*This Question Paper contains 8 Printed Pages.*

# 050(E)

**(MARCH, 2008)**

***Time : 3.00 Hours]*** ***[Maximum Marks : 100***

***Instructions :***

1. **All** the questions are **compulsory.**
2. Write your answers according to the instructions given below with the questions.
3. Begin each section from a new page.

## SECTION - A

*Given below are **1** to **15** multiple choice questions, each carrying **ONE** mark. Write the letter of the correct option (A) or (B) or (C) or (D).* **15**

**1.** Find the value of $a$, if P(2, 3) is circumcentre of the triangle with vertices A($a$, 6), B(5, 1) and C(4, 6).

(A) $-4$ (B) 1

(C) 4 (D) 0

**2.** Find $\alpha$ if a line $x + y + 1 = 0$ is converted in the form of a line $x \cos\alpha + y \sin\alpha = p$.

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$ (D) $\frac{7\pi}{4}$

**3.** If the circle $x^2 + y^2 + 4x + Ky - 4 = 0$ touches both the axes, then find out K.

(A) $\pm 8$ (B) $\pm 4$

(C) $\pm 2$ (D) $\pm 1$

**4.** Obtain the equation of a Parabola having focus (0, –2) and the equation of directrix is $y = 2$ and (0, 0) is the vertex of Parabola.

(A) $x^2 = -8y$ (B) $y^2 = 8x$

(C) $x^2 = 8y$ (D) $y^2 = -8x$

5. Find the radius of a director-circle of an ellipse $4x^2 + 9y^2 = 36$.

(A) $\sqrt{5}$ (B) $\sqrt{13}$

(C) $\sqrt{10}$ (D) 5

6. If $|\bar{a}| = 10$, $|\bar{b}| = 2$ and $\bar{a} \cdot \bar{b} = 12$, then find $|\bar{a} \times \bar{b}|$.

(A) 12 (B) 14

(C) 16 (D) 18

7. Find magnitude of projection of vector $\bar{i} + \bar{j} + \bar{k}$ on $\bar{j}$.

(A) –1 (B) 0

(C) 1 (D) 2

8. Find the measure of the angle between plane $\bar{r} \cdot (1, 2, 1) = 1$ and $\frac{x}{2} = \frac{y}{1} = \frac{z}{-1}$.

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) None of them

9. Find $\lim_{x \to 0} \frac{(1+x)^{1/3} - 1}{x}$.

(A) 0 (B) 1

(C) $1/3$ (D) None of them

10. Find $\frac{d}{dx}\left[\tan^{-1}\left(\frac{1-\cos x}{1+\cos x}\right)^{\frac{1}{2}}\right]$; $\pi < x < 2\pi$.

(A) 0 (B) $1/2$

(C) $-1/2$ (D) 1

11. Find $c$ applying Rolle's theorem to $f(x) = 1 + \sin x$, $x \in [0, \pi]$

(A) 0 (B) $\pi/4$

(C) $\pi$ (D) $\pi/2$

12. Evaluate : $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx$

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

13. Find the area of the region bounded by the curve $y = \tan x$, X-axis and the lines $x = 0$ and $x = \pi/4$.

(A) $\log 2$ (B) $\frac{3}{2}\log 2$

(C) $\frac{1}{2}\log 2$ (D) $2 \log 2$

14. Determine the degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

(A) 1 (B) 2

(C) 0 (D) not defined

15. A stone falls from a tower of height 40 m. What will be its velocity, when it reaches on the land ?

(A) 14 m/s (B) 28 m/s

(C) 21 m/s (D) 7 m/s

## SECTION - B

*Answer the following **15** questions. (No. **16** to **30**)*
*Each question carries **ONE** mark.* **15**

16. Find the point A on the X-axis which is at the distance of 5 units from point B(2, –3).

17. Obtain the equation of a circle which touches the X-axis, given that the equations of lines containing two of the diameters of the circle are $3x - 2y - 5 = 0$ and $x + y - 5 = 0$.

18. Find the focus of a Parabola $y^2 + 6y - 2x + 5 = 0$.

**19.** The equations of the asymptotes of Hyperbola are $3x + 4y = 2$ and $4x - 3y = 2$. Find the eccentricity.

**20.** Find the unit vector in the direction of vector (1, 2, 3).

**21.** Find the area of a Parallelogram, if its diagonals are $2\bar{i} + \bar{k}$ and $\bar{i} + \bar{j} + \bar{k}$.

**22.** Represent the equation of line $\frac{3-x}{1} = \frac{2-y}{3} = \frac{1-z}{4}$ in the vector-form.

**23.** Find the length of a chord, cut by sphere $x^2 + y^2 + z^2 - x - y - z = 0$ on any axis.

**24.** If $f'(x) = f(x)$ and $f(0) = 1$, then find out the value of $\lim_{x \to 0} \frac{f(x)-1}{x}$.

**25.** Evaluate :

$\int x^{4x}(1 + \log x)\,dx, \quad x > 0$.

**26.** Evaluate :

$\int \left(\frac{1+x}{x^2}\right) e^{-x}\,dx$.

**27.** If $\int_1^k f(x)\,dx = 47$ ; $\quad f(x) = \begin{cases} 2x + 8, & \text{if } 1 \le x \le 2 \\ 6x, & \text{if } 2 \le x \le k \end{cases}$

then find $k$ .

