Mathematics

Class X

Board Paper - 2010

Time: **2½** hour | Total Marks: 80

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1. Answer to this paper must be written on the paper provided separately.
2. You will NOT be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections.
   Attempt all questions from Section A and any four questions from Section B.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
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7. Mathematical tables are provided.

Section - A (40 Marks)

Q.1.

(a) Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6},\ x \in R$$ (3)

(b). Tarun bought and article for Rs. 8000 and spent Rs. 1000 for transportation. He marked the article Rs. 11,700 and sold it to a customer. If the customer had to pay 10% sales tax, find:

(i) the customer's price

(ii) Tarun's profit percent. (3)

(C) Mr. Gupta opened a recurring deposit account in a bank. He deposited Rs. 2500 per month for two years. At the time of maturity he got Rs. 67,500. Find:

(i) the total interest earned by Mr. Gupta.

(ii) the rate of interest per annum.
(4)

Q.2.

(a) Given $A = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$ and $D = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. Find AB + 2C − 4D.
(3)

(b) Nikita invests Rs. 6000 for two years at a certain rate of interest compounded annually. At the end of first year it amounts to Rs. 6720. Calculate:

(i) the rate of interest.

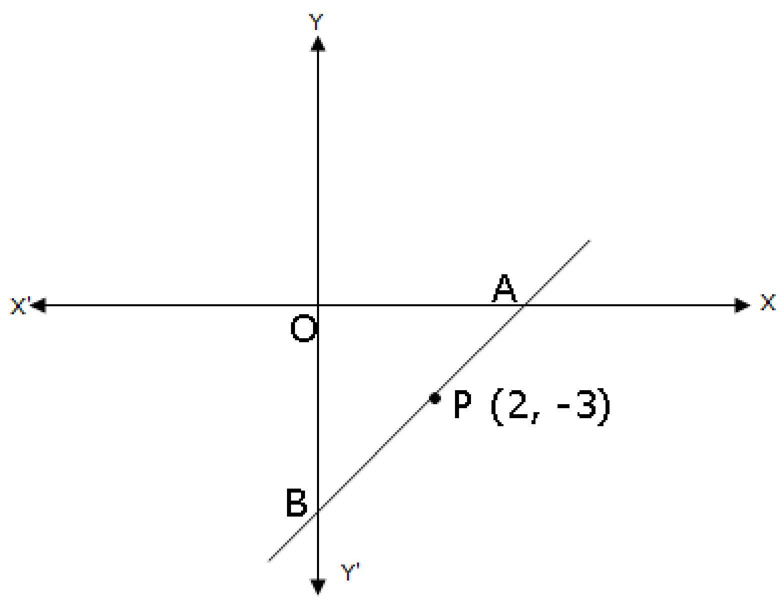
(ii) the amount at the end of the second year.
(3)

(c) A and B are two points on the x − axis and y-axis respectively. P (2, −3) is the mid- point of AB. Find the:

(i) coordinates of A and B

(ii) slope of line AB.

(iii) equation of line AB. (4)



Q.3.

(a) Cards marked with numbers 1, 2, 3, 4,…,20 are well shuffled and a card is drawn at random. What is the probability that the number on the card is:

(i) A prime number,

(ii) A number divisible by 3,

(iii) A perfect square? (3)

(b) Without using trigonometric tables evaluate

$$\frac{\sin 35^o \cos 55^o + \cos 35^o \sin 55^o}{\operatorname{cosec}^2 10^o - \tan^2 80^o} \quad (3)$$

(c) (Use graph paper for this question)

A(0, 3), B(3, −2) and O(0,0) are the vertices of triangle ABO.

(i) Plot the triangle on a graph sheet taking 2 cm = 1 unit on both the axes.

(ii) Plot D the reflection of B in the Y axis, and write its co-ordinates.

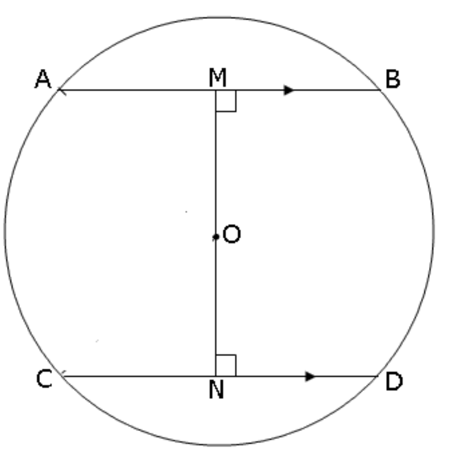
(iii) Give the geometrical name of the figure ABOD.

(iv) Write the equation of the line of symmetry of the figure ABOD. (4)

Q.4.

(a) When divided by $x - 3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p + 3)x - 6$ leave the same remainder. Find the value of 'p'. (3)

(b) In the figure given below AB and CD are two parallel chords and O is the centre. If the radius of the circle is 15 cm, find the distance MN between the two chords of length 24 cm and 18 cm respectively. (3)



(c) The distribution given below shows the marks obtained by 25 students in an optitude test. Find the mean. Median mode of the distribution. (4)

Marks obtained	5	6	7	8	9	10
No. of students	3	9	6	4	2	1

SECTION – B(40 marks)

Q.5.

