

SYLLABUS

FIRST SEMESTER

B.Sc MATHEMATICS-I

(5 Lecture hours per week+2 hours of problem working classes)

1. Mathematical Logic

Open sentences-compound open sentences-quantifiers-Truth sets-connectives involving quantifiers methods of proof methods of disproof. 7 lecture hours

2. Relations and Functions

relations on a set-equivalence relations-equivalence classes -partition of a set. Functions (mappings)-set theoretic properties of functions-existence of inverse of a function and properties of inverse functions- composition of functions-associativity-inverse of composition. 8 lecture hours

3. Differential Calculus

Successive differentiation-nth derivatives of the functions. $(ax+b)^m$, $\log(ax+b)$, $e^{ax} \sin(ax+b)$ $\cos(ax+b)$, $e^{ax} \sin(bx+c)$, $e^{ax} \arccos(bx+c)$ -Leibnitz theorem and its applications.

Partial differentiation-first and higher derivatives-Differentiation of homogeneous functions-Euler's theorem-Total derivative and total differential Differentiation of implicit functions and composite functions-Jacobians. 20 lecture hours

4. Integral Calculus

Reduction formulas for $\int \sin^n x \, dx$, $\int \cos^n x \, dx$, $\int \tan^n x \, dx$, $\int \cot^n x \, dx$, $\int \sec^n x \, dx$, $\int \operatorname{cosec}^n x \, dx$, $\int \sin^m x \cos^n x \, dx$. Differentiation under the integral sign. 10 lecture hours

5. Analytic Geometry of three-dimensions

Relation between Cartesian coordinates and position vectors-Distance and Division formulas-Direction cosines of a line (as components of unit vector)-Direction ratios-Angle between two lines-area of a triangle and volume of a tetrahedron with given vertices.

Equation of a line in different forms-Perpendicular from a point onto a line. Equation of a plane in different forms-Perpendicular from a point onto a plane.

Angle between two planes-Line of intersection of two planes-Plane coaxial with given planes-Planes bisecting the angle between two planes-Angle between a line and a plane-Coplanarity of two lines-Shortest distance between two lines.

Equation of the sphere in general and standard forms-equation of a sphere with given ends of a diameter.

Standard equations of right circular cone and right circular cylinder.

Standard equations of quadric surfaces (paraboloid, ellipsoid and hyperboloid of one and two sheets).
 hours 30 lecture

Note: All the derivations (book works) must be through vector methods with reduction to corresponding Cartesian equivalents.

Book for study/reference

1. Y.F. Lin and S.Y. Lin: Set Theory-Intuitive Approach (Houghton Mifflin Co.USA)
2. Lipschutz; Set Theory and Related Topics (Schaum Series), TMH Edition, Delhi
3. S. Shantinaraya, Differential Calculus (S. Chand, Delhi)
4. S. Shantinarayan, Integral Calculus (S. Chand, Delhi)
5. S. Shantinarayan, Elements of Analytical Solid Geometry (S. Chand, Delhi)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Logic 2 Relations and Functions 2 Differential Calculus: 6 Integral Calculus: 2 Geometry (up to skew lines: 5; remaining part-3) 8 Total: 20	15	2	30
II	Logic: 2 Relations and Functions: 2 Total : 4	2	5	10
III	Differential Calculus: 5	3	5	15

IV	Integral Calculus:	3	2	5	10
V	Geometry (up to skew lines)	5	3	5	15
VI	Geometry (sphere, Cone, cylinder):	3	2	5	10

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

SECOND SEMESTER

MATHEMATICS-II

(5 Lecture hours per week+2 hours of problem working classes)

1. Matrices

Elementary row and column transformations (operations)-equivalent matrices finding the inverse of a non-singular matrix by elementary operations. Rank of matrix-Invariance of rank under elementary transformations-Determination of rank of a matrix by reducing it to the echelon and normal forms.

Homogeneous and non-homogeneous systems of m linear equations in n unknowns-consistency criterion-Solution by elimination method.

Eigenvalues and eigenvectors of a square matrix-standard properties -Cayley-Hamilton theorem (with proof)-Applications 20 lecture hours

2. Differential Calculus

Polar coordinates Angle between the radius vector and the tangent-Angle of intersection of curves (polar form)-Polar sub tangent and polar subnormal- Perpendicular from pole on the tangent-Pedal equations. Derivative of an arc in Cartesian, parametric and polar forms.