**28.** Find the length of sub tangent of $y = e^{x/c}$.

**29.** If a distance of 150 cm. is travelled in 30 seconds with an initial velocity of 10 cm/s, find the constant accleration (retardation).

**30.** If the maximum horizontal range is 200 m, find the minimum velocity for that.

## SECTION - C

*Answer the following 10 questions* ***(31 to 40).***
*Each question carries* ***TWO*** *marks. Do as directed :* **20**

**31.** A line passing through (2, 4) intersects the X-axis and Y-axis at A and B respectively. Find the equation of the locus of the mid-point of $\overline{AB}$.

**32.** For the Parabola $x^2 = 12y$, find the area of the triangle, whose vertices are the vertex of the parabola and the two end-points of its latus rectum.

**33.** Find the equation of Ellipse, which is passing through the points (1, 4) and (– 6, 1).

**34.** Find the equation of Hyperbola for which the distance from one vertex to two foci are 9 and 1.

**OR**

Find the measure of angle between the asymptotes of hyperbola $3x^2 - 2y^2 = 1$.

**35.** If $\bar{x}.\bar{y} = \bar{x}.\bar{z}$, $\bar{x} \times \bar{y} = \bar{x} \times \bar{z}$ and $\bar{x} \neq \bar{0}$, then prove that $\bar{y} = \bar{z}$.

**36.** If $\bar{a}.\bar{b} = \bar{a}.\bar{c} = 0$, $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$, then prove that

$\bar{a} = \pm 2\left(\bar{b} \times \bar{c}\right)$, where $\left(\bar{b} \wedge \bar{c}\right) = \pi/6$.

**37.** Find the equation of a sphere given that its centre is (1, 1, 0) and that it touches the plane $2x + 2y + z + 5 = 0$.

**38.** If $y = \tan^{-1}\left(\dfrac{5x}{1-6x^2}\right)$, then find $\dfrac{dy}{dx}$.

**OR**

$f(x) = [x]$. Is $f$ continuous and differentiable at $x = 1$ ?

**39.** Find the measure of the angle between the curves
$y = \sin x$ and $y = \cos x$, $0 < x < \pi$.

40. Obtain $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx$ ; $x \neq \frac{k\pi}{2}$, $\tan x > 0$.

**OR**

Obtain $\int \frac{1}{\sin^4 x + \cos^4 x}\, dx$.

## SECTION - D

*Answer the following 10 questions (**41** to **50**).*

*Each question carries **3 (THREE)** marks. Do as directed.* **30**

41. A is $(2\sqrt{2},\ 0)$ and B is $(-2\sqrt{2},\ 0)$. If $|AP - PB| = 4$, find the equation of locus of P.

42. Find the equation of the incircle of the triangle formed by the following lines – $x = 2$, $4x + 3y = 5$ and $4x - 3y + 13 = 0$.

**OR**

Get the equation of the circle that passes through the origin and that cuts chords of length 5 on the lines $y = \pm x$.

43. Prove by vectors, that if the median on the base of a triangle is also altitude on the base, the triangle is isosceles.

**OR**

There are two forces (2, 5, 6) and (–1, 2, 1) that act on a particle and as a result of which the particle moves from A(4, –3, –2) to B(6, 1, –3). Find the work done.

44. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other and also find the point of intersection.

45. Obtain the equation of a plane that passes through the points (2, 3, – 4) and (1, –1, 3), and that is parallel to X-axis.

**46.** Find $\lim_{x \to e^3} \frac{\log x - 3}{x - e^3}$.

**47.** Prove that of all the rectangles having the same area, the square has minimum perimeter.

**OR**

$y = ax^3 + bx^2 + cx + 5$ touches X-axis at (–2, 0) and the slope of the tangent where it meets Y-axis is 3, then find $a, b, c$.

**48.** Evaluate :

$$\int_0^1 \frac{\log(1+x)}{(1+x)^2}\, dx .$$

**49.** Find the area of the region bounded by the curve $y = 2\sqrt{1-x^2}$ and X-axis.

**OR**

Evaluate :

$\int_2^3 e^{-x}\, dx$ as a limit of the sum.

**50.** Solve the differential equation.

$x\, dy + y\, dx = xy\, dx,\ y(1) = 1.$

## SECTION - E

*Answer the following 4 questions (**51** to **54** ).*

*Each question carries **FIVE** marks. Do as directed* **20**

**51.** The equation of the line containing one of the sides of an equilateral triangle is $x + y = 2$ and one of the vertices of the triangle is (2, 3). Find the equations of lines containing the remaining sides of the triangle.

**OR**

A is (1, 3) in $\Delta ABC$ and the lines $x - 2y + 1 = 0$ and $y - 1 = 0$ contain two medians of the triangle. Find the co-ordinates of B and C.

**52.** Find $\lim_{x \to 1} \frac{x^n - 1 - n(x-1)}{(x-1)^2}$ ; $x \neq 1$.

**53.** If $y = \log(1 + \sin x)$, then prove that $e^y \cdot \frac{d^2y}{dx^2} + 1 = 0$.

**54.** Evaluate :

$$\int \left( \frac{2007x + 2008}{2008x + 2007} \right) dx .$$

**OR**

Evaluate :

$$\int \frac{1}{\sin x + \sec x} \, dx .$$