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Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours]

[Maximum Marks : 200

SECTION - A

- N. B. :
- i) All questions are compulsory.
 - ii) Each question carries one mark.
 - iii) Choose the most suitable answer from the given four alternatives. 40 × 1 = 40

1. If $\vec{PR} = 2\vec{i} + \vec{j} + \vec{k}$, $\vec{QS} = -\vec{i} + 3\vec{j} + 2\vec{k}$ then the area of the quadrilateral PQRS is

- | | |
|---------------------------|------------------|
| a) $5\sqrt{3}$ | b) $10\sqrt{3}$ |
| c) $5 \frac{\sqrt{3}}{2}$ | d) $\frac{3}{2}$ |

2. If $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j} + 2\vec{k}$ then a unit vector perpendicular to \vec{a} and \vec{b} is

- | | |
|---|---|
| a) $\frac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}$ | b) $\frac{\vec{i} - \vec{j} + \vec{k}}{\sqrt{3}}$ |
| c) $\frac{-\vec{i} + \vec{j} + 2\vec{k}}{\sqrt{3}}$ | d) $\frac{\vec{i} - \vec{j} - \vec{k}}{\sqrt{3}}$ |

[Turn over

3. The point of intersection of the lines

$$\vec{r} = \left(-\vec{i} + 2\vec{j} + 3\vec{k} \right) + t \left(-2\vec{i} + \vec{j} + \vec{k} \right) \text{ and}$$

$$\vec{r} = \left(2\vec{i} + 3\vec{j} + 5\vec{k} \right) + s \left(\vec{i} + 2\vec{j} + 3\vec{k} \right) \text{ is}$$

- a) (2, 1, 1) b) (1, 2, 1)
- c) (1, 1, 2) d) (1, 1, 1).

4. The distance from the origin to the plane $\vec{r} \cdot \left(2\vec{i} - \vec{j} + 5\vec{k} \right) = 7$ is

- a) $\frac{7}{\sqrt{30}}$ b) $\frac{\sqrt{30}}{7}$
- c) $\frac{30}{7}$ d) $\frac{7}{30}$.

5. If \vec{a} , \vec{b} , \vec{c} are three mutually perpendicular unit vectors then

$$\left| \vec{a} + \vec{b} + \vec{c} \right| \text{ is}$$

- a) 3 b) 9
- c) $3\sqrt{3}$ d) $\sqrt{3}$.

6. If the line $5x - 2y + 4k = 0$ is a tangent to $4x^2 - y^2 = 36$, then k is

- a) $\frac{4}{9}$ b) $\frac{2}{3}$
- c) $\frac{9}{4}$ d) $\frac{81}{16}$.

7. The eccentricity of the hyperbola with asymptotes $x + 2y - 5 = 0$,

$$2x - y + 5 = 0 \text{ is}$$

- a) 3 b) $\sqrt{2}$
- c) $\sqrt{3}$ d) 2.

B

8. The point of contact of the parabola $y^2 = 4ax$ and the tangent $y = mx + c$ is

a) $\left(\frac{2a}{m^2}, \frac{a}{m}\right)$

b) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

c) $\left(\frac{a}{m}, \frac{2a}{m^2}\right)$

d) $\left(-\frac{a}{m^2}, -\frac{2a}{m}\right)$.

9. The slope of the normal to the curve $y = 3x^2$ at the point whose x -coordinate = 2, is

a) $\frac{1}{13}$

b) $\frac{1}{14}$

c) $-\frac{1}{12}$

d) $\frac{1}{12}$.

10. If $s = t^3 - 4t^2 + 7$, the velocity when the acceleration is zero, is

a) $\frac{32}{3}$ m/sec

b) $-\frac{16}{3}$ m/sec

c) $\frac{16}{3}$ m/sec

d) $-\frac{32}{3}$ m/sec.

11. If p is true and q is false then which of the following is not true ?

a) $p \rightarrow q$ is false

b) $p \vee q$ is true

c) $p \wedge q$ is false

d) $p \leftrightarrow q$ is true.

12. Given $E(X + C) = 8$ and $E(X - C) = 12$ then the value of C is

a) -2

b) 4

c) -4

d) 2 .

13. In 5 throws of a die, getting 1 or 2 is a success. The mean number of successes is

a) $\frac{5}{3}$

b) $\frac{3}{5}$

c) $\frac{5}{9}$

d) $\frac{9}{5}$.

14. If $f(x)$ is a *p.d.f.* of a normal distribution with mean μ then $\int_{-\infty}^{\infty} f(x) dx$ is

a) 1

b) 0.5

c) 0

d) 0.25 .

15. If X is a discrete random variable then $P(X \geq a)$ is equal to

a) $P(X < a)$

b) $1 - P(X \leq a)$

c) $1 - P(X < a)$

d) 0 .