(a) Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.

$$px^2 - 4x + 3 = 0.$$ (3)

(b) Rohit borrows Rs. 86,000 from Arun for two years at 5% per annum simple interest. He immediately lends out this money to Akshay at 5% compound interest compounded annually for the same period. Calculate Rohit's profit in the transaction at the end of two years. (3)

(c) Mrs. Kapoor opened a Savings Bank Account in State Bank of India on 9th January 2008. Her pass book entries for the year 2008 are given below:

Date	Particulars	Withdrawals (in Rs.)	Deposits (in Rs.)	Balance (in Rs.)
Jan 9,2008	By Cash	-	10,000	10,000
Feb 12, 2008	By Cash	-	15,500	25,500
April 6, 2008	To Cheque	3500	-	22,000
April 30, 2008	To Self	2000	-	20,000
July 16, 2008	By Cheque	-	6500	26,500
August 4, 2008	To Self	5500	-	21,000

August 20, 2008	To Cheque	1200	-	19,800
Dec. 12, 2008	By Chash	-	1700	21,500

Mrs. Kapoor closes the account on 31st December, 2008. If the bank pays interest at 4% per annum, find the interest Mrs. Kapoor receives on closing the account. Give your answer correct to the nearest rupee. (4)

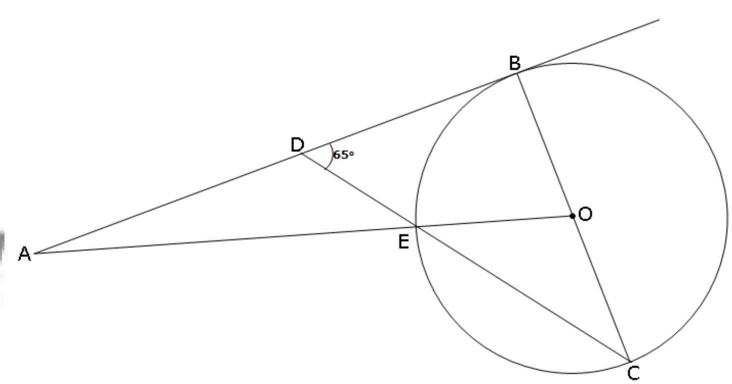
Q.6.

(a) A manufacturer marks an article for Rs. 5000. He sells it to a wholesaler at a discount of 25% on the marked price and the wholesaler sells it to a retailer at a discount of 15% on the marked price. The retailer sells it to a consumer at the marked price and at each stage the VAT is 8%. Calculate the amount of VAT received by the government from:

(i) the wholesaler,

(ii) the retailer. (3)

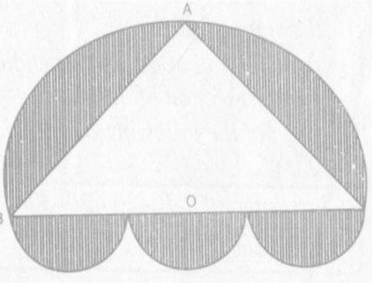
(b) In the following figure O is the centre of the circle and AB is a tangent to it at point B. $\angle BDC = 65^{\circ}$. Find $\angle BAO$. (3)



(c) A doorway is decorated as shown in the figure. There are four semi-circles. BC, the diameter of the larger semi-circle is of length 84 cm. Centres of the three equal semi-circles lie on BC. ABC is an isosceles triangle with AB = AC.

If BO = OC, find the area of the shaded region. $\left(\text{Take } \pi = \frac{22}{7}\right)$

(4)



Q.7.

(a) Use ruler and compasses only for this question:

(i) Construct $\triangle ABC$, where AB = 3.5 cm, BC = 6 cm and $\angle ABC = 60^{\circ}$.

(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

(iv) Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.

(3)

(b) The equation of a line $3x + 4y - 7 = 0$. Find

(i) The slope of the line.

(ii) The equation of a line perpendicular to the given line and passing through the intersection of the lines $x - y + 2 = 0$ and $3x + y - 10 = 0$.

(3)

(c) The Mean of the following distribution is 52 and the frequency of class interval 30-40 is 'f. Find 'f.

(4)

Class Interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	3	f	7	2	6	13

Q.8.

(a) Use the Remainder Theorem to factorise the following expression:

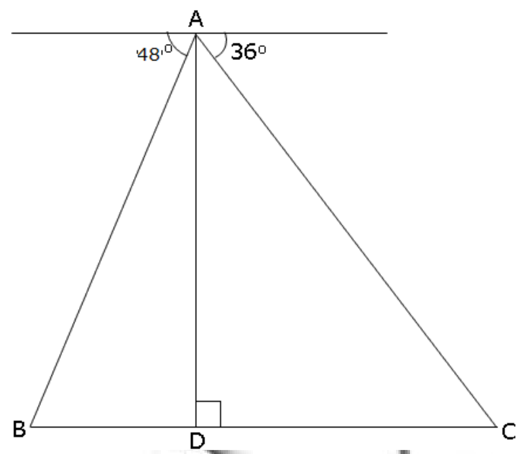
$$2x^3 + x^2 - 13x + 6$$ (3)

(b) If x, y, z are in continued proportion , prove that $\frac{(x+y)^2}{(y+z)^2} = \frac{x}{z}$.

(3)

(c) From the top of a light house 100 m high the angles of depression of two ships on opposite sides of it are 48° and 36° respectively. Find the distance between the two ships to the nearest metre.

(4)



Q.9.

(a) Evaluate:

$$\begin{bmatrix} 4\sin 30^\circ & 2\cos 60^\circ \\ \sin 90^\circ & 2\cos 0^\circ \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$ (3)

(b) In the given figure ABC is a triangle with $\angle EDB = \angle ACB$.

