A

1023

Register Number

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION - A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. If $\left[\vec{a} \times \vec{b}\,,\ \vec{b} \times \vec{c}\,,\ \vec{c} \times \vec{a} \right] = 64$ then $\left[\vec{a}\,,\ \vec{b}\,,\ \vec{c} \right]$ is

a) 32 b) 8

c) 128 d) 0.

2. The shortest distance of the point (2, 10, 1) from the plane $\vec{r} \cdot \left(3\vec{i} - \vec{j} + 4\vec{k} \right) = 2\sqrt{26}$ is

a) $2\sqrt{26}$ b) $\sqrt{26}$

c) 2 d) $\frac{1}{\sqrt{26}}$.

[Turn over

3. The point of intersection of the line $\vec{r} = \left(\vec{i} - \vec{k}\right) + t\left(3\vec{i} + 2\vec{j} + 7\vec{k}\right)$ and the plane $\vec{r} \cdot \left(\vec{i} + \vec{j} - \vec{k}\right) = 8$ is

a) (8, 6, 22)

b) (− 8, − 6, − 22)

c) (4, 3, 11)

d) (− 4, − 3, − 11).

4. The centre and radius of the sphere $\left| \vec{r} - \left(2\vec{i} - \vec{j} + 4\vec{k}\right) \right| = 5$ are

a) (2, − 1, 4) and 5

b) (2, 1, 4) and 5

c) (− 2, 1, 4) and 6

d) (2, 1, − 4) and 5.

5. The non-parametric vector equation of a plane passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$ is

a) $\left[\vec{r} - \vec{a},\ \vec{u},\ \vec{v} \right] = 0$

b) $\left[\vec{r},\ \vec{u},\ \vec{v} \right] = 0$

c) $\left[\vec{r},\ \vec{a},\ \vec{u} \times \vec{v} \right] = 0$

d) $\left[\vec{a},\ \vec{u},\ \vec{v} \right] = 0$.

6. The angle between the asymptotes to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is

a) $\pi - 2\tan^{-1}\left(\frac{3}{4}\right)$

b) $\pi - 2\tan^{-1}\left(\frac{4}{3}\right)$

c) $2\tan^{-1}\left(\frac{3}{4}\right)$

d) $2\tan^{-1}\left(\frac{4}{3}\right)$.

7. The length of the latus rectum of the rectangular hyperbola $xy = 32$ is

a) $8\sqrt{2}$ b) 32

c) 8 d) 16.

8. The foci of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ are

a) $(\pm 5, 0)$

b) $(0, \pm\sqrt{5})$

c) $(0, \pm 5)$

d) $(\pm\sqrt{5}, 0)$.

9. The equation of the tangent to the curve $y = \frac{x^3}{5}$ at the point $\left(-1, -\frac{1}{5}\right)$ is

a) $5y + 3x = 2$ b) $5y - 3x = 2$

c) $3x - 5y = 2$ d) $3x + 3y = 2$.

10. The value of c in Rolle's theorem for the function $f(x) = \cos\frac{x}{2}$ on $[\pi, 3\pi]$ is

a) 0 b) 2π

c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$.

11. The area bounded by the parabola $y^2 = x$ and its latus rectum is

a) $\frac{4}{3}$ b) $\frac{1}{6}$

c) $\frac{2}{3}$ d) $\frac{8}{3}$.

12. The length of the arc of the curve $x^{2/3} + y^{2/3} = 4$ is

a) 48 b) 24

c) 12 d) 96.

13. The surface area obtained by revolving the area bounded by the curve $y = f(x)$, the two ordinates $x = a$, $x = b$ and x-axis, about x-axis is

a) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

b) $\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$

c) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

d) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dx$.

14. Solution of $\frac{dx}{dy} + mx = 0$, where $m < 0$ is

a) $x = c\, e^{my}$

b) $x = c\, e^{-my}$

c) $x = my + c$

d) $x = c$.

15. If $\frac{dy}{dx} = \frac{x - y}{x + y}$ then

a) $2xy + y^2 + x^2 = c$

b) $x^2 + y^2 - x + y = c$

c) $x^2 + y^2 - 2xy = c$

d) $x^2 - y^2 - 2xy = c$.

16. ' + ' is not a binary operation on

a) N b) Z

c) C d) $Q - \{ 0 \}$.

17. $Var (4x + 3)$ is

a) 7 b) 16 $Var\, x$

c) 19 d) 0.

18. In a Poisson distribution if $p (X = 2) = p (X = 3)$ then the value of its parameter λ is

a) 6 b) 2

c) 3 d) 0.

19. The distribution function $F (X)$ of a random variable X is

a) a decreasing function

b) a non-decreasing function

c) a constant function

d) increasing first and then decreasing.

