



Given AB = 80 m

Let AP = x m, therefore, PB = (80 - x) m

In ΔAPC,

$$\tan 30^o = \frac{AC}{AP}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \text{... (1)}$$

In ΔBPD,

$$\tan 60^o = \frac{BD}{AB}$$

$$\sqrt{3} = \frac{h}{80 - x} \qquad \text{... (2)}$$

Dividing (1) by (2),

$$\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{\frac{h}{x}}{\frac{h}{80-x}}$$

$$\Rightarrow \frac{1}{3} = \frac{80-x}{x}$$

$$\Rightarrow x = 240 - 3x$$

$$\Rightarrow 4x = 240$$

$$\Rightarrow x = 60$$

From (1),

$$\frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow h = \frac{60}{\sqrt{3}} = 20\sqrt{3}m$$

Thus, the height of both the poles is $20\sqrt{3}m$ and the distances of the point from the poles are 60 m and 20 m.

34. The given vertices are A(x,y), B(1,2) and C(2,1).

It is know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\frac{1}{2}|\, x_1 \ \ y_2 - y_3 \ + x_2 \ \ y_3 - y_1 \ + x_3 \ \ y_1 - y_2 \ \ |$$

$\therefore$ Area of Δ ABC

$$= \frac{1}{2}|\, x \ \ 2-1 \ +1\times \ 1-y \ +2 \ \ y-2 \ |$$

$$= \frac{1}{2}|\, x+1-y+2y-4 \,|$$

$$= \frac{1}{2}|\, x+y-3 \,|$$

The area of Δ ABC is given as 6 sq units.

$$\Rightarrow \frac{1}{2}[x+y-3] = 6 \Rightarrow x+y-3 = 12$$

$$\therefore x+y = 15$$