ICSE Paper 2005

MATHEMATICS

SECTION A [40 Marks]

(Answer all questions from this Section.)

Question 1.

- (a) (x-2) is a factor of the expression $x^3 + ax^2 + bx + 6$. When this expression is divided by (x-3), it leaves the remainder 3. Find the values of a and b. [3]
- (b) What number must be added to each of the numbers 6, 15, 20 and 43 to make them proportional?

 [3]
- (c) If the interest is compounded half yearly, calculate the amount when the Principal is ₹ 7,400, the rate of interest is 5% per annum and the duration is one year.

 [4]

Solution.

ď.,

 \Rightarrow

...

(a) Let

$$p(x) = x^3 + ax^2 + bx + 6$$

As (x-2) is a factor of p(x), p(2) = 0

$$8 + 4a + 2b + 6 = 0$$

$$2a+b+7=0$$

...(1)

As p(x) leaves the remainder 3 when divided by (x-3),

$$p(3) = 3$$

$$27 + 9a + 3b + 6 = 3$$

$$3a+b+10=0$$

...(2)

Solving (1) and (2), we get

$$2a+b+7=0$$

$$\frac{3a}{2} + b + 10 = 0$$

$$-a-3=0 \Rightarrow a=-3$$

from (1),

 \Rightarrow

$$2(-3) + b + 7 = 0$$

$$b = -1$$

$$a = -3, b = -1$$

Ans.

(b) Let the number x be added from each number.

$$(6+x):(15+x)::(20+x):(43+x)$$

$$(6+x)(43+x) = (15+x)(20+x)$$

$$\Rightarrow$$
 49x + 258 = 35x + 300

$$\Rightarrow 14x = 42$$

$$x = 3$$

Ans.

(c) Given: P = 7,400, r = 5%, n = 1 year.

Since, interest is compounded half yearly.

$$r = \frac{5}{2}$$
% and $n = 2$ years.

We know that,

$$A = P\left(1 + \frac{r}{100}\right)^{n}$$

$$= 7,400 \left(1 + \frac{5}{200}\right)^{2}$$

$$= \frac{7,400 \times 41 \times 41}{40 \times 40}$$

$$= 7,774.625$$

$$= ₹7,774.63$$

Ans.

Question 2.

- (a) Mr. R. K. Nair gets ₹ 6,455 at the end of one year at the rate of 14% per annum in a recurring deposit account. Find the monthly instalment. [3]
- (b) $A = \{x : 11x 5 > 7x + 3, x \in R\}$ and $B = \{x : 18x 9 \ge 15 + 12x, x \in R\}$ Find the range of set $A \cap B$ and represent it on a number line. [3]
 - ber [3] C. [4] E

(c) In the given figure, AB and DE are perpendicular to BC. If AB = 9 cm, DE = 3 cm and AC = 24 cm, calculate AD.

Solution.

(a) Let the monthly instalment be ₹x.
 Given: Maturity amount = ₹6,455, Time (n) = 1 year = 12 months, Rate (R) = 14% p.a.

Principle for one month =
$$P \frac{n(n+1)}{2}$$

= $\frac{x \times 12(12+1)}{2}$
= $78x$
Interest = $\frac{PRT}{100}$
= $\frac{78x \times 14 \times 1}{100 \times 12}$

Actual sum deposite = 12x

Maturity amount = Interest + Actual sum deposite

$$6,455 = \frac{91x}{100} + 12x$$

$$6,455 = \frac{1,291x}{100}$$

$$x = \frac{6,45,500}{1,291}$$

$$= 500$$

∴ Monthly instalment be ₹ 500.

Ans.

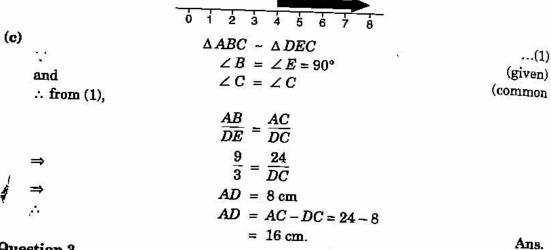
(b) Let $A = \{x: 11x-5 > 7x+3, x \in R\}$ $\{x: 4x > 8, x \in R\}$

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from equation (1) and (2), we get

$$A \cap B = \{x : x \ge 4, x \in R\}$$

Number line:



Question 3.

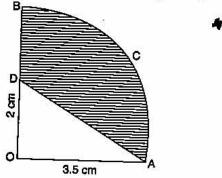
(a) Use a graph paper for this question. (Take 10 small divisions = 1 unit on both

P and Q have co-ordinates (0, 5) and (-2, 4).