20. For a standard normal distribution the mean and variance are

a) μ, σ^2 b) μ, σ

c) 0, 1 d) 1, 1.

21. If $A = [\ 2\ \ 0 \cdot 1\ ]$, then rank of $A\,A^T$ is

a) 1

b) 2

c) 3

d) 0.

22. If A is a scalar matrix with scalar $k \neq 0$, of order 3, then A^{-1} is

a) $\frac{1}{k^2}\ I$

b) $\frac{1}{k^3}\ I$

c) $\frac{1}{k}\ I$

d) $k\,I$.

23. If A and B are any two matrices such that $AB = 0$ and A is non-singular, then

a) $B = 0$

b) B is singular

c) B is non-singular

d) $B = A$.

24. Cramer's rule is applicable only (with three unknowns) when

a) $\Delta \neq 0$

b) $\Delta = 0$

c) $\Delta = 0,\ \Delta_x \neq 0$

d) $\Delta_x = \Delta_y = \Delta_z = 0$.

25. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a}+\vec{b}\right)$ is a unit vector if

a) $\theta = \frac{\pi}{3}$

b) $\theta = \frac{\pi}{4}$

c) $\theta = \frac{\pi}{2}$

d) $\theta = \frac{2\pi}{3}$.

26. The modulus and amplitude of the complex number $\left[e^{3-\frac{i\pi}{4}}\right]^3$ are respectively

a) $e^9, \frac{\pi}{2}$

b) $e^9, -\frac{\pi}{2}$

c) $e^6, -\frac{3\pi}{4}$

d) $e^9, -\frac{3\pi}{4}$.

27. If $x = \cos\theta + i\sin\theta$, the value of $x^n + \frac{1}{x^n}$ is

a) $2\cos n\theta$

b) $2i\sin n\theta$

c) $2\sin n\theta$

d) $2i\cos n\theta$.

28. If $-i+2$ is one root of the equation $ax^2 - bx + c = 0$, then the other root is

a) $-i-2$

b) $i-2$

c) $2+i$

d) $2i+1$.

29. The number of values of $(\cos \theta + i \sin \theta)^{p/q}$ where p and q are non-zero integers prime to each other, is

a) p

b) q

c) $p + q$

d) $p - q$.

30. The point of intersection of the tangents at $t_1 = t$ and $t_2 = 3t$ to the parabola $y^2 = 8x$ is

a) $\left(6t^2 , 8t \right)$

b) $\left(8t , 6t^2 \right)$

c) $\left(t^2 , 4t \right)$

d) $\left(4t , t^2 \right)$.

31. Which of the following curves is concave down ?

a) $y = - x^2$

b) $y = x^2$

c) $y = e^x$

d) $y = x^2 + 2x - 3$.

32. $\lim\limits_{x \to 0} \dfrac{x}{\tan x}$ is

a) 1

b) -1

c) 0

d) ∞ .

33. If $u = \dfrac{1}{\sqrt{x^2 + y^2}}$, then $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y}$ is equal to

a) $\frac{1}{2} u$

b) u

c) $\frac{3}{2} u$

d) $- u$.

34. The curve $ay^2 = x^2(3a - x)$ cuts the y-axis at

a) $x = -3a,\ x = 0$

b) $x = 0,\ x = 3a$

c) $x = 0,\ x = a$

d) $x = 0$.

35. The value of $\int_0^{\pi/4} \cos^3 2x \, dx$ is

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) 0

d) $\frac{2\pi}{3}$.

36. The integrating factor of the differential equation $\frac{dy}{dx} - y \tan x = \cos x$ is

a) $\sec x$

b) $\cos x$

c) $e^{\tan x}$

d) $\cot x$.

37. The order and degree of the differential equation

$$\frac{d^2y}{dx^2} - y + \left(\frac{dy}{dx} + \frac{d^3y}{dx^3}\right)^{3/2} = 0 \text{ are}$$

a) 2, 3

b) 3, 3

c) 3, 2

d) 2, 2.

38. Which of the following is a tautology ?

a) $p \vee q$

b) $p \wedge q$

c) $p \vee \sim p$

d) $p \wedge \sim p$.

39. A monoid becomes a group if it also satisfies the

a) closure axiom

b) associative axiom

c) identity axiom

d) inverse axiom.

40. In the multiplicative group of n^{th} roots of unity, the inverse of ω^{k} ($k < n$) is

a) $\omega^{1/k}$

b) ω^{-1}

c) ω^{n-k}

d) $\omega^{n/k}$.

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. Solve the following non-homogeneous equations of three unknowns using determinants :

$2x + 2y + z = 5$

$x - y + z = 1$

$3x + y + 2z = 4$.