Curvature of plane curves-formula for radius of curvature in Cartesian, parametric, polar and pedal forms-center of curvature-evolutes.

Concavity, convexity, Inflexion-singular points-Asymptotes-Envelopes

Tracing of standard Cartesian, parametric and polar curves (Cissoids, witch of Agnesis, Strophoid, Astroid, Folium of Descartes, Catenary, Cycloid, Cardioid, Lemniscate, equiangular Spiral, three leaved rose and four leaved rose) 30 lecture hours

3. Integral Calculus

Applications of Integral Calculus: Computation of lengths of arcs, plane areas, and surface area and volume of solids of revolutions for standard curves in Cartesian and polar forms.

10 lecture

hours

4. Differential equations

Solutions of ordinary differential equations of first order and first degree:

- (i) Variable separable and reducible to variable separable forms.
- (ii) Homogeneous and reducible to homogeneous forms
- (iii) Linear equations, Bernoulli equation and those reducible to these
- (iv) Exact equations, equations reducible to exact form with standard integrating factors.

Equations of first order and higher degree-solvable for p, solvable for y-solvable for x- Clairaut's equations- singular solution-Geometrical meaning.

Orthogonal trajectories in Cartesian and polar forms.
hours

15 lecture

Book for study/reference

1. F. Ayres Matrices (Schaum Publishing Co, U.S.A)
2. S. Shantinarayan, Differential Calculus (S. Chand, Delhi)
3. S. Shantinarayan, Integral Calculus, (S. Chand, Delhi)
4. F. Ayres, Differential Equations (Schaum Series)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Matrices: 5 Differential Calculus: (up to evolutes: 5; remaining part 3) 8 Integral Calculus: 2 Differential Equations: 5 Total: 20	15	2	30
II	Matrices: 5	3	5	15
III	Differential Calculus: 4 (up to evolutes):	2	5	10
IV	Differential Calculus: 4 (remaining part):	2	5	10
V	Integral Calculus: 3	2	5	10
VI	Differential Equations: 4	3	5	15

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

THIRD SEMESTER

MATHEMATICS-III

(5 Lecture hours per week+2 hours of problem working classes)

1. Group theory

Recapitulation of the definition and standard properties of groups and subgroups Cyclic groups-properties-order of an element of a group-properties related to order of an element-subgroup generated by an element of group-coset decomposition of a group-moduls relation-index of a group- Lagrange's theorem-consequences. 20 lecture hours

2. Sequences of Real Numbers

Definition of sequence-Bounded sequences- limit of a sequence-convergent, divergent and oscillatory sequences-Monotonic sequences and their properties- Cauchy's criterion.

12 lecture hours

3. Series of Real Numbers

Definition of convergence, divergence and oscillation of series-properties of convergent series-properties of series of positive terms-Geometric series.

Tests for convergence of series-p-series-comparison tests-Cauchy's root test-D' Alembert's test, Raabe's test, Absolute and conditional convergence-D' Alembert test for absolute convergence-Alternating series-Leibnitz test

Summation of Binomial, Exponential, and Logarithmic series. 18 lecture hours

4. Differential Calculus

Definition of the limit of a function in ϵ - δ form-continuity-types of discontinuities- properties of continuous functions on a closed interval (boundedness, attainment of bounds and taking every value between bounds). Differentiability. Differentiability implies Continuity-converse not true. Rolle's Theorem-Lagrange's and Cauchy's First Mean Value Theorems Taylor's Theorem (Lagrange's form)- Maclaurin's expansion-Evaluation of limits by L'Hospital's rule. 15 lecture hours

5. Fourier Series

Trigonometric Fourier series of functions with period 2π and period $2L$ - Half-range cosine and sine series. 10 lecture hours

Book for study/reference

1. L.N. Herstein, Topics in Algebra (Vikas)
2. J.B. Fraleigh, First course in Abstract Algebra (Addision-Weskey)
3. Murry R. Seigal: Advanced Calculus (Schaum Series)
4. S. Shantinarayan, Differential Calculus (S. Chand, Delhi)
5. H. Churchill: Fourier Series and Boundary Value problems.