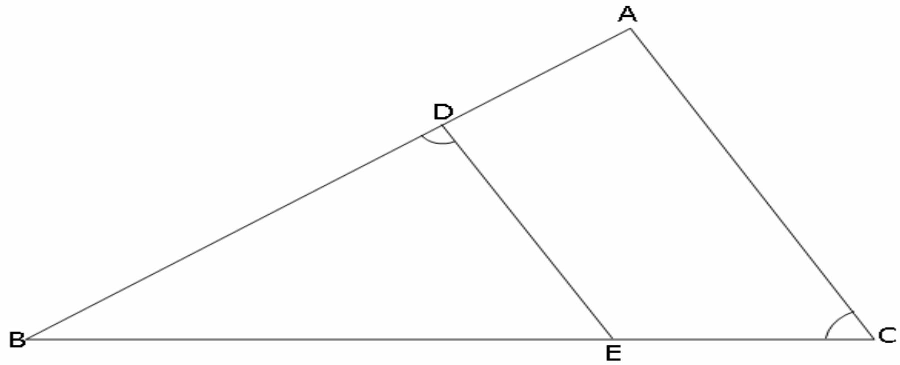
Prove that $\Delta ABC \sim \Delta EBD$.

If BE = 6 cm, EC = 4 cm, BD = 5 cm

And area of $\Delta BED = 9\ cm^2$. Calculate the

(i) length of AB

(ii) area of ΔABC. (3)



(C) Vivek invests Rs 4500 in 8%. Rs. 10 shares at Rs. 15. He sells the shares when the price rises to Rs. 30, and invests the proceeds in 12% Rs. 100 shares at Rs. 125. Calculate.

(i) the sale proceeds

(ii) the number of Rs. 125 shares he buys.

(iii) the change in his annual income from dividend. (4)

Q.10.

(a) A positive number is divided into two parts such that the sum of the squares of the two parts is 208. The square of the larger part is 18 times the smaller part. Taking x as the smaller part of the two parts, find the number. (4)

(b) The monthly income of a group of 320 employees in a company is given below:

Monthly Income	No. of Employees
6000-7000	20
7000-8000	45
8000-9000	65
9000-10000	95
10000-11000	60
11000-12000	30
12000-13000	5

Draw an ogive the given distribution on a graph sheet taking 2 cm = Rs. 1000 on one axis and 2 cm = 50 employees on the other axis. From the graph determine:

(i) the median wage

(ii) the number of employees whose income is below Rs 8500.

(iii) if the salary of a senior employee is above Rs. 11,500, find the number of senior employees in the company.

(iv) the upper quartile. (6)

Q.11.

(a) Construct a regular hexagon of side 4 cm. Construct a circle circumscribing the hexagon.
(3)

(b) A hemispherical bowl of diameter 7.2 cm is filled completely with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone.
(3)

(c) Given : $x = \dfrac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$

Use componendo and dividendo to prove that $b^2 = \dfrac{2a^2x}{x^3 + 1}$.

(4)

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Solution

Section - A (40 Marks)

Sol.1.

(a)

$$-3 < -\frac{1}{2} - \frac{2x}{3} \le \frac{5}{6}$$

Multiply by 6, we get

$$\Rightarrow -18 < -3 - 4x \le 5$$

$$\Rightarrow -15 < -4x \le 8$$

Dividing by -4, we get

$$\Rightarrow \frac{-15}{-4} > x \ge \frac{8}{-4}$$

$$\Rightarrow -2 \le x < \frac{15}{4}$$

$$\Rightarrow x \in \left[-2, \frac{15}{4}\right[$$

(b) Cost to Tarun = Rs. 8000 + Rs. 1000 = Rs. 9000

Marked Price = Rs. 11700

Sales Tax% = 10%

Sale Tax = $11700 \times \frac{10}{100}$ = Rs. 1170

(i) The customer's price = 11700 + 1170 = Rs. 12870

(ii) Profit = 11700 – 9000 = Rs. 2700

$\therefore$ Profit% = $\frac{2700 \times 100}{9000} = 30\%$

(c)

(i) Monthly instalment = Rs. 2500

n = 24, Amount deposited = 2500 $\times$ 24 = Rs. 60000

Maturity value = Rs. 67500

$\therefore$ Interest on his deposit = Rs. (67500 – 60000) = Rs. 7500

(ii) Now, Interest = $\frac{n(n+1)}{2} \times \frac{\text{Instalment} \times \text{Rate}}{100 \times 12}$

$\Rightarrow$ $7500 = \frac{24 \times 25}{2} \times \frac{2500 \times \text{Rate}}{100 \times 12}$

$\therefore$ Rate = $\frac{7500 \times 100 \times 24}{24 \times 25 \times 2500}$ = 12% p.a.

Sol.2.

(a) AB = $\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 18-2 \\ -6+4 \end{bmatrix} = \begin{bmatrix} 16 \\ -2 \end{bmatrix}$

$\therefore \quad AB + 2C - 4D = \begin{bmatrix} 16 \\ -2 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} - \begin{bmatrix} 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) (i) P = Rs. 6000, Amount at the end of first year = Rs. 6720

S.I. for first year = Rs. (6720 – 6000) = Rs. 720

Let r% be the rate of interest p.a

$$720 = \frac{6000 \times r \times 1}{100}$$

$$\therefore \quad r = \frac{720}{60} = 12\%$$

(ii) S.I. for second year $= \dfrac{6720 \times 12 \times 1}{100}$

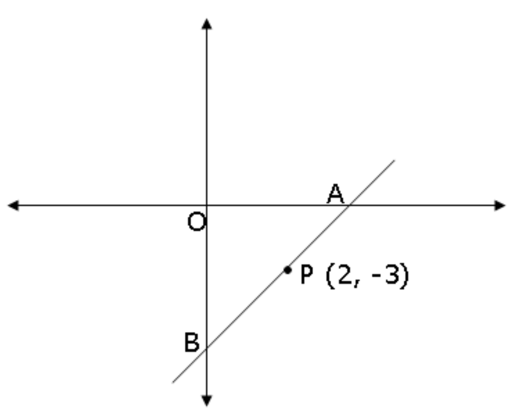
= Rs. 806.40

$\therefore$ Amount at the end of second year

= Rs. (6720 + 806.40)

= Rs. 7526.40

(c)



(i)

Let coordinates be A(x,0) and B(0,y).

