MATHS PAPER-4 ASSIGNMENT-1

FDN/4/BMS/2008/T-1

Paper-4: Maths & Stats.
FDN/4/BMS/2008/T-1

1(a)

$$\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y} = K$$

on comparing we have

$x = K(y+z)$, $y = K(z+x)$, $z = K(x+y)$

on adding each eqn given above

$$x+y+z = K(y+z) + K(z+x) + K(x+y)$$

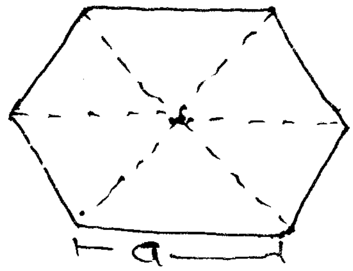
$$x+y+z = K[y+z+z+x+x+y]$$

$$\cancel{(x+y+z)} = 2K\,\cancel{(x+y+z)}$$

$K = \frac{1}{2}$ hence prove

1(b)

2(a) Since, hexagone can be divided into 6 equilateral triangles by joining each point to the center.



Let side of hexagone = a

∴ it also equal to side of each Δ.

Area of 6 equil Δs = area of hexagone

$$\frac{\cancel{6}\sqrt{3}}{4}a^2 = \cancel{54}\,\cancel{\sqrt{3}}$$

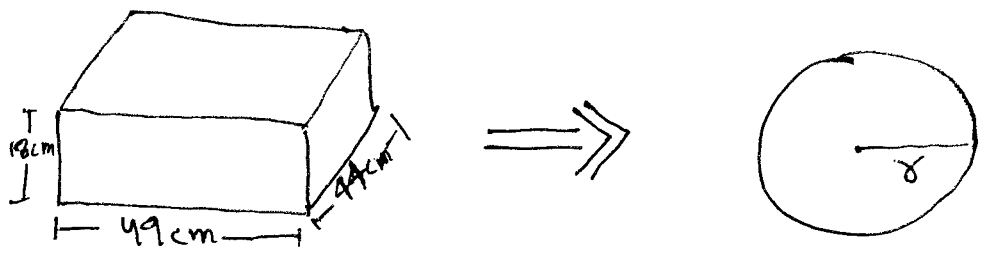
$$a^2 = 36$$

$$a = 6\,cm$$

Length of each side = 6 cm

∴ perimeter = 6a = 6×6 = 36 cm

2(b)



∵, when block is melt to form sphere then volume of sphere & block are equal.

Let r be the radius of sphere

∴ Vol. of block = Vol. of sphere

$$18 \times 49 \times 44 = \frac{4}{3} \times \pi r^3$$

$$\cancel{18} \times 49 \times \cancel{44} \times 3 = \cancel{4} \times \frac{\cancel{22}}{7} r^3$$

$$r^3 = 9 \times 3 \times 49 \times 7$$

$$r = 7 \times 3 = 21\,cm$$

radius of sphere = 21 cm

3 (a) Let the co-ordinate of point is (x, y)

distance of point to $(3,0)$ $= \sqrt{(x-3)^2+(y-0)^2}$

" " " to $(-3,0)$ $= \sqrt{(x+3)^2+(y-0)^2}$

according to quest.

$$(x-3)^2 + (y-0)^2 + (x+3)^2 + (y-0)^2 = 36$$

$$\Rightarrow x^2+9-\cancel{6x}+y^2+x^2+9+\cancel{6x}+y^2 = 36$$

$$\Rightarrow 2x^2+2y^2+18-36 = 0$$

$$\Rightarrow x^2+y^2-9=0$$

$$\Rightarrow x^2+y^2=9$$

so, the locus will be a circle of radius 3 & center $(0,0)$

3 (b) equation of circle given $x^2+y^2-2x-2y-38=0$

compare it to genral form of circle

$$x^2+y^2+2gx+2fy+c=0$$

$$g=-1, \quad f=-1$$

$\therefore$ center $(-g,-f) = (1,1)$

Now radius of required circle $= \sqrt{(2-1)^2+(-4-1)^2} = \sqrt{26}$

center of circle $(2,-4)$

$\therefore$ eqn of circle $(x-h)^2+(y-k)^2 = r^2$ [(h,k) center]

$$(x-2)^2+(y+4)^2 = 26$$

$$x^2+4-4x+y^2+16+8y-26=0$$

$$\rightarrow x^2+y^2-4x+8y-6=0$$

req. eqn of of circle

4(a) (i) $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$= \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \times \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$= \lim_{x\to 0} \frac{(1+x)-(1-x)}{(\sqrt{1+x}+\sqrt{1-x})\cdot x}$

$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{1+x}+\sqrt{1-x})}$

$= \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{1+1} = 1$ Ans

(ii) $\lim_{x\to -2} \frac{x^2-4}{x+2}$

$= \lim_{x\to -2} \frac{x^2-2^2}{x+2}$

$= \lim_{x\to -2} \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}}$

$= \lim_{x\to -2} x-2$

$= -2-2 = -4$ Ans

4(b) (i) $I = \int x(3x^2+7)^7\,dx$

Let $3x^2+7 = t$

diff. w.r.t x

$6x = \frac{dt}{dx}$

$x\cdot dx = \frac{dt}{6}$

$I = \int t^7 \cdot \frac{dt}{6}$

$= \frac{t^8}{8\times 6} + C$

$= \frac{t^8}{48} + C$

$= \frac{(3x^2+7)^8}{48} + C$

soln

(ii) $I = \int_1^e x\log x\,dx$

Let $I_1 = \int x\log x\,dx$

on applying ILATE method

$I\int II\,dx - \int\left(\frac{d}{dx} I \int (II\,dx)\right)dx$

here $I = \log x$, $II = x$

$I_1 = \log x\int x\,dx - \int\left(\frac{d}{dx}\log x\int x\cdot dx\right)\cdot dx$

