

# ICSE Paper 2011

## MATHEMATICS

### SECTION A [40 Marks]

(Answer all questions from this Section.)

#### Question 1.

(a) Find the value of 'k' if  $(x - 2)$  is a factor of:

$$x^3 + 2x^2 - kx + 10$$

Hence determine whether  $(x + 5)$  is also a factor. [3]

(b) If  $A = \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ , is the product  $AB$  possible? Give a reason. If yes, find  $AB$ . [3]

(c) Mr. Kumar borrowed ₹ 25,000 for two years. The rate of interest for the two successive years are 8% and 10% respectively. If he repays ₹ 6,200 at the end of the first year, find the outstanding amount at the end of the second year. [4]

#### Solution :

(a) Let  $f(x) = x^3 + 2x^2 - kx + 10$

∵  $(x - 2)$  is a factor,

$$\therefore f(2) = 0$$

$$f(2) = 8 + 8 - 2k + 10 = 0$$

$$\Rightarrow k = 13$$

To check for  $(x + 5)$  is a factor,

$$\begin{aligned} f(-5) &= (-5)^3 + 2(-5)^2 - 13(-5) + 10 \\ &= -125 + 50 + 65 + 10 = 0 \end{aligned}$$

∴  $(x + 5)$  is a factor.

(b)  $A_{2 \times 2} \cdot B_{2 \times 1}$

From the order of both the matrix A and B, it is clear that  $AB$  is possible because the number of columns of A are equal to the number of rows of B.

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 5 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 + 20 \\ 8 - 8 \end{bmatrix} \\ &= \begin{bmatrix} 26 \\ 0 \end{bmatrix} \end{aligned}$$

Ans.

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(c) Given : Principal = ₹ 15,000

We know that  $A = P \left( 1 + \frac{r}{100} \right)^n$

$$\begin{aligned} \text{Amount after 1}^{\text{st}} \text{ year} &= 15,000 \left( 1 + \frac{8}{100} \right) \\ &= ₹ 16,200 \end{aligned}$$

Principal after repayment = 16,200 - 6,200 = ₹ 10,000

$$\begin{aligned} \text{Amount outstanding at the end of second year} \\ &= 10,000 \left( 1 + \frac{10}{100} \right) \\ &= ₹ 11,000 \end{aligned}$$

Ans.

**Question 2.**

(a) From a pack of 52 playing cards all cards whose numbers are multiples of 3 are removed. A card is now drawn at random.

What is the probability that the card drawn is :

(i) a face card (King, Jack or Queen)

(ii) an even numbered red card ?

[3]

(b) Solve the following equation :

$$x - \frac{18}{x} = 6. \text{ Give your answer correct to two significant figures.}$$

[3]

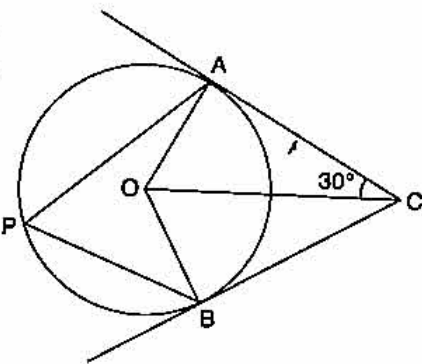
(c) In the given figure O is the centre of the circle. Tangents at A and B meet at C. If  $\angle AOC = 30^\circ$ , find

(i)  $\angle BCO$

(ii)  $\angle AOB$

(iii)  $\angle APB$

[4]



**Solution :**

(a) Number of cards which are multiples of 3 = 12

$$\text{Cards left in the pack} = 40$$

(i) Number of face cards = 12

$$P(\text{face card}) = \frac{12}{40} = \frac{3}{10}$$

Ans.

(ii) Even numbered red cards = 10

$$P(\text{even number red card}) = \frac{10}{40} = \frac{1}{4}$$

Ans.