Mid point of A and B is given by $\left(\dfrac{x+0}{2}, \dfrac{y+0}{2}\right) = \left(\dfrac{x}{2}, \dfrac{y}{2}\right)$

$\Rightarrow \quad (2,-3)=\left(\frac{x}{2},\frac{y}{2}\right)$

$\Rightarrow \quad \frac{x}{2}=2 \quad \text{and} \quad \frac{y}{2}=-3$

$\Rightarrow \quad x=4 \text{ and } y=-6$

$\therefore \quad A(4,0) \text{ and } B(0,-6)$

(ii) Slope of line AB, m = $\frac{y_2-y_1}{x_2-x_1}=\frac{-6-0}{0-4}=\frac{3}{2}$

(iii) Equation of line AB, using A(4, 0)

$$y-0=\frac{3}{2}(x-4)$$

$\Rightarrow \quad 3x - 2y = 12$

Sol.3.

(a) Total numbers = 20

(i) the prime numbers are 2, 3, 5, 7, 11, 13, 17, 19 respectively

Favourable cases = 8

Probability of getting a prime number = $\frac{8}{20}=\frac{2}{5}$

(ii) the numbers divisible by 3 are 3 ,6 ,9 ,12 ,15 ,18 respectively

Favourable cases =6

Probability of getting a number divisible by 3 = $\frac{6}{20}=\frac{3}{10}$

(iii) the perfect squares are 1, 4, 9, 16 respectively

Favourable cases = 4

Probability of getting a perfect square number = $\frac{4}{20}=\frac{1}{5}$

(b) $\frac{\sin 35^o \cos 55^o + \cos 35^o \sin 55^o}{\operatorname{cosec}^2 10^o - \tan^2 80^o}$

$$= \frac{\sin 35^o . \cos\left(90^o - 35^o\right) + \cos 35^o . \sin\left(90^o - 35^o\right)}{\operatorname{cosec}^2\left(90^o - 80^o\right) - \tan^2 80^o}$$

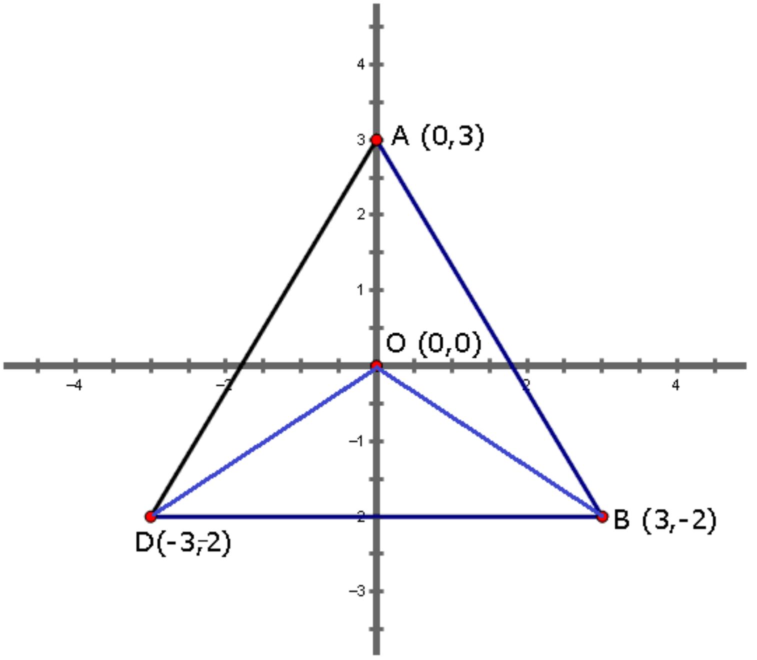
$$= \frac{\sin 35^o . \sin 35^o + \cos 35^o . \cos 35^o}{\sec^2 80^o - \tan^2 80^o}$$

$$= \frac{\sin^2 35^o + \cos^2 35^o}{\sec^2 80^o - \tan^2 80^o} = \frac{1}{1} = 1$$

(c) (i) $\triangle$AOB is shown in the graph.

(ii) the reflection of B in Y- axis is given by D (−3, −2)

(iii) figure ABOD is a tetrahedron



(iv) Equation of line of symmetry of figure ABOD is x = 0, i.e. y-axis.

Sol.4.