$= \log x\cdot\frac{x^2}{2} - \int \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\,dx$

$= \log x\cdot\frac{x^2}{2} - \frac{x^2}{4}$

now taking limit

$I = \left[\log x\cdot\frac{x^2}{2} - \frac{x^2}{4}\right]_1^e$

$= \left(\frac{e^2}{2}\cdot\log e - \frac{e^2}{4}\right) - \left(\frac{1}{2}\log 1 - \frac{1}{4}\right)$

$= \frac{e^2}{4} + \frac{1}{4} = \frac{e^2+1}{4}$ soln

5(a)

5(a) expenses on wages = Rs 125
" " materials = Rs 110
taxes = Rs 180
distributed points = Rs 65
administration = Rs 20

Total = Rs 500

Percentage of each

$$\text{wages} = \frac{125}{500} \times 100 = 25\%$$

$$\text{materials} = \frac{110}{500} \times 100 = 22\%$$

$$\text{taxes} = \frac{180}{500} \times 100 = 36\%$$

$$\text{distributed points} = \frac{65}{500} \times 100 = 13\%$$

$$\text{adminsitration} = \frac{20}{500} \times 100 = 4\%$$

now 1% corresponds to 3.6°, so corresponding sector of each (in degree)

wages = $25 \times 3.6° = 90°$
materials = $22 \times 3.6° = 79.2°$
taxes = $36 \times 3.6 = 129.6°$
distributed = $13 \times 3.6° = 46.8°$
administration = $4 \times 3.6 = 14.4°$

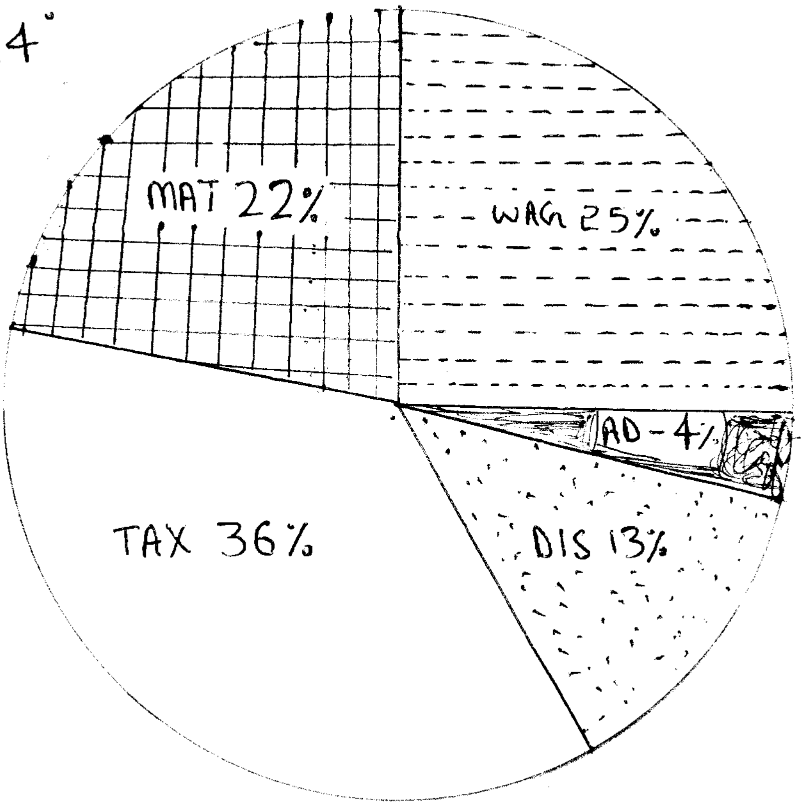


FIG:- PIE CHART OF GIVEN DATA

5(b)

Marks	No. of students
Less than 10	5
" " 20	9
" " 30	15
" " 40	18
" " 50	20

frequency distribution of data

Marks	students (f)	c.f
0 − 10	5	5
10 − 20	4 (=9-5)	9
20 − 30	6 (=15-9)	15
30 − 40	3 (=18-15)	18
40 − 50	2 (= 20-18)	20

$N = 20$

median = value of $\frac{N}{2}$ th term = value of $\frac{20}{2}$ th = 10 th term

$\therefore$ median class = (20-30)

$l_1 = 20$, $l_2 = 30$, $f_m = 6$, $\frac{N}{2} = 10$, $c = 9$

$$\text{median} = l_1 + \frac{l_2 - l_1}{f_m}\left[\frac{N}{2} - c\right)$$

$$= 20 + \frac{30-20}{6}\left[10-9\right) = 20 + \frac{10}{6} = \frac{130}{6}$$

median = 21.67 marks