(b) Let  $x^2 - 6x - 18 = 0$

Compare with equation  $ax^2 + bx + c = 0$ , we get

$$a = 1, b = -6, c = -18$$

Now,

$$\begin{aligned} x &= \frac{b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{36 + 72}}{2} \end{aligned}$$

$$= \frac{6 \pm 6\sqrt{3}}{2} \Rightarrow x = 3 \pm 3\sqrt{3}$$

$\therefore$   
Taking +ve and -ve sign respectively, we get

$$x = 8.196 \text{ or } x = -2.196$$

**Ans.**

- (c)  $\Delta ACO \cong \Delta BCO$  (R.H.S.)  
 $\therefore \angle BCO = \angle ACO$  (C.P.C.T.)  
 (i)  $\angle BCO = 30^\circ$   
 In  $\Delta ACO$ ,  $\angle OAC = 90^\circ$  (Radius is perpendicular to tangent)  
 $\therefore \angle AOC = 60^\circ$   
 Also  $\angle BOC = 60^\circ$  (C.P.C.T.)  
 (ii)  $\angle AOB = 120^\circ$   
 (iii)  $\angle APB = 60^\circ$  (Angle at circumference is half the angle at the centre)

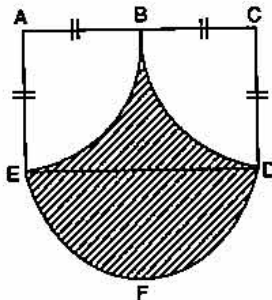
**Question 3.**

(a) Ahmed has a recurring deposit account in a bank. He deposits ₹ 2,500 per month for 2 years. If he gets ₹ 66,250 at the time of maturity, find

- (i) The interest paid by the bank.  
 (ii) The rate of interest. [3]

(b) Calculate the area of the shaded region, if the diameter of the semi circle is equal to 14 cm.

Take  $\pi = \frac{22}{7}$  [3]



(c) ABC is a triangle and G(4, 3) is the centroid of the triangle. If A = (1, 3), B = (4, b) and C = (a, 1), find 'a' and 'b'. Find the length of side BC. [4]

**Solution :**

(a) (i) Interest =  $66,250 - 2,500 \times 24$   
 $= 66,250 - 60,000$   
 $= ₹ 6,250$  **Ans.**

(ii) Principal =  $\frac{n(n+1)}{2} \times \text{sum of deposited per month}$   
 $= \frac{24(24+1)}{2} \times 2,500$   
 $= \frac{24 \times 25}{2} \times 2,500$

$$I = \text{Principal} \times \frac{R}{100} \times \frac{1}{12}$$

$$R = \frac{6,250 \times 2 \times 100 \times 12}{2,500 \times 24 \times 25}$$

$$= 10\% \text{ p.a.}$$

Ans.

- (b) Area of shaded portion = Complete area – area of the two quadrants  
 = (Area of ACDE + Area of semi circle EFD)  
 – (Area of Quadrant ABE + Area of Quadrant BCD)

$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{4} (7)^2 + \frac{\pi}{4} (7)^2 \right\}$$

$$= \left\{ 14 \times 7 + \frac{\pi}{2} (7)^2 \right\} - \left\{ \frac{\pi}{2} (7)^2 \right\}$$

$$= 98 \text{ cm}^2.$$

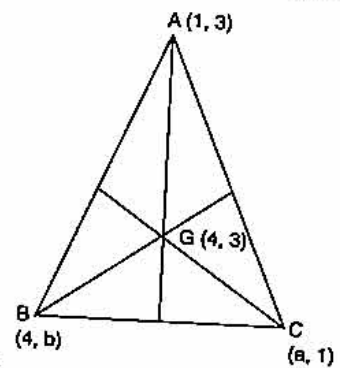
Ans.

- (c) Coordinate of centroid G  $\left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$

$$\Rightarrow \frac{1 + 4 + a}{3} = 4 \Rightarrow a = 7$$

$$\frac{3 + b + 1}{3} = 3 \Rightarrow b = 5$$

Now,  $BC = \sqrt{(4-7)^2 + (5-1)^2}$   
 $= \sqrt{9 + 16} = 5 \text{ units.}$



Ans.

**Question 4.**

- (a) Solve the following inequation and represent the solution set on the number line  
 $2x - 5 \leq 5x + 4 < 11$ , where  $x \in I$ . [3]  
 (b) Evaluate without using trigonometric tables: [3]

$$2 \left( \frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left( \frac{\cot 55^\circ}{\tan 35^\circ} \right)^2 - 3 \left( \frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right)$$

- (c) A Mathematics aptitude test of 50 students was recorded as follows:

| Marks           | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|-----------------|-------|-------|-------|-------|--------|
| No. of students | 4     | 8     | 14    | 19    | 5      |

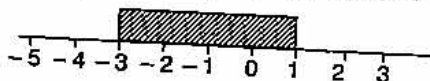
Draw a histogram for the above data using a graph paper and locate the mode. [4]