(a) If (x – 3) divides $f(x) = x^3 - px^2 + x + 6$, then

Remainder = $f(3) = 3^3 - p(3)^2 + 3 + 6 = 36 - 9p$

If (x−3) divides $g(x) = 2x^3 - x^2 - (p + 3) x - 6$, then

Remainder = $g(3) = 3(3)^3 - 3^2 - (p + 3)(3) - 6 = 30 - 3p$

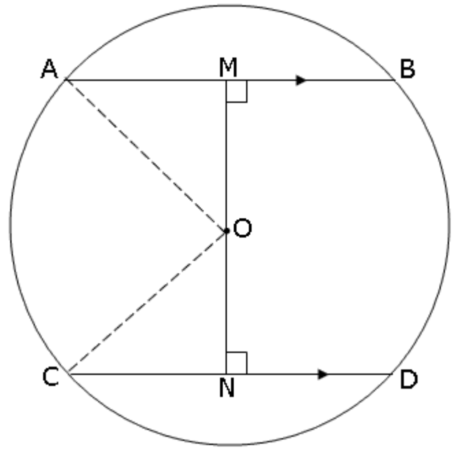
Now $f(3) = g(3)$

$\Rightarrow$ $36 - 9p = 30 - 3p$

$\Rightarrow$ $-6p = -6$

$\Rightarrow$ $p = 1$

(b) Construction : join OA and OB



As $OM \perp AB$ and $ON \perp CD$

$\therefore$ AM = MB = 24/2 cm = 12 cm

CN = ND = 18/2 cm = 9 cm

$\therefore$ $OM = \sqrt{OA^2 - AM^2} = \sqrt{15^2 - 12^2} = 9$ cm

$ON = \sqrt{OC^2 - CN^2} = \sqrt{15^2 - 9^2} = 12$ cm

$\therefore$ MN = OM + ON = 9 + 12 = 21 cm

(c)

Marks Obtained (x)	No. of Students (f)	c.f	fx
5	3	3	15
6	9	12	54
7	6	18	42
8	4	22	32
9	2	24	18
10	1	25	10

	$\sum f = 25$		$\sum fx = 171$

Mean $\bar{x} = \frac{\sum fx}{\sum f} = \frac{171}{25} = 6.84$

Median = $\left(\frac{25+1}{2}\right)$ the term = 13th term = 7

Since the number 6 has maximum frequency 9.

$\therefore$ Mode = 6

SECTION – B (40 marks)

Sol.5.

(a) Given the roots of the equation $px^2 - 4x + 3 = 0$ are equal.

Here a = p, b= −4 , c = 3

Since roots are equal.

$\therefore$ $b^2 - 4ac = 0$

$\Rightarrow$ $16 - 4p(3) = 0$

$\Rightarrow$ $12p = 16$

$\therefore$ $p = \frac{4}{3}$

(b) Given, P = Rs. 86000

R = 5%

T = 2 years

$$S.I. = \frac{P \times R \times T}{100} = \frac{86000 \times 5 \times 2}{100}$$

= Rs. 8600

$$C.I. = P\left[\left(1+\frac{r}{100}\right)^2 - 1\right]$$

$$= 86000\left[\left(1+\frac{5}{100}\right)^2 - 1\right]$$

$$= 86000 \times \frac{41}{20 \times 20} = \text{Rs. } 8815$$

$\therefore$ Rohit's profit = Rs. (8815 – 8600) = Rs. 215

(c) The minimum balance in the account for each month is given below:

Month	Minimum Balance
January	10,000
February	10,000
March	25,500
April	20,000
May	20,000
June	20,000
July	20,000
August	19,800
September	19,800
October	19,800
November	19,800

Total = Rs. 2,04,700

$$\text{S.I.} = \frac{PRT}{100} = \frac{204700 \times 1 \times 4}{100 \times 12}$$

= Rs. 682.33 ~ Rs. 682 appox.

Sol.6.

(a) (i) M.P. of an artical = Rs. 5000

Discount% for wholesaler = 25%

Cost price to wholesaler $= 5000 - 5000 \times \frac{25}{100} = \text{Rs. } 3750$

$$VAT(8\%) = 3750 \times \frac{8}{100} = Rs.\ 300$$

Therefore, VAT paid by manufacturer = Rs. 300

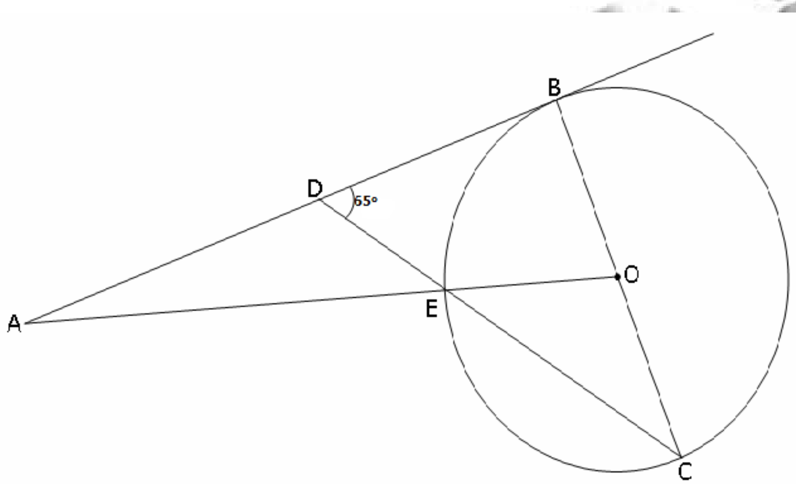
Now discount to retailer = 15%

Cost price to retailer = $5000 - 5000 \times \frac{15}{100}$ = Rs. 4250

$$VAT\ (8\%) = 4250 \times \frac{8}{100} = Rs.\ 340$$

Therefore, VAT paid by retailer = Rs. (340 – 300) = Rs. 40

(b)



Form the given figure, OB is radius and $OB \perp AB$, therefore in triangle BDC,

$$\begin{aligned} &\angle DBC + \angle BDC + \angle BCD = 180^\circ \\ \Rightarrow\quad & 90^\circ + 65^\circ + \angle BCD = 180^\circ \\ \Rightarrow\quad & \angle BCD = 25^\circ \end{aligned}$$