20. If \vec{a} and \vec{b} are two unit vectors and θ is the angle between them, then $(\vec{a} + \vec{b})$ is a unit vector if

- a) $\theta = \frac{\pi}{3}$
- b) $\theta = \frac{\pi}{4}$
- c) $\theta = \frac{\pi}{2}$
- d) $\theta = \frac{2\pi}{3}$.

21. A particular integral of $(D^2 - 4D + 4)y = e^{2x}$ is

- a) $\frac{x^2}{2} e^{2x}$
- b) xe^{2x}
- c) xe^{-2x}
- d) $\frac{x}{2} e^{-2x}$.

22. The order and degree of the differential equation $y' + (y'')^2 = x(x + y'')^2$ respectively are

- a) 1, 1
- b) 2, 1
- c) 2, 2
- d) 1, 2.

23. Which of the following is a contradiction ?

- a) $p \vee q$
- b) $p \wedge q$
- c) $p \vee (\sim p)$
- d) $p \wedge (\sim p)$.

24. The order of $[7]$ in $(\mathbb{Z}_9, +_9)$ is

- a) 9 b) 6
c) 3 d) 1.

25. Which of the following is correct ?

- a) An element of a group can have more than one inverse
b) If every element of a group is its own inverse, then the group is Abelian
c) The set of all 2×2 real matrices forms a group under matrix multiplication
d) $(a * b)^{-1} = a^{-1} * b^{-1}$ for all $a, b \in G$.

26. The volume generated by rotating the triangle with vertices at $(0, 0)$, $(3, 0)$ and $(3, 3)$ about x -axis, is

- a) 18π b) 2π
c) 36π d) 9π .

27. The arc length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

a) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

b) $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

c) $\int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx$

d) $2\pi \int_a^b y \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dx.$

28. The surface area of the solid of revolution of the region bounded by $y = \sqrt{2x}$, $x = 0$ and $x = 2$ about x -axis, is

- a) $8\sqrt{5} \pi$
- b) $2\sqrt{5} \pi$
- c) $\sqrt{5} \pi$
- d) $4\sqrt{5} \pi$.

29. The differential equation obtained by eliminating a and b from $y = ae^{3x} + be^{-3x}$ is

- a) $\frac{d^2 y}{dx^2} + ay = 0$
- b) $\frac{d^2 y}{dx^2} - 9y = 0$
- c) $\frac{d^2 y}{dx^2} - 9 \frac{dy}{dx} = 0$
- d) $\frac{d^2 y}{dx^2} + 9x = 0$.

30. Integrating factor of $\frac{dy}{dx} + \frac{1}{x \log x} \cdot y = \frac{2}{x^2}$ is

- a) e^x
- b) $\log x$
- c) $\frac{1}{x}$
- d) e^{-x} .

31. Which of the following functions is increasing in $(0, \infty)$?

- a) e^x
- b) $\frac{1}{x}$
- c) $-x^2$
- d) x^{-2} .

32. One of the critical numbers of $x^{3/5} (4 - x)$ is

- a) 4
b) $\frac{4}{3}$
c) 0
d) $\frac{3}{5}$.

33. The percentage error in the 11th root of the number 28 is approximately times the percentage error in 28.

- a) $\frac{1}{28}$
b) $\frac{1}{11}$
c) 11
d) 28.

34. The curve $9y^2 = x^2 (4 - x^2)$ is symmetrical about

- a) y -axis only
b) x -axis only
c) $y = x$
d) both the axes.

35. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

- a) $\frac{3\pi}{16}$
b) $\frac{3}{16}$
c) 0
d) $\frac{3\pi}{8}$.

36. The polar form of the complex number $(i^{25})^3$ is

- a) $\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$
- b) $\cos \pi + i \sin \pi$
- c) $\cos \pi - i \sin \pi$
- d) $\cos \frac{\pi}{2} - i \sin \frac{\pi}{2}$.

37. If $z_1 = 4 + 5i$ and $z_2 = -3 + 2i$ then $\frac{z_1}{z_2}$ is

- a) $\frac{2}{13} - \frac{22}{13}i$
- b) $\frac{-2}{13} + \frac{22}{13}i$
- c) $\frac{-2}{13} - \frac{23}{13}i$
- d) $\frac{2}{13} + \frac{22}{13}i$.

38. The value of $\sqrt{z\bar{z}}$ is

- a) $|z|^2$
- b) $|z|$
- c) $2|z|$
- d) $2|z|^2$.

39. If $a = 3 + i$ and $z = 2 - 3i$ then the points on the Argand diagram representing az , $3az$ and $-az$ are

- a) vertices of a right-angled triangle
- b) vertices of an equilateral triangle
- c) vertices of an isosceles triangle
- d) collinear.

40. The radius of the director circle of the conic $9x^2 + 16y^2 = 144$ is

- a) $\sqrt{7}$
- b) 4
- c) 3
- d) 5.

SECTION - B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks.