**Solution :**

- (a) Given :  $2x - 5 \leq 5x + 4$  and  $5x + 4 < 11$   
 $-3x \leq 9$   $5x < 7$   
 $x \geq -3$   $x < 1.4$   
 $-3 \leq x$

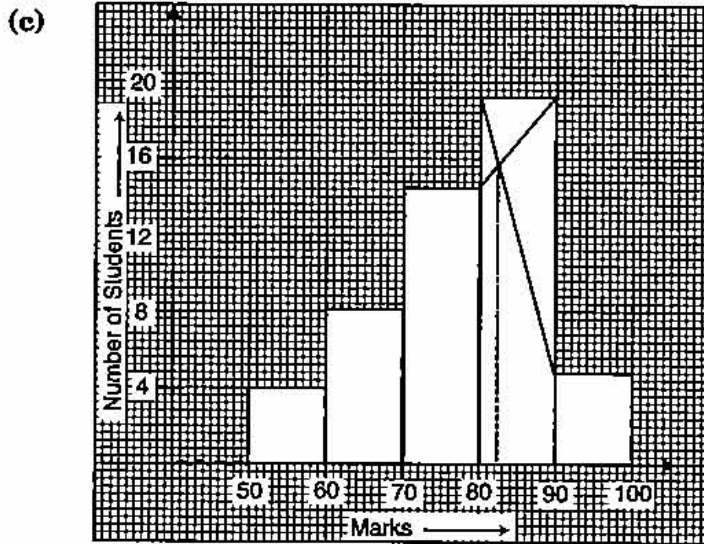
Solution set,

$$x \in [-3, -2, -1, 0, 1]$$



$$\begin{aligned}
 \text{(b) Given : } & 2 \left( \frac{\tan 35^\circ}{\cot 55^\circ} \right)^2 + \left( \frac{\cot 55^\circ}{\tan 35^\circ} \right) - 3 \left( \frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ} \right) \\
 &= 2 \left( \frac{\tan (90^\circ - 55^\circ)}{\cot 55^\circ} \right)^2 + \left( \frac{\cot (90^\circ - 35^\circ)}{\tan 35^\circ} \right) - 3 \left( \frac{\sec (90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ} \right) \\
 &= 2 \left( \frac{\cot 55^\circ}{\cot 55^\circ} \right)^2 + \left( \frac{\tan 35^\circ}{\tan 35^\circ} \right) - 3 \left( \frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ} \right) \\
 &= 2 + 1 - 3 = 0
 \end{aligned}$$

Ans.



Mode from graph = 82.5.

**SECTION B [40 Marks]**

Answer any four Questions in this Section.

**Question 5.**

- (a) A manufacturer sells a washing machine to a wholesaler for ₹ 15,000. The wholesaler sells it to a trader at a profit of ₹ 1,200 and the trader in turn sells it to a consumer at a profit of ₹ 1,800. If the rate of VAT is 8% find :
- The amount of VAT received by the State Government on the sale of this machine from the manufacturer and the wholesaler.
  - The amount that the consumer pays for the machine. [3]
- (b) A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed. [3]
- (c) ABCD is a parallelogram where A(x, y), B (5, 8), C (4, 7) and D (2, -4). Find
- Coordinates of A
  - Equation of diagonal BD. [4]

**Solution :**

(a) (i) VAT received by Govt. from manufacturer =  $15,000 \times \frac{8}{100} = ₹ 1,200$

$$\text{VAT from wholesaler} = 1200 \times \frac{8}{100} = ₹ 96$$

Total VAT from manufacturer and wholesaler

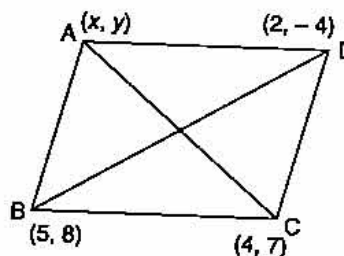
$$= 1200 + 96 = ₹ 1296 \text{ Ans.}$$

(ii) Amount that customer pays =  $(15000 + 1200 + 1800) + \text{VAT}$   
 $= 18,000 + \frac{18,000 \times 8}{100}$   
 $= 18,000 + 1440 = ₹ 19,440$  **Ans.**

(b) Number of spheres =  $\frac{\text{Volume of cone}}{\text{Volume of each sphere}}$   
 $= \frac{\frac{1}{3} \pi (5)^2 (8)}{\frac{4}{3} \pi (0.5)^3} = \frac{50 \times 10^3}{5 \times 5 \times 5}$   
 $= 400$  **Ans.**

(c) In a parallelogram, mid point of diagonal BD co-incides with the mid point of diagonal AC.