Now, OE =OC = radius, $\angle OEC = \angle OCE = 25^O$ (as $\angle OCE = \angle BCD$)

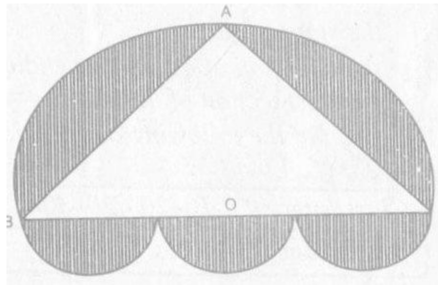
$\Rightarrow \angle AED = \angle OEC = 25^O$ (Vartically opposite angles)

Also, $\angle ADE = 180^O - 65^O = 115^O$

Therefore, in triangle AED,

$\angle BAO = 180^O - 115^O - 25^O = 40^O$

(c)



Here, radius of larger semi circle = $\frac{84}{2} = 42\,\text{cm}$

And, radius of smaller semi-circle = $\frac{84}{3\times 2} = 14\,\text{cm}$

Area of the shaded region = $\frac{\pi(42)^2}{2} + 3\times\frac{\pi(14)^2}{2} - \frac{1}{2}\times 84\times 42$

$= \frac{22}{7}[21\times 42 + 3\times 14\times 7] - 42\times 42$

$= 22\,[3\times 42 + 42] - 42\times 42$

$= 42\times[88 - 42] = 42\times 46$

$= 1932\ \text{cm}^2$

Sol.7.

(a) Steps of constructions:

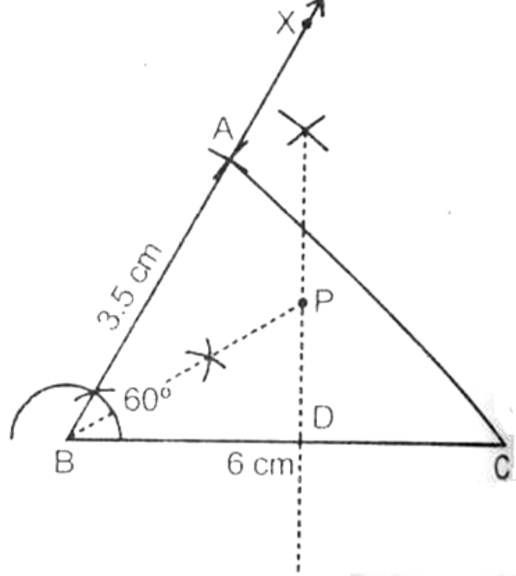
(i) draw a line BC = 6 cm and $\angle$CBX = 60°. Cut off AB = 3.5 cm. Join AC, ΔABC is the required triangle.

(ii) Draw perpendicular bisector of BC and bisector of $\angle$B.

(iii) Bisector of $\angle$B meets bisector of BC at P, therefore BP is the required length,

Where BP = 3.5 cm.

(iv) P is the point which is equidistant from BA and BC, also equidistant from B and C.



(b) $3x + 4y - 7 = 0$ .....(1)

(i) Slope of the line $m = -\frac{\text{coefficient of } x}{\text{coefficient of } y} = -\frac{3}{4}$

(ii) Equation of line perpendicular to the given line

$4x - 3y = \lambda$ .....(2)

Solving the equations $x - y + 2 = 0$ and $3x + y - 10 = 0$, point of intersection

is (2,4).

Line (2) passes through points (2,4).

$4(2) - 3(4) = \lambda \Rightarrow \lambda = -4$

Hence equation of required line is:

$$4x - 3y + 4 = 0$$

(c)

C.I.	Frequency	Mid Value (x)	fx
10-20	5	15	75
	3	25	75
20-30	f	35	35f
30-	7	45	315

40	2	55	110
40-50	6	65	390
	13	75	975
50-60			
60-70			
70-80			
	$\sum f = 36 + f$		$\sum fx = 1940 + 35f$

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$52 = \frac{1940 + 35f}{36 + f}$$

$\Rightarrow$ $1872 + 52f = 1940 + 35f$

$\Rightarrow$ $17f = 68$

$\Rightarrow$ $f = 4$

Sol.8.

(a) $f(x) = 2x^3 + x^2 - 13x + 6$

Factors of constant term 6 are $\pm1, \pm2, \pm3, \pm6$..

By hit and trail, putting $x = 2$, $f(2) = 2(2)^3 + 2^2 - 13(2) + 6 = 0$,

Hence $(x - 2)$ is a factor of $f(x)$ using factor theorem

So $f(x) = 2x^2(x - 2) + 5x(x - 2) - 3(x - 2)$

$= (x - 2)(2x^2 + 5x - 3)$

$= (x - 2)[2x^2 + 6x - x - 3]$

$= (x - 2)[2x(x + 3) - (x +)]$

$= (x - 2)(x + 3)(2x - 1)$

(b) $\because$ x,y,z are in continued proportion,

$$\therefore \quad \frac{x}{y} = \frac{y}{z} \Rightarrow y^2 = zx \qquad ...(1)$$