- (i) P is invariant when reflected in an axis. Name the axis.
- (ii) Find the image of Q on reflection in the axis found in (i).
- (iii) (0, k) on reflection in the origin is invariant. Write the value of k.
- (iv) Write the co-ordinates of the image of Q, obtained by reflecting it in the origin followed by reflection in x-axis.
- (b) If the mean of the following distribution is 7.5, find the missing frequency 'f':

Variable :				- The the messing frequen				
and the second s	0	6	7	8	9	10	11	12
Frequency:	20	17	f	10	R	6	7	
11. 6.	140 540			-		_ •		- 6

(c) In the figure given below, OACB is a quadrant of a circle. The radius OA = 3.5cms. OD = 2 cm. Calculate the area of the shaded portion. [3]



Solution.

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III 11 11 11 11 11 11 11 11 11 11 11 11
Scrie:
11111111111111111111111111111111111111
P (0, 5)
E(-2, 4) Qellitt 4
######################################
######################################

4 Q' (2, -4)

- (i) Y-axis.
- (ii) Q'(2, 4)
- (iii) k=0
- (iv) Q'(2, 4).

Variable (x)	Frequency (f)	$f \times x$
5	20	100
` 6	17	102
7	f	7 <i>f</i>
8	. 10	80
9	8	72
10	6	60
11	7	77
12	6	72
	$\Sigma f = 74 + f$	$\Sigma f x = 563 + 7f$

$$\begin{aligned}
\text{Mean} &= \frac{\Sigma f x}{\Sigma f} \\
\Rightarrow & 7.5 &= \frac{563 + 7f}{74 + f} \\
\Rightarrow & 555 + 7.5f &= 563 + 7f \\
\Rightarrow & 0.5f &= 8 \\
f &= 16 \text{ Ans.}
\end{aligned}$$

(c) Area of quadrant OACB =
$$\frac{1}{4}\pi r^2$$

= $\frac{1}{4} \times \frac{22}{7} \times (3.5)^2$
= 9.625 cm^2
Area of \triangle OAD = $\frac{1}{2} \times 2 \times 3.5$
= 3.5 cm^2
Required shaded portion = $9.625 - 3.5$
= 6.125 cm^2 .

Question 4.

(a) Draw a histogram to represent the following data:

Pocket money in ₹	No. of Students	
150 — 200	10	
200 — 250	5	
250 300	7	
300 — 350	4	
350 — 400	3	

(b) Prove that
$$(1 + \tan A)^2 + (1 - \tan A)^2 = 2\sec^2 A$$
. [3]

- (c) The catalogue price of a computer set is ₹ 45,000. The shopkeeper gives a discount of 7% on the listed price. He gives a further off-season discount of 4% on the balance. However, sales tax at 8% is charged on the remaining amount. Find:
 - (i) The amount of sales tax a customer has to pay,
 - (ii) The final price he has to pay for the computer set.

[4]

[3]

Solution.

(b) L.H.S. =
$$(1 + \tan A)^2 + (1 - \tan A)^2$$

= $1 + \tan^2 A + 2 \tan A + 1 + \tan^2 A - 2 \tan A$
= $2 + 2 \tan^2 A$
= $2 (1 + \tan^2 A)$
= $2 \sec^2 A$ (7. $\sec^2 A - \tan^2 A = 1$)

Hence proved.

(c) Given:
$$P = 45,000$$
, $r_1 = 7\%$, $r_2 = 4\%$

$$S.P. after discount = \left(45,000 - \frac{7}{100} \times 45,000\right) - further discount$$

$$= 41,850 - \frac{4}{100} \times 41,850 = ₹40,176.00$$
(i) Sales tax = $\frac{8}{100} \times 40,176$

= **₹**3,214.08

(ii) Final price he has to pay =
$$\frac{7}{40}$$
,176 + 3,214.08 = $\frac{7}{43}$,390.08.

Ans

SECTION B [40 Marks]

(Answer any four questions from this Section)

Question 5.

- (a) Solve the following equation and give your answer up to two decimal places : $x^2 - 5x - 10 = 0$
- (h) PQR is a right-angled triangle with PQ = 3 cm and QR = 4 cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle.
- (c) In the given figure, write
 - (i) the co-ordinates of A, B and C.
 - (ii) the equation of the line through A and // to BC. [4]

Solution.

...

(a) Comparing $x^2 - 5x - 10 = 0$ with $ax^2 + bx$ +c = 0, we have a = 1, b = -5, c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25 + 40}}{2 \times 1} = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5 + 8.06}{2} \text{ or } \frac{5 - 7.08}{2}$$

x = 6.53 or -1.53.