Mid point of BD =  $\left( \frac{5+2}{2}, \frac{8-4}{2} \right) = \left( \frac{7}{2}, 2 \right)$   
 Mid point of AC =  $\left( \frac{x+4}{2}, \frac{y+7}{2} \right)$   
 Equating,  $\frac{x+4}{2} = \frac{7}{2} \Rightarrow x = 3$   
 $\frac{y+7}{2} = 2 \Rightarrow y = -3$



(i) Co-ordinates of A (3, -3). **Ans.**

(ii)  $m \text{ of } BD = \frac{8 - (-4)}{5 - 2} = \frac{12}{3} = 4$

Equation of BD,

$$y - y_1 = m(x - x_1)$$

$$y + 4 = 4(x - 2)$$

$\Rightarrow$

$$y = 4x - 4$$
 **Ans.**

**Question 6.**

(a) Use a graph paper to answer the following questions. (Take 1 cm = 1 unit on both axes).

- (i) Plot A(4, 4), B(4, -6) and C(8, 0), the vertices of a triangle ABC.
- (ii) Reflect ABC on the y-axis and name it as A'B'C'.
- (iii) Write the coordinates of the image A', B' and C'.
- (iv) Give a geometrical name for the figure AA'C'B'BC.
- (v) Identify the line of symmetry of AA'C'B'BC.

[5]

(b) Mr. Choudhury opened a Saving's Bank Account at State Bank of India on 1st April 2007. The entries of one year as shown in his pass book are given below :

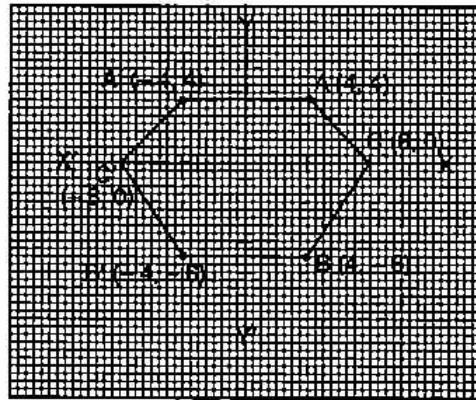
| Date            | Particulars | Withdrawals<br>(in ₹) | Deposits<br>(in ₹) | Balance<br>(in ₹) |
|-----------------|-------------|-----------------------|--------------------|-------------------|
| 1st April 2007  | By Cash     | —                     | 8550.00            | 8550.00           |
| 12th April 2007 | To Self     | 1200.00               | —                  | 7350.00           |
| 24th April 2007 | By Cash     | —                     | 4550.00            | 11900.00          |

|                 |           |         |         |          |
|-----------------|-----------|---------|---------|----------|
| 8th July 2007   | By Cheque | —       | 1500.00 | 13400.00 |
| 10th Sept. 2007 | By Cheque | —       | 3500.00 | 16900.00 |
| 17th Sept. 2007 | To Cheque | 2500.00 | —       | 14400.00 |
| 11th Oct. 2007  | By Cash   | —       | 800.00  | 15200.00 |
| 6th Jan. 2008   | To Self   | 2000.00 | —       | 13200.00 |
| 9th March 2008  | By Cheque | —       | 950.00  | 14150.00 |

If the bank pays interest at the rate of 5% per annum, find the interest paid on 1st April, 2008. Give your answer correct to the nearest rupee. [5]

**Solution :**

- (a) (i) In the given diagram.  
 (ii) In the given diagram.  
 (iii)  $A'(-4, 4)$   
 $B'(-4, -6)$   
 $C'(-8, 0)$   
 (iv) Hexagon.  
 (v) Y-Axis is the line of symmetry.



|           |   |
|-----------|---|
| (b)       | <b>Minimum Balances</b>                                 |
| April     | 7,350.00  |
| May       | 11,900.00   |
| June      | 11,900.00   |
| July      | 13,400.00   |
| August    | 13,400.00   |
| September | 14,400.00   |
| October   | 14,400.00   |
| November  | 15,200.00   |
| December  | 15,200.00   |
| January   | 13,200.00   |
| February  | 13,200.00   |
| March     | 14,150.00   |
|           | 1,57,700.00   |
|           | $\Rightarrow P = ₹ 1,57,700$                            |
|           | $I = \frac{1,57,700 \times 5}{100} \times \frac{1}{12}$ |
|           | $= \frac{7885}{12} = ₹ 657.08$                          |