$$\frac{x+y}{y} = \frac{y+z}{z} \qquad \text{(By componendo)}$$

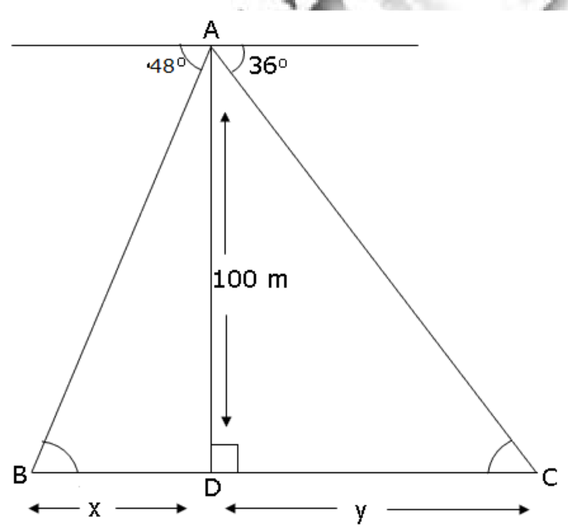
$$\Rightarrow \quad \frac{x+y}{y+z} = \frac{y}{z} \qquad \text{(By alternaedo)}$$

$$\Rightarrow \quad \frac{(x+y)^2}{(y+z)^2} = \frac{y^2}{z^2}$$

$$\Rightarrow \quad \frac{(x+y)^2}{(y+z)^2} = \frac{zx}{z^2}$$

$$\Rightarrow \quad \frac{(x+y)^2}{(y+z)^2} = \frac{x}{z} \qquad \text{Hence Proved.}$$

(c)



From rt. angle ΔADC,

$$\frac{AD}{CD} = \tan 36^o$$

$$\Rightarrow \quad \frac{100}{y} = \tan 36^o$$

$\Rightarrow \qquad y = \frac{100}{\tan 36^o} = \frac{100}{0.7265}$

$\Rightarrow \qquad y = 137.638$ m

From r. angle ΔADB.

$$\frac{100}{x} = \tan 48^o$$

$\Rightarrow \qquad x = \frac{100}{1.1106} = 90.04$ m.

$\therefore$ Distance between the ships = x + y

= 137.638 + 90.04

= 227.678 m.

= 228 m. (appro.)

Sol.9.

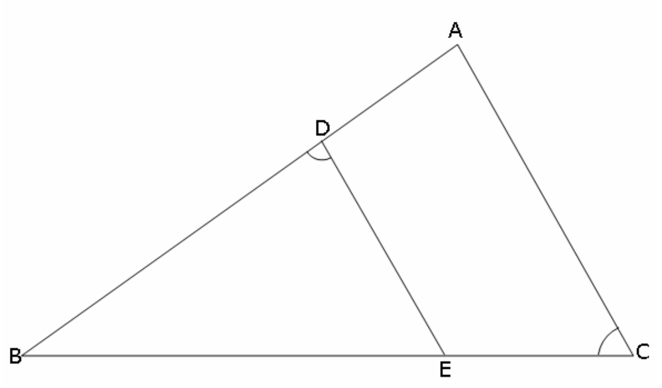
(a) $\begin{bmatrix} 4\sin 30^o & 2\cos 60^o \\ \sin 90^o & 2\cos 0^o \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 4\times\frac{1}{2} & 2\times\frac{1}{2} \\ 1 & 2\times 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 8+5 & 10+4 \\ 4+10 & 5+8 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 14 \\ 14 & 13 \end{bmatrix}$$

(b)



In ΔABC and ΔEBD.

$\angle ACB = \angle EDB$ (given)

$\angle ABC = \angle EBD$ (common)

$\Delta ABC \sim \Delta EBD$ (by AA- similarity).

(i) We have, $\frac{AB}{BE} = \frac{BC}{BD} \Rightarrow AB = \frac{6 \times 10}{5} = 1$ cm.

(ii) $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BED} = \left(\frac{AB}{BE}\right)^2$

$\Rightarrow$ Area of $\Delta ABC = \left(\frac{12}{6}\right)^2 \times 9 \text{ cm}^2$

$= 4 \times 9 \text{ cm}^2 = 36 \text{ cm}^2$

(c) (i) Number of Rs.15 shares bought $= \frac{4500}{15} = 300$

Total FV. of shares = F.V of each share x no. of shares= 300×10 = Rs. 3000

Dividend = 8% 0f total F.V $= \frac{8}{100} \times 3000$ = Rs. 240

S.P. of shares = 300×30 = Rs. 9000

Investment = Rs. 9000

(ii) The number of Rs. 125 shares $= \frac{9000}{125} = 72$

(iii) Total FV of Rs. 100 shares = Rs. 72 × 100 = Rs. 7200

New dividend $= \frac{12}{100} \times 7200$ = Rs. 864

The change in his annual income from dividend

= Rs. 864 – Rs. 240

= Rs. 624

Sol.10.