Ans.

4 cm

£

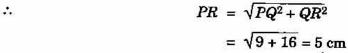
(b) Let 0 be the cent www.10 years question paper.com

$$\angle OMQ = \angle ONQ = 90^{\circ}$$

- : radius is always L to tangent.
- :. OMQN is a square.

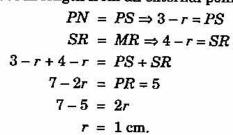
Let the radius be r cm.

$$PR = \sqrt{PQ^2 + QR^2}$$
$$= \sqrt{9 + 16} = 5 \text{ cm}$$



We have, PN = PS, SR = MR, QN = QM

Tangents are equal in length from an external point.



Ans.

(c) (i) A(2, 3), B(-1, 2), C(3, 0)

Ans.

(ii) Slope of BC =
$$\frac{2-0}{-1-3} = -\frac{1}{2}$$

Since line is parallel i.e., $m = m_1$

- ... The equation
- $y-y_1 = m(x-x_1)$

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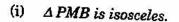
$$y-3 = -\frac{1}{2}(x-2)$$

$$\Rightarrow \qquad 2y-6 = -x+2$$

$$\Rightarrow \qquad x+2y = 8$$

Question 6.

(a) In the figure, PM is a tangent to the circle and PA = AM. Prove that:



(ii)
$$PA.PB = MB^2$$

[3]



(b) Find the value of x given that $A^2 = B$

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

(c) Write down the relation denoted by the arrow diagram, by listing the ordered pairs. State the domain, co-domain and the range of the relation.

Is the relation a function ? If so, state its type. **

[4]

[3]

Ans.

Solution.

(a) (i)
$$\angle AMP = \angle APM$$
 ('.' $PA = AM$ given) but $\angle AMP = \angle PBM$

... Angle between the tangent and chord is equal to angle subtended by same chord in alternate segment.

$$\angle APM = \angle PBM$$

∴ Δ PMB is an isosceles.

Proved

(ii)
$$\triangle PMB \sim \triangle PAM$$
 ...(1)
$$\angle PBM = \angle AMP$$
 (as proved above)
$$\angle MPB = \angle MPA$$
 (common)
$$\therefore \text{ from (1), } \frac{MP}{PA} = \frac{MB}{AM} = \frac{PB}{PM}$$

$$\frac{mr}{PA} = \frac{mB}{AM} = \frac{PB}{PM}$$

$$PM^2 = PA PB$$

$$\Rightarrow MB^2 = PA PB$$

$$MB^- = PA_*P_*$$

PM = MB

Proved

Ans.

(b) Given:
$$A^2 = B$$

$$\Rightarrow A \cdot A = B$$

$$\begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow x = 36$$

Solution has not given due to out of present syllabus.

www.10yearsquestionpaper.com Question 7.

- (s) Mr. Tiwari invested ₹ 29,040 in 15% ₹ 100 shares quoted at a premium of 20%. Calculate:
 - (i) The number of shares bought by Mr. Tiwari.
 - (ii) Mr. Tiwari's income from the investment.

(iii) The percentage return on his investment.

[3]

(b) From the top of a cliff 92 m high, the angle of depression of a buoy is 20°. Calculate to the nearest metre, the distance of the buoy from the foot of the cliff.

(c) A circle with center O, diameter AB and a chord AD is drawn. Another circle is drawn with AO as diameter to cut AD at C.

Prove that BD = 2OC.

[4]

Solution.

Market value of one share = 100 + 20 = ₹ 120(a) (i)

No. of shares =
$$\frac{\text{Invested money}}{\text{Market value}}$$
$$= \frac{29,040}{120} = 242$$

Ans.

(ii), Income =
$$\frac{15}{100} \times 100 \times 242$$

= $\frac{3}{630}$

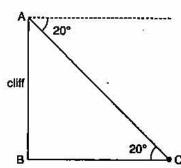
Ans.

(iii) % Return =
$$\frac{3,630}{29,040} \times 100 = 12.5\%$$
.

Ans.

(b) The distance of buoy from foot = BC

$$= \frac{92}{\tan 20^{\circ}}$$
= \cdot 20^{\circ} \times 92
= \tan 70^{\circ} \times 92
= 2.748 \times 92
= 252.8 m

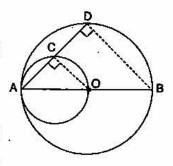


(c) In A OAC and A BAD.

$$\angle OCA = \angle BAD = 90^{\circ}$$

Angle is semi-circle is right angle.

and
$$\angle OAC = \angle BAD$$
 (common)
 $\therefore \qquad \triangle OCA \sim \triangle BAD$
 $\therefore \qquad \frac{OA}{BA} = \frac{OC}{BD}$
 $\Rightarrow \qquad \frac{OA}{BOA} = \frac{OC}{BD}$ ($\therefore BA = 2AO$)



Question 8.