42. Show that the adjoint of $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$ is A itself.

43. Show that the two lines $\vec{r} = \left(\vec{i} - \vec{j} \right) + t \left(2 \vec{i} + \vec{k} \right)$ and $\vec{r} = \left(2 \vec{i} - \vec{j} \right) + s \left(\vec{i} + \vec{j} - \vec{k} \right)$ are skew lines and find the distance between them.

44. a) If $\vec{a}$, $\vec{b}$ are any two vectors, then prove that

$$\left| \vec{a} \times \vec{b} \right|^2 + \left(\vec{a} \cdot \vec{b} \right)^2 = \left| \vec{a} \right|^2 \left| \vec{b} \right|^2 .$$

b) Find the value of λ if the points (3, 2, – 4), (9, 8, – 10) and (λ, 4, – 6) are collinear.

45. p represents the variable z. Find the locus of p if $Re \left(\frac{z + 1}{z - i} \right) = 0$.

46. Prove that $(1 + i)^n + (1 - i)^n = 2^{\frac{n+2}{2}} \cos \frac{n\pi}{4}$; $n \in N$.

47. Find the equation of the hyperbola if the asymptotes are $2x + 3y - 8 = 0$ and $3x - 2y + 1 = 0$ and (5, 3) is a point on the hyperbola.

48. Obtain the Maclaurin's series for $\frac{1}{1 + x}$.

49. Find the intervals on which $f (x) = x^3 - 3x + 1$ is increasing or decreasing.

50. If $w = x + 2y + z^2$ and $x = \cos t$; $y = \sin t$; $z = t$, find $\frac{dw}{dt}$.

51. Evaluate $\int_0^{\pi/2} \log (\tan x) \, dx$.

52. Solve : $\frac{dy}{dx} + y = x$.

53. Construct the truth table for $(p \vee q) \wedge r$.

54. Find the mean and variance for the following probability density function :

$$f(x) = \begin{cases} xe^{-x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

55. a) State and prove cancellation laws on groups.

OR

b) Four coins are tossed simultaneously. What is the probability of getting

i) exactly 2 heads

ii) at least 2 heads

iii) at most 2 heads ?

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Show that the equations $2x + 5y + 7z = 52$, $x + y + z = 9$, $2x + y - z = 0$ are consistent and solve them by using rank method.

57. Prove by vector method $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

58. Find the vector and cartesian equations to the plane through the point $(-1, -2, 1)$ and perpendicular to the planes

$x + 2y + 4z + 7 = 0$ and $2x - y + 3z + 3 = 0$.

59. A comet is moving in a parabolic orbit around the sun which is at the focus of the parabola. When the comet is 80 million km from the sun, the line segment from the sun to the comet makes an angle of $\frac{\pi}{3}$ radians with the axis of the orbit. Find (i) the equation of the comet's orbit (ii) how close the comet comes nearer to the sun.

(Take the orbit as open rightward).

60. Find the eccentricity, centre, foci and vertices of the hyperbola

$x^2 - 3y^2 + 6x + 6y + 18 = 0$ and also trace the curve.

61. Find the equation of the rectangular hyperbola which has for one of its asymptotes the line $x + 2y - 5 = 0$ and passes through the points $(6, 0)$ and $(-3, 0)$.

62. A water tank has the shape of an inverted circular cone with base radius 2 metres and height 4 metres. If water is being pumped into the tank at a rate of $2\ m^3/min$, find the rate at which the water level is rising, when the water is 3 m deep.

63. Show that the volume of the largest right circular cone that can be inscribed in a sphere of radius a is $\frac{8}{27}$ (volume of the sphere).

64. Using Euler's theorem, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$, if

$u = \sin^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$.

65. Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$.

66. Find the volume of a right circular cone with radius r and height h.

67. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{3x}$

when $x = \log 2,\ y = 0$ and when $x = 0,\ y = 0$.

68. The rate at which the population of a city increases at any time is proportional to the population at that time. If there were 1,30,000 people in a city in 1960 and 1,60,000 in 1990. what population may be anticipated in 2020 ?

$$\left[\log_e\left(\frac{16}{13}\right) = 0{\cdot}2070,\ e^{0{\cdot}42} = 1{\cdot}52\right].$$

69. Show that the set of all matrices of the form $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$, $a \in R - \{0\}$ forms an Abelian group under matrix multiplication.

70. a) Find all the values of $\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)^{3/4}$ and hence prove that the product of the values is 1.

OR

b) Find c, μ and σ^2 of the normal distribution whose probability function is given by $f(x) = ce^{-x^2+3x}$, $-\infty < x < \infty$.