$10 \times 6 = 60$

41. Find the rank of the matrix $\begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

42. Solve the following non-homogeneous system of linear equations by determinant method :

$$4x + 5y = 9, \quad 8x + 10y = 18.$$

43. a) Show that the vectors $3\vec{i} - 2\vec{j} + \vec{k}$, $\vec{i} - 3\vec{j} + 5\vec{k}$ and $2\vec{i} + \vec{j} - 4\vec{k}$ form a right-angled triangle.

b) A force of magnitude 5 units acting parallel to $2\vec{i} - 2\vec{j} + \vec{k}$ displaces the point of application from $(1, 2, 3)$ to $(5, 3, 7)$. Find the work done.

44. Show that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$.

45. Find the square root of $(-8 - 6i)$.

46. The headlight of a motor vehicle is a parabolic reflector of diameter 12 cm and depth 4 cm. Find the position of the bulb on the axis of the reflector for effective functioning of the headlight.

B

[Turn over

47. a) Using Rolle's theorem find the value of c for $f(x) = \sqrt{1-x^2}$, $-1 \leq x \leq 1$.

b) Obtain the Maclaurin's series for e^x .

48. Evaluate: $\lim_{x \rightarrow 1} x^{\frac{1}{x-1}}$.

49. Given $w = \frac{x}{x^2 + y^2}$ where $x = \cos t$; $y = \sin t$. Find $\frac{dw}{dt}$.

50. Evaluate: $\int_0^{2\pi} \sin^9 \frac{x}{4} dx$.

51. Solve: $\frac{dy}{dx} + \frac{4x}{x^2 + 1} y = \frac{1}{(x^2 + 1)^2}$.

52. Construct the truth table for $\sim [(\sim p) \wedge (\sim q)]$.

53. Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

54. In a Poisson distribution if $P(X=2) = P(X=3)$, find $P(X=5)$.

[Given $e^{-3} = 0.050$].

55. a) Find the mean and variance for the probability density function

$$f(x) = \begin{cases} \frac{1}{24} & , -12 \leq x \leq 12 \\ 0 & , \text{otherwise} \end{cases}$$

OR

b) Simplify: $\frac{(\cos \theta + i \sin \theta)^4}{(\sin \theta + i \cos \theta)^5}$.

SECTION - C

N. B. : i) Answer *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. 10 × 10 = 100

56. Solve by matrix inversion method :

$$2x - y + 3z = 9, \quad x + y + z = 6, \quad x - y + z = 2.$$

57. Prove by vector method : Altitudes of a triangle are concurrent.

58. Find the vector and Cartesian equations of the plane through the points

(1, 2, 3) and (2, 3, 1) and perpendicular to the plane $3x - 2y + 4z - 5 = 0$.

59. Find the axis, vertex, focus, equation of the directrix, equation of the latus

rectum and length of the latus rectum of the parabola $y^2 + 8x - 6y + 1 = 0$ and

hence sketch its graph.

60. An arch is in the form of a semi-ellipse whose span is 48 feet wide. The height of

the arch is 20 feet. How wide is the arch at a height of 10 feet above the base ?

61. Find the equation of the rectangular hyperbola which has one of its asymptotes

$x + 2y - 5 = 0$ and passes through the points (6, 0) and (- 3, 0).

62. Show that the equation of the normal to the curve $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$ at θ is $x \cos \theta - y \sin \theta = a \cos 2\theta$.
63. Trace the curve $y = x^3$.
64. Find the area between the curve $y = x^2 - x - 2$, x -axis and the lines $x = -2$, $x = 4$.
65. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance a and b from the centre of the sphere is $2\pi r (b - a)$ and hence deduce the surface area of the sphere ($b > a$).
66. Solve : $(D^2 - 2D + 2) y = \sin 2x + 5$.
67. The sum of Rs. 1,000 is compounded continuously at the nominal rate of interest 4 per cent per annum. In how many years will the amount be twice the original principal ? $(\log_e 2 = 0.6931)$.
68. Prove that the set of four functions f_1, f_2, f_3, f_4 on the set of non-zero complex numbers $\mathbb{C} - \{0\}$ defined by $f_1(z) = z$, $f_2(z) = -z$, $f_3(z) = \frac{1}{z}$, $f_4(z) = -\frac{1}{z}$, $\forall z \in \mathbb{C} - \{0\}$ forms an Abelian group with respect to composition of functions.

69. A random variable X has the following probability mass function :

x	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

- i) Find k
- ii) Evaluate $P(X < 4)$, $P(X \geq 5)$ and $P(3 < X \leq 6)$
- iii) Find the smallest value of x for which $P(X \leq x) > \frac{1}{2}$.

70. a) If α and β are the roots of $x^2 - 2x + 4 = 0$, prove that

$$\alpha^n - \beta^n = i 2^{n+1} \sin \frac{n\pi}{3} \text{ and deduce } \alpha^9 - \beta^9.$$

OR

- b) A poster is to have an area of 180 cm^2 with 1 cm margins at the bottom and sides and a 2 cm margin on the top. What dimensions will give the largest printed area ?

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