**Ans.**

**Question 7.**

- (a) Using componendo and dividendo, find the value of  $x$

$$\frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = 9$$

[3]

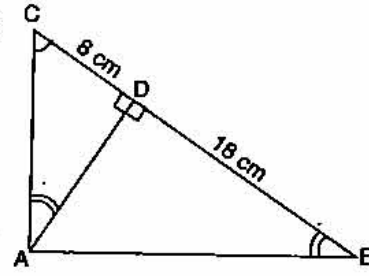
(b) If  $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$  and  $I$  is the identity matrix of the same order and  $A^t$  is the transpose of matrix  $A$ , find  $A^t \cdot B + BI$ . [3]

(c) In the adjoining figure  $ABC$  is a right angled triangle with  $\angle BAC = 90^\circ$ .

(i) Prove  $\triangle ADB \sim \triangle CDA$ .

(ii) If  $BD = 18$  cm,  $CD = 8$  cm, find  $AD$ .

(iii) Find the ratio of the area of  $\triangle ADB$  is to area of  $\triangle CDA$ . [4]



**Solution :**

(a) Given : 
$$\frac{\sqrt{3x+4} + \sqrt{3x-5}}{\sqrt{3x+4} - \sqrt{3x-5}} = \frac{9}{1}$$

Applying componendo and Dividendo,

$$\frac{\sqrt{3x+4} + \sqrt{3x-5} + \sqrt{3x+4} - \sqrt{3x-5}}{\sqrt{3x+4} + \sqrt{3x-5} - \sqrt{3x+4} + \sqrt{3x-5}} = \frac{9+1}{9-1}$$

$$\frac{2\sqrt{3x+4}}{2\sqrt{3x-5}} = \frac{10}{8}$$

$$\frac{\sqrt{3x+4}}{\sqrt{3x-5}} = \frac{5}{4}$$

Squaring both sides, 
$$\frac{3x+4}{3x-5} = \frac{25}{16}$$

Applying Componendo and Dividendo,

$$\frac{3x+4 + 3x-5}{3x+4 - 3x+5} = \frac{25+16}{25-16}$$

$$\frac{6x-1}{9} = \frac{41}{9}$$

$$6x = 42$$

$$x = 7$$

**Ans.**

(b) Transpose of matrix  $A$ , 
$$A^t = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$A^t \cdot B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8-1 & -4+3 \\ 20-3 & -10+9 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix}$$

$$B \cdot I = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$



$$A^t \cdot B + B \cdot I = \begin{bmatrix} 7 & -1 \\ 17 & -1 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -3 \\ 6 & 2 \end{bmatrix}$$

Ans.

(c) Let

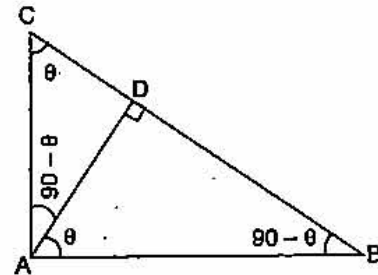
$$\angle DAB = \theta$$

$\therefore$

$$\angle DAC = 90 - \theta$$

$$\angle DBA = 90 - \theta$$

$$\angle DCA = \theta$$



$\therefore$  All three angles of  $\Delta ADB$  are equal to all angles of  $\Delta CDA$ .

(i)  $\therefore \Delta ADB \sim \Delta CDA$

Proved

(ii)  $\therefore \frac{CD}{AD} = \frac{AD}{BD}$

$\Rightarrow AD^2 = CD \times BD$

$= 8 \times 18 \Rightarrow AD = 12$

Ans.

(iii)  $\frac{\Delta ADB}{\Delta CDA} = \frac{\frac{1}{2} AD \times BD}{\frac{1}{2} AD \times CD}$

$= \frac{BD}{CD} = \frac{18}{8}$

$= \frac{9}{4}$

Ans.

**Question 8.**

(a) (i) Using step-deviation method, calculate the mean marks of the following distribution.