(a) Mr. Rakesh Sharma receives his annual salary as given below ;

BD = 2OC

Basic Salary

 $\mathbf{7}$ 6,000 per month.

Hence proved.

Ans.

(common)

Dearness Allowance

₹ 5,000 per month.

Solution has not given due to out of present syllabus.

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 Contribution towards provident Fund : ₹ 13,200 per year.

Contribution towards L.I.C. premium.

: ₹5,000 per year. Donations: To Prime Minister's Relief Fund : ₹2,000 (eligible for 100% tax exemption)

Calculate:

Mr. Sharma's taxable income, (ii) The Mrs. Sharma has to pay for the financial year.

Tax slab:

Upto ₹ 50,000 No tax.

₹ 50,001 to ₹ 60,000 10% of income exceeding ₹50,000

₹ 60,001 to ₹ 1,50,000 ₹ 1,000 + 20% of the income exceeding ₹60,000. Above ₹ 1.50.000 ₹ 19,000 + 30% of the income exceeding

₹ 1,50,000. Standard Deduction ₹ 20,000.

Rebate in tax 20% of the total savings or ₹ 14,000 whichever

CESS 2% of the tax payable after rebate.

[6] (b) A metallic sphere of radius 10.5 cm is melted and then recast into small cones. each of radius 3.5 cm and height 3 cm. Find the number of cones thus obtained. [4]

Solution:

(b) No. of cones =
$$\frac{\text{Volume of sphere}}{\text{Volume of one cone}} = \frac{\frac{4}{3} \pi R^3}{\frac{1}{3} \pi r^2 h} = \frac{4R^3}{r^2 h} = \frac{4 \times (10.5)^3}{(3.5)^2 \times 3}$$

Question 9.

(a) Use a graph paper for this question. The graph of a linear equation in x and y, passes through A (-1, -1) and B (2, -1)5). From your graph, find the values of h and k, if the line passes through (h, 4) and (1/2, k). [3]

Ans.

(b) In an isosceles triangle ABC, with AB = AC, BD is the perpendicular from B to the side AC. Prove that $BD^2 - CD^2 = 2CD.AD$.

(c) A page from the passbook of Mrs. Rama Bhalla is given below:

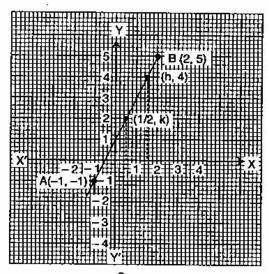
= 126.

Date	Particulars	Withdrawals	Deposit	Balance	Signature
Year 2004		₹ Ps.	₹ Ps.	₹ Ps.	3000
January 1	B/F	(s)		20000.00	
January 9	By cash	-	200.00	2200.00	1
February 10	To cheque	500.00		1700.00	
February 24	By cheque	H	300.00	2000.00	
July 29	To cheque	200.00		1800.00	
November 7	By cash	a a	300.00	2100.00	1
December 8	By cash	0	200.00	2300.00	

Calculate the interest due to Mrs. Bhalla for the period from January 2004 to December 2004, at the rate of 5% per annum. [4] Solution.

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(a)



$$h=\frac{3}{2},\,k=2$$

Ans.

(b) In $\triangle ABD$, In $\triangle BDC$,

$$BD^2 = AB^2 - AD^2 \qquad ...(i)$$

 $CD^2 = BC^2 - BD^2$...(ii)

Substracting (ii) from (i),

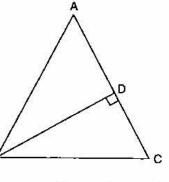
$$\begin{split} BD^2 - CD^2 &= (AB^2 - AD^2) - (BC^2 - BD^2) \\ &= AB^2 - (AC - DC)^2 - BC^2 + BD^2 \\ &= AB^2 - AC^2 - DC^2 + 2ACDC - BC^2 + BD^2 \end{split}$$

$$= 2ACDC - DC^2 - (BC^2 - DB^2)$$

$$= 2ACDC - 2DC^2$$

$$= 2DC (AC - DC)$$

= 2DCAD



Hence Proved.

(c)

2,300 December

 $(\cdot,\cdot AB = AC)$

Total
$$\frac{23,500}{P \times R \times T}$$

S.I. = $\frac{P \times R \times T}{100}$
= $\frac{23,500 \times 5 \times 1}{12 \times 100} = 97.916$
= $\frac{397.92}{23500} = \frac{397.92}{23500} = \frac{39$

Ans.

