Roll No. ....

Total No. of Questions : 09]

May-08

[Total No. of Pages : 02

Paper ID [AM201]

(Please fill this Paper ID in OMR Sheet)

B.Tech. (Sem. - 3rd/4th)

MATHEMATICS - III (AM - 201)

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Four** questions from Section - B.
3) Attempt any **Two** questions from Section - C.

MAY 2008

Section - A

Q1) *(10 x 2 = 20)*

a) Can $f(x) = \tan x$ be expanded as a Fourier series in the interval $(-\pi, \pi)$.

b) Give the sufficient condition for the existence of Laplace transform of $f(t)$.

c) If $L\{f(t)\} = \frac{1}{s} e^{-\frac{1}{s}}$, find the Laplace transform of $e^{-t} f(3t)$.

d) Find the $L^{-1}\left\{\frac{e^{-3s}}{s^2}\right\}$.

e) Write algorithm of power series method for solution of differential equations.

f) For the Bessel's function prove the recurrence relation

$$\frac{d}{dx}\left\{x^p J_p(x)\right\} = x^p J_{p-1}(x).$$

g) Form the partial differential equation from $F(xy + z^2, x + y + z) = 0$.

h) Is the function $u(x, y) = 2xy + 3xy^2 - 2y^3$, a harmonic function?

i) Give the definition of Conformal transformation.

j) Find the poles and singularity from $\frac{e^z}{1 + z^2}$.

Section - B

(4 x 5 = 20)

Q2) Solve the system of differential equations using Laplace transform

$$2\frac{dx}{dt}+\frac{dy}{dt}-x-y=e^{-t} \text{ and } \frac{dx}{dt}+\frac{dy}{dt}+2x+y=e^{t}.$$

Given that x(0) = 2, y(0) = 1.

Q3) Using convolution theorem find the inverse of $\frac{1}{s^2\left(s^2+1\right)}$.

Q4) Find the series solution of differential equation $(1-x^2)\,y''-2x\,y'+6y=0$.

Q5) Solve $\left(D_x^4-D_x^3D_y+2D_x^2D_y^2-5D_xD_y^3+3D_y^4\right)z=0$.

Q6) Evaluate $\oint_C |z|^2\,dz$, around the square with vertices at (0,0), (1,0), (1,1) and (0,1).

Section - C

(2 x 10 = 20)

Q7) Find the Fourier series expansion of $f(x)=\begin{cases}1+\dfrac{2x}{\pi}, -\pi\le x\le \pi \\ 1-\dfrac{2x}{\pi}, 0\le x\le \pi\end{cases}$.

Also deduce that $\frac{\pi^2}{8}=\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$

Q8) Solve Laplace's equation in rectangle with $u(0, y) = 0$, $u(a, y) = 0$, $u(x, b) = 0$ and $u(x, 0) = f(x)$.

Q9) Find the Laurent series of $f(z)=\frac{1}{(1-z)(z-2)}$ for the following intervals

(a) $1<|z|<2$ (b) $|z|>2$