(ii) State the modal class :

[5]

| Class Interval | Frequency |
|----------------|-----------|
| 50-55          | 5         |
| 55-60          | 20        |
| 60-65          | 10        |
| 65-70          | 10        |
| 70-75          | 9         |
| 75-80          | 6         |
| 80-85          | 12        |
| 85-90          | 8         |

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(b) Marks obtained by 200 students in an examination are given below :

| Marks  | No. of Students |
|--------|-----------------|
| 0-10   | 5               |
| 10-20  | 11              |
| 20-30  | 10              |
| 30-40  | 20              |
| 40-50  | 28              |
| 50-60  | 37              |
| 60-70  | 40              |
| 70-80  | 29              |
| 80-90  | 14              |
| 90-100 | 6               |

Draw an ogive for the given distribution taking 2 cm = 10 marks on one axis and 2 cm = 20 students on the other axis. Using the graph, determine

- (i) The median marks.
- (ii) The number of students who failed if minimum marks required to pass is 40.
- (iii) If scoring 85 and more marks is considered as grade one, find the number of students who secured grade one in the examination. [5]

Solution :

| (a) (i) | C.I.  | $f$             | $x$  | $d = x - 67.5$ | $u$ | $f.u$             |
|---------|-------|-----------------|------|----------------|-----|-------------------|
|         | 50-55 | 5               | 52.5 | -15            | -3  | -15               |
|         | 55-60 | 20              | 57.5 | -10            | -2  | -40               |
|         | 60-65 | 10              | 62.5 | -5             | -1  | -10               |
|         | 65-70 | 10              | 67.5 | 0              | 0   | 0                 |
|         | 70-75 | 9               | 72.5 | 5              | 1   | 9                 |
|         | 75-80 | 6               | 77.5 | 10             | 2   | 12                |
|         | 80-85 | 12              | 82.5 | 15             | 3   | 36                |
|         | 85-90 | 8               | 87.5 | 20             | 4   | 32                |
|         |       | $\Sigma f = 80$ |      |                |     | $\Sigma f.u = 24$ |

A.M. = 67.5

$$\bar{x} = \text{A.M.} + \frac{\Sigma fu}{\Sigma f} \times i$$

$$= 67.5 + \frac{24}{80} \times 5$$

$$= 67.5 + 1.5 = 69$$

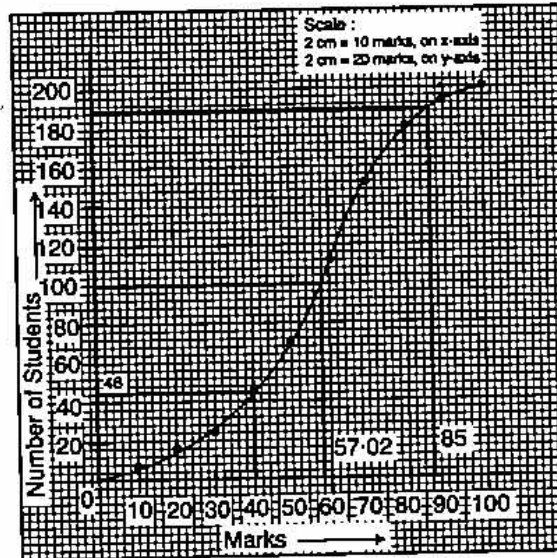
Ans.

(ii) Modal class is 55-60 (class with highest freq.)

Ans.

(b)

| C.I.   | <i>f</i> | C.F. |
|--------|----------|------|
| 0-10   | 5        | 5    |
| 10-20  | 11       | 16   |
| 20-30  | 10       | 26   |
| 30-40  | 20       | 46   |
| 40-50  | 28       | 74   |
| 50-60  | 37       | 111  |
| 60-70  | 40       | 151  |
| 70-80  | 29       | 180  |
| 80-90  | 14       | 194  |
| 90-100 | 6        | 200  |



(i)  $n = 200$

$$\begin{aligned} \text{Median} &= \left(\frac{n}{2}\right)^{\text{th}} \text{ observation} \\ &= \left(\frac{200}{2}\right)^{\text{th}} \text{ observation} \\ &= 100^{\text{th}} \text{ observation} = 57.02 \end{aligned}$$

Ans.

(ii) Number of students who failed = 46

Ans.

(iii) Number of students who secured grade one =  $200 - 188 = 12$ .

Ans.

**Question 9.**

(a) Mr. Parekh invested ₹ 52,000 on 100 shares at a discount of ₹ 20 paying 8% dividend. At the end of one year he sells the shares at a premium of ₹ 20. Find

(i) The annual dividend.

(ii) The profit earned including his dividend.

[3]

(b) Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.