Question 10. www.10yearsquestionpaper.com

- (a) Using a ruler and compass only:
 - (i) Construct a triangle ABC with BC = 6 cm, ∠ABC = 120° and AB = 3.5 cm.
 - (ii) In the above figure, draw a circle with BC as diameter. Find a point 'P' on the circumference of the circle which is equidistant from AB and BC.
 Measure ∠BCP.
 [4]
- (b) Using a graph paper, draw an Ogive for the following distribution which shows a record of the weight in kilograms of 200 students.

Weight	40-45	45-50	50–55	55–60	60–65	65–70	70–75	75-80
Frequency	5	17	22	34	51	31	20	— <u> </u>

Use your Ogive to estimate the following:

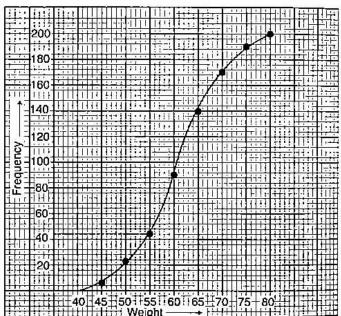
- (i) The percentage of students weighing 55 kg or more.
- (ii) The weight above which the heaviest 30% of the students fall,
- (iii) The number of students who are:
 - (1) under-weight and
 - (2) over-weight, if 55.70 kg is considered as standard weight.

[6]

Solution.

(a) Refer Answer 10. (a), 2013.

(b)	Weight	f	c.f.
	40–45	5	5
	45-50	17	. 22
	50–55	22	44
	55–60	45	89
	60-65	51	140
	65–70	31	171
	70–75	20	191
	75–80	9	200



(i) Number of students weigting 55 kg or more = 200 - 44 = 156.

Percentage (%) =
$$\frac{156}{200} \times 100$$

= 78% (app.)

Ans.

(ii)
$$\frac{30}{100} \times 200 = 60$$

Heaviest weight = weight of (200 - 60) students = 140 students.

= 140 Students

: the weight of 140 students = 65 kg

(iii) (1) Under weight
$$= 45$$

(2) Over weight
$$= 200 - 45 = 155$$
.

Ans.

Question 11. www.10yearsquestionpaper.com

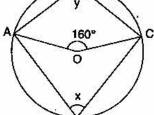
(a) In the alongside figure, O is the centre of the circle and $\angle AOC = 160^{\circ}$.

Prove that $3 \angle y - 2 \angle x = 140^{\circ}$.



(b) Without using mathematical tables, find the value of x if

 $\cos x = \cos 60^{\circ}\cos 30^{\circ} + \sin 60^{\circ}\sin 30^{\circ}$.



(c) By increasing the speed of a car by 10 km/hr, the time of journey for a distance of 72 km. is reduced by 36 minutes. Find the original speed of the car.

[4]

Solution.

$$\angle x = \frac{1}{2}(160^{\circ}) = 80^{\circ}$$
 $\angle y = \frac{1}{2}(360^{\circ} - 160^{\circ}) = 100^{\circ}$

The angle at the centre is twice the angle at the circumference.

$$3 \angle y - 2 \angle x = 3 \times 100^{\circ} - 2 \times 80^{\circ}$$
$$= 300^{\circ} - 160^{\circ} = 140^{\circ}$$

Hence proved.

$$\cos x = \cos 60^{\circ} \cos 30^{\circ} + \sin 60^{\circ} \sin 30^{\circ}$$

$$\cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$
$$= 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\cos x = \cos 30^{\circ}$$

$$x = 30^{\circ}$$
.

Ans.

Time =
$$\frac{72}{x}$$
, when original speed

Time = $\frac{72}{x+10}$, when speed increased by 10 km/hr.

$$\frac{72}{x} - \frac{72}{x + 10} = \frac{36}{60} = \frac{3}{5}$$

$$72\left[\frac{x+10-x}{x(x+10)}\right] = \frac{3}{5}$$

$$\Rightarrow$$

:

$$240 \times 5 = x^2 + 10x$$

$$x^2 + 10x - 1200 = 0$$

$$x^2 + (40 - 30) x - 1200 = 0$$

$$x^{2} + 40x - 30x - 1200 = 0$$

$$x(x + 40) - 30(x + 40) = 0$$

$$(x+40)(x-30) = 0$$

$$x - 30 = 0$$
 or $x + 40 = 0$

$$x = 30$$
 $x = -40$ (not possible)

: Speed of car be 30 km/hr.

Ans.