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Groups: 6 Sequences: 3 Series: 5 Differential Calculus: 4 Fourier Series: 2 Total: 20	15	2	30
II	Groups: 5	3	5	15
III	Sequences: 3	2	5	10
IV	Series: 5	3	5	15
V	Differential Equations: 4	2	5	10
VI	Fourier Series: 3	2	5	10

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

FOURTH SEMESTER

MATHEMATICS-IV

(5 Lecture hours per week+2 hours of problem working classes)

1. Group Theory

Normal subgroups examples and problems-Quotient group-Homomorphism and Isomorphism of groups-Kernel and image of a homomorphism- Normality of the kernel - Fundamental theorem of homomorphism-properties related to isomorphism-Permutation group-Cay ley's theorem.

15 lecture hours

2. Differential Calculus

Continuity and differentiability of functions of two and three variables-Taylor's theorem and expansions of functions of two variables-Maxima and minima of functions of two variables. Method of Lagrange multipliers 10 lecture hours

3. Integral Calculus

Gamma and Beta functions-results following definitions-Relations connecting the two functions-duplication formula - Applications to valuation of integrals.

4. Differential Equations

Second and higher order ordinary linear differential equations with constant coefficients-complementary function-particular integrals (standard types)-Cauchy-Euler differential equation. Simultaneous linear differential equations (two variables) with constant coefficients.

Solutions of second order ordinary linear differential equations with variable coefficients by the following methods:

- (i) When a part of complementary function is given
- (ii) Changing the independent variable
- (iii) Changing the dependent variable
- (iv) Variation of parameters
- (v) When the equation is exact.

Total differential equations - Necessary condition for the equation $Pdx+Qdy+Rdz=0$ be integrable Simultaneous equations of the form $dx/P=dy/Q=dz/R$ 18 lecture hours

5. Laplace Transforms

Definition and basic properties-Laplace transforms of some common functions and standard results-Laplace transform of periodic functions-Laplace transform of derivatives and the integral of a function-Laplace transforms the Heaviside function and Dirac-delta function-Convolution theorem (No proof)-Inverse Laplace transforms-Laplace transform method of solving ordinary linear differential equations of first and second orders with constant coefficients. 12 lecture hour

6. Linear Programming

Linear inequalities and their graphs Statement of the linear programming problem in standard form-classification of solutions- solution of linear programming problems by graphical method.

Illustrative examples on the solution of linear programming problems in two and three variables by the simplex method. 10 lecture hours

Books for study/reference

1. L.N. Herstein: Topics in Algebra (Vikas)
2. J.B. Frayleigh: First course in Abstract Algebra (addition-Wesley)
3. S. Shantinarian: Differential Calculus (S. Chand)
4. S. Shantinarian: Integral Calculus (S. Chand)
5. F. Ayres: Differential equations (Schaum)
6. M.G. Smith: Laplace Transform Theory (Van-No strand)
7. L.S. Srinath: Linear Programming (East-West)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be	No. of subdivisions to	Marks for each subdivision	Maximum Marks for
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	set in the topic	be answered		the Question
I	Groups: 5 Differential Calculus 3 Integral Calculus 3 Differential Equations: 4 Laplace Transforms: 3 Linear Programming: 2 Total: 20	15	2	30
II	Groups: 4	2	5	10
III	Differential and Integral Calculus (3+2)=: 5	3	5	15
IV	Differential Equations: 5	3	5	15
V	Laplace Transforms: 3	2	5	10
VI	Linear Programming: 3	2	5	10

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

FIFTH SEMESTER

MATHEMATICS-V

(4 Lecture hours per week+2 hours of problem working classes)

1. Rings, Integral Domains, Fields

Ring Types of rings-properties of rings-Rings of integers modulo n-Subrings-Ideals-Principal and Maximal ideals in a commutative ring-examples and standard properties Homomorphism and Isomorphism -Properties of homomorphism- Quotient rings Integral domains-fields-properties following the definition-field is an integral domain-finite integral domain is a field. 18 lecture hours

2. Geometry of space curves

Vector function of a single scalar variable-its interpretation as a space curve-derivative-tangent, normal and binormal vectors to a space curve-Serret Frenet formulas-simple geometrical applications.

Vector function of two scalar Variables-its interpretation as a surface-Tangent plane and Normal to a surface-parametric curves on a surface-parametric curves on the surfaces of a right circular cylinder and sphere-cylindrical and spherical polar coordinates.

Orthogonal curvilinear coordinates-Relations between base vectors and normal vectors-reciprocity of these vectors-Arc, area and volume elements-Specialization to Cartesian, cylindrical and spherical coordinates. 15 lecture hours

3. Vector Differential Calculus

Scalar field-gradient of a scalar field geometrical meaning directional derivative-vector field-divergence and curl of a vector field-solenoid and irrotational field- scalar and vector potentials Laplacian of a scalar field-vector identities.