[3]

(c) Prove that  $(\text{cosec } A - \sin A)(\sec A - \cos A) \sec^2 A = \tan A$ .

[4]

**Solution :**

(a) Investment = ₹ 52,000, N.V = ₹ 100, M.V. of one share = ₹  $(100 - 20) = ₹ 80$ ,  
Dividend = 8%

$$\begin{aligned} \text{No. of shares} &= \frac{\text{Investment}}{\text{MV}} \\ &= \frac{52,000}{80} \\ &= ₹ 650 \end{aligned}$$

(i) Annual Dividend =  $\frac{8}{100} \times 650 \times 100$   
= ₹ 5,200

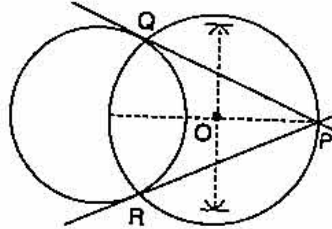
Ans.

(ii)

$$\begin{aligned} \text{Profit} &= \text{Total S.P.} + \text{Dividend} - \text{Investment} \\ &= 650 \times 120 + 5,200 - 52,000 \\ &= 78,000 + 5,200 - 52,000 \\ &= ₹ 31,200 \end{aligned}$$

Ans.

(b) Length of the tangent = 4.8 cm.



(c)

$$\begin{aligned} \text{L.H.S.} &= (\operatorname{cosec} A - \sin A)(\sec A - \cos A) \cdot \sec^2 A \\ &= \left( \frac{1}{\sin A} - \sin A \right) \left( \frac{1}{\cos A} - \cos A \right) \cdot \sec^2 A \\ &= \left( \frac{1 - \sin^2 A}{\sin A} \right) \left( \frac{1 - \cos^2 A}{\cos A} \right) \cdot \sec^2 A \\ &= \frac{\cos^2 A}{\sin A} \cdot \frac{\sin^2 A}{\cos A} \times \frac{1}{\cos^2 A} \\ &= \frac{\sin A}{\cos A} \\ &= \tan A = \text{R.H.S.} \end{aligned}$$

Hence Proved

**Question 10.**

- (a) 6 is the mean proportion between two numbers  $x$  and  $y$  and 48 is the third proportional of  $x$  and  $y$ . Find the numbers. [3]
- (b) In what period of time will ₹ 12,000 yield ₹ 3,972 as compound interest at 10% per annum, if compounded on an yearly basis? [3]
- (c) A man observes the angle of elevation of the top of a building to be  $30^\circ$ . He walks towards it in a horizontal line through its base. On covering 60 m the angle of elevation changes to  $60^\circ$ . Find the height of the building correct to the nearest metre. [4]

**Solution :**

(a)

$$\begin{aligned} xy &= 6^2 \\ \Rightarrow xy &= 36 \end{aligned}$$

$$x : y :: y : 48$$

$$\frac{x}{y} = \frac{y}{48}$$

$$\Rightarrow y^2 = 48x \quad \dots(2)$$

Substituting the value of  $x$  from (1),

$$y^2 = 48 \times \frac{36}{y}$$

$$\Rightarrow y^3 = 48 \times 36$$

$$y^3 = 6 \times 8 \times 6 \times 6$$

$$\therefore y = 12$$

$$\therefore x = 3$$

Ans.

(b) Given : P = ₹ 12,000, C.I. = ₹ 3,972, R% = 10% p.a.

Let  $A = P \left( 1 + \frac{R}{100} \right)^n$

$$\Rightarrow 15,972 = 12,000 \left( 1 + \frac{10}{100} \right)^n$$

$$\frac{1331}{1000} = \left( \frac{11}{10} \right)^n$$

$$\therefore \left( \frac{11}{10} \right)^3 = \left( \frac{11}{10} \right)^n$$

$$\Rightarrow n = 3 \text{ years.}$$

Ans.