(a) Let the smaller part be x

Then, $(\text{larger part})^2 = 18x$

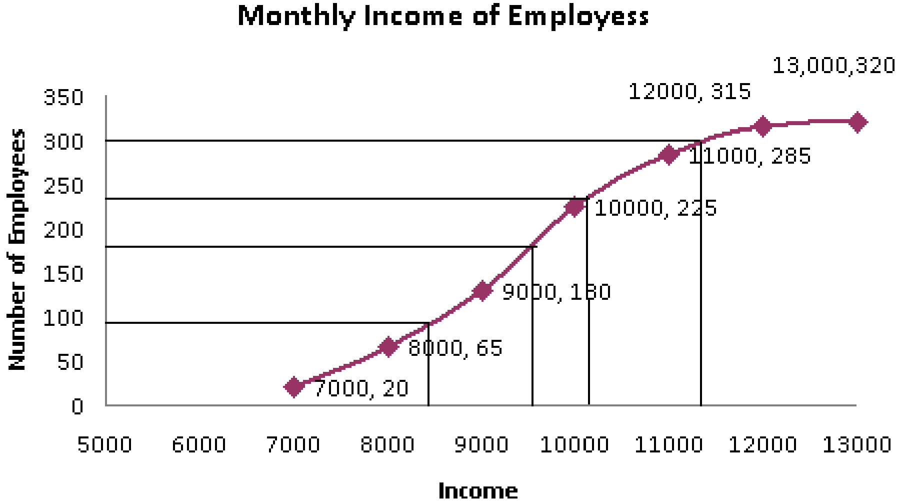
$\therefore$ larger part = $\sqrt{18x}$

Now, the sum of the squares of both the terms is given to be 208

$$x^2 + \left(\sqrt{18x}\right)^2 = 208$$
$$\Rightarrow x^2 + 18x = 208$$
$$\Rightarrow x^2 + 18x - 208 = 0$$
$$\Rightarrow x^2 + 26x - 8x - 208 = 0$$
$$\Rightarrow x(x+26) - 8(x+26) = 0$$
$$\therefore \Rightarrow (x+26)(x-8) = 0$$
$$\therefore \text{either } (x+26) = 0 \text{ or } (x-8) = 0$$
$$\Rightarrow x = -26 \text{ or } x = 8$$
$$x = -26 \text{ is rejected as it is negative}$$
$$\therefore x = 8$$
$$\text{smaller part} = 8$$
$$\text{larger part} = \sqrt{18 \times 8} = 12$$

(b)

Monthly Income	No. of Employees	c.f.
6000-7000	20	20
7000-8000	45	65
8000-9000	65	130
9000-10000	95	225
10000-11000	60	285
11000-12000	30	315
12000-13000	5	320



Here n = 320

(i) median = $\frac{n}{2}$ th term = 160 th term

From the graph, the corresponding x co-ordinate is 9400

median wage = 9400 (approx.)

(ii) The number of employees whose income is below Rs. 8500 = 90 (appro.)

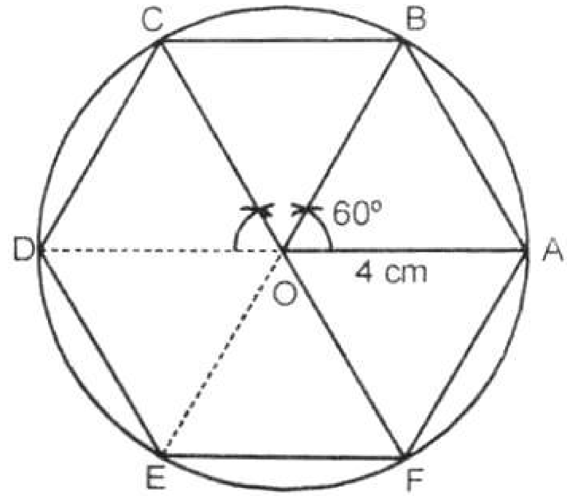
(ii) The number of senior employees whose salary is above Rs. 11500 = 320-300= 20 (approx.)

(iv) The upper quartile Q_3 = $\frac{3n}{4}$ th term = 240th term

From the graph, the corresponding x co-ordinate is 10250 (approx.)

Sol.11.

(a)



Steps of constructions:

(i) Draw a circle of radius 4 cm with center O.

(ii) Since $\frac{360^\circ}{6} = 60^\circ$, draw radii OA and OB, such that $\angle AOB = 60^\circ$.

(iii) Cut off arcs BC, CD, DE, EF and each equal to arc AB on given circle.

(iv) Join AB, BC, CD, DE, EF and FA to get required regular hexagon ABCDEF in a

given circle.

(b) Volume of hemispherical bowl $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3.6)^3$ cm^3

Volume of cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (4.8)^2 \times h\,\text{cm}^3$

But, Volume of bowl = Volume of cone

$$\Rightarrow \quad \frac{2}{3}\pi \times (3.6)^3 = \frac{1}{3}\pi \times (4.8)^2 \times h$$

$$\Rightarrow \quad h = \frac{2 \times 3.6 \times 3.6 \times 3.6}{4.8 \times 4.8} = 4.05 \text{ cm}$$

(c)
$$x = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}$$

By componendo and dividendo,

$$\frac{x+1}{x-1} = \frac{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2} + \sqrt{a^2 + b^2} - \sqrt{a^2 - b^2}}{\sqrt{a^2 + b^2} + \sqrt{a^2 - b^2} - \sqrt{a^2 + b^2} + \sqrt{a^2 - b^2}}$$

$$\frac{x+1}{x-1} = \frac{2\sqrt{a^2 + b^2}}{2\sqrt{a^2 - b^2}}$$

Squaring both sides,

$$\frac{x^2 + 2x + 1}{x^2 - 2x + 1} = \frac{a^2 + b^2}{a^2 - b^2}$$

By componendo and dividendo.

$$\frac{(x^2 + 2x + 1) + (x^2 - 2x + 1)}{(x^2 + 2x + 1) - (x^2 - 2x + 1)} = \frac{(a^2 + b^2) + (a^2 - b^2)}{(a^2 + b^2) - (a^2 - b^2)}$$

$$\Rightarrow \quad \frac{2(x^2 + 1)}{4x} = \frac{2a^2}{2b^2}$$

$$\Rightarrow \quad \frac{x^2 + 1}{2x} = \frac{a^2}{b^2}$$

$$\Rightarrow \quad b^2 = \frac{2a^2x}{x^2 + 1}$$

Hence Proved.