Expressions for $\Delta\phi$ $\text{div } f$ $\Delta^2\phi$ and $\text{curl } f$ in orthogonal curvilinear coordinates and specialization to Cartesian, cylindrical and spherical polar coordinates. 15 lecture hours

4. Special Functions

Polynomial solution of Legendre differential equation -Legendre polynomials-generating function-Recurrence relations-Rodrigue's formula-orthogonality.

Series Solution of Bessel differential equation Bessel function (x)- Recurrence relations-generating function-orthogonality 12 lecture hours

For study/reference:

1. I.N. Herstein: Topic in Algebra (Vikas)
2. J.B. Frayleigh: A First course in Abstract Algebra (Addison-wesley)
3. B. Spain: vector Analysis (ELBS)
4. Chorlton F: Differential and Deference Equations (Van Nortand)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Rings, Integral Domains, Fields 6 Geometry of space curves: 5 Vector Differential Calculus: 5 Special functions: 4 Total: 20	15	2	30
II	Rings, Integral Domains, Fields 6	4	5	20
III	Geometry of space curves: 5	3	5	15
IV	Vector Differential Calculus: 5	3	5	15
V	Special functions: 4	2	5	10

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

MATHEMATICS-VI

(4 Lecture hours per week+2 hours of problem working classes)

1. Partial Differential Equations

Formation of partial differential equations-equations of First Order Lagrange's linear equation-Charpit's method-Standard types of first order non-linear partial differential equations.

Solution of second order linear partial differential equations in two variables with constant coefficients by finding complementary function and particular integral canonical form for parabolic, elliptic and hyperbolic equations-Solution by separation of variables.

Solutions of one-dimensional heat and wave equations and two-dimensional Laplace equation using Fourier series. 15 lecture hours

2. Numerical Analysis

Finite Differences-Definition and properties of Δ , Δ^2 , S and E and the relations between them-The n th differences of a polynomial.

Newton-Gregory forward and backward interpolation formulas-Lagrange's and Newton's interpolation formulas for unequal intervals-Inverse interpolation.

Numerical differentiation using forward and backward difference formulas-Computation of first and second derivatives.

Numerical Integration: Quadrature formula-Trapezoidal rule-Simpson's $1/3$ and $3/8$ rules-Weddle's rule. 15 lecture hours

3. Particle Dynamics

Newton's Laws of motion-Conservative forces and potential energy-Definitions of work, kinetic energy and power.

Motion of a particle in a uniform force field-simple harmonic motion-Two dimensional motion of projectiles.

Tangential and Normal components of velocity and acceleration-Radial and Transverse components of velocity and acceleration-Constrained Motion of a particle under gravity along inside and outside of a circle

Motion of a particle in a central force field-Determination of orbit from central forces and vice versa.

Linear momentum, angular momentum and energy of a system of particles Principles of linear momentum, angular momentum and energy-Motion of centroid and motion relative to centroid. 30 lecture hours

1. I.N. Sneddon: An introduction to Partial differential equations (Mc Graw Hill)
2. M.K. Jain, SRK Iyengar and R.K. Jain Numerical methods for scientific and Engineering Computation (Wiley Eastern)
3. F. Choriton: Text book of dynamics (Van No strand)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
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I	Partial Differential Equations:	5	15	2	30
	Numerical Analysis:	5			
	Particle Dynamics (up to motion outside And inside circle: 6;				
	Remaining part:4)	10			
	Total:	20			
II	Partial Differential Equations:	5	3	5	15
III	Numerical Analysis:	5	3	5	15
IV	Particle Dynamics (up to motion outside		4	5	20
	And inside circle:	6			
V	Particle Dynamics		2	5	10
	(remaining part):	4			

Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

SIXTH SEMESTER

MATHEMATICS-VII

(4 Lecture hours per week+2 hours of problem working classes)

1. Linear Algebra

Vector space-Examples-Properties-Subspaces-criterion for a subset to be a subspace-Linear combination-linear independent and dependent subsets-Basis and dimension-Standard results-Examples illustrating concepts and results.

Linear transformations-properties-matrix of a linear transformation-change of basis-range and kernel-rank and nullity-Rank-Nullity theorem-Non-singular linear transformations.

Eigenvalues and Eigenvectors of a linear transformation-Interpretation in terms of matrices-Examples illustration the concepts. 18 lecture