(c) Let BC = x and AB = h  
In right angled  $\Delta ADB$

$$\tan 30^\circ = \frac{h}{60 + x}$$

$$\Rightarrow 60 + x = h\sqrt{3}$$

Now right angled  $\Delta ACB$

$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

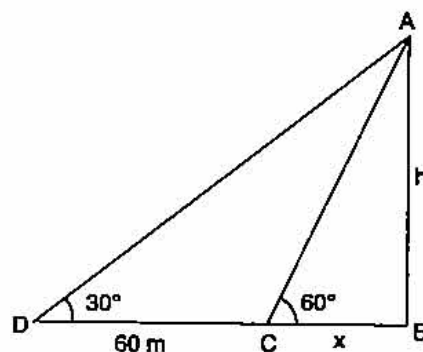
Equating 'x',  $h\sqrt{3} - 60 = \frac{h}{\sqrt{3}}$

$$\Rightarrow 3h - 60\sqrt{3} = h$$

$$2h = 60\sqrt{3}$$

$$\therefore h = 30\sqrt{3}$$

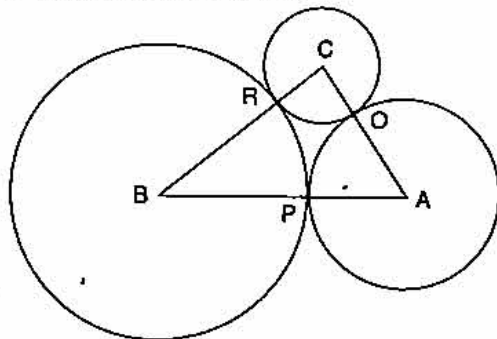
$$= 51.96 \text{ m.}$$



Ans.

**Question 11.**

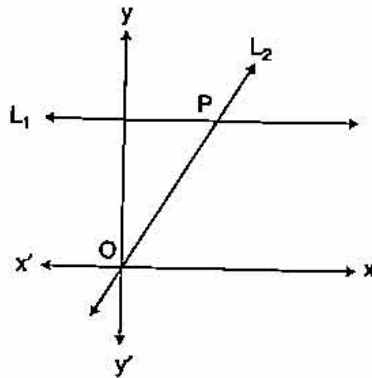
(a) ABC is a triangle with AB = 10 cm, BC = 8 cm and AC = 6 cm (not drawn to scale). Three circles are drawn touching each other with the vertices as their centres. Find the radii of the three circles. [3]



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(b) ₹ 480 is divided equally among 'x' children. If the number of children were 20 more then each would have got ₹ 12 less. Find 'x'. [3]

(c) Given equation of line  $L_1$  is  $y = 4$ .



(i) Write the slope of line  $L_2$  if  $L_2$  is the bisector of angle O.

(ii) Write the co-ordinates of point P.

(iii) Find the equation of  $L_2$ .

[4]

**Solution :**

(a) Let the three radii be  $x, y, z$  respectively.

$$x + y = 10 \quad \dots(1)$$

$$y + z = 8 \quad \dots(2)$$

$$x + z = 6 \quad \dots(3)$$

Adding equation's (1), (2) and (3),  $2x + 2y + 2z = 24$

$$\therefore x + y + z = 12 \quad \dots(4)$$

Subtracting each equation (1), (2) and (3) from equation (4), we get

$$z = 2 \text{ cm, } x = 4 \text{ cm, } y = 6 \text{ cm.}$$

**Ans.**

(b) Initial share of each child =  $\frac{480}{x}$

New share of each child =  $\frac{480}{x + 20}$

Difference in share is ₹ 12

$$\therefore \frac{480}{x} - \frac{480}{x + 20} = 12$$

$$\frac{1}{x} - \frac{1}{x + 20} = \frac{12}{480} = \frac{1}{40}$$

$$\frac{x + 20 - x}{x(x + 20)} = \frac{1}{40}$$

$$x^2 + 20x = 800$$

$$x^2 + 20x - 800 = 0$$

$$x^2 + 40x - 20x - 800 = 0$$

$$x(x + 40) - 20(x + 40) = 0 \text{ (not possible)}$$

$$x = 20 \text{ or } x = -40$$

$$\therefore x = 20$$

**Ans.**

- (c) (i) Slope of  $L_2$  is  $m = \tan 45^\circ$   
 $\therefore m = 1$  ( $L_2$  makes an angle of  $45^\circ$  with X axis)
- (ii) Equation of line  $L_2$   
 $y - 0 = 1(x - 0)$  It passes through  $(0, 0)$   
 $\therefore$  Equation of  $L_2$  is  $y = x$   
 P can be obtained by solving  $L_1$  and  $L_2$  simultaneously,  
 $L_1$   $y = 3$   
 $L_2$   $y = x$   
 On solving, we get  $x = 3, y = 3$   
 Co-ordinate of P  $(3, 3)$
- (iii) Equation of  $L_2$  is  $y = x$  [as solved above part (ii)].

**Ans.**

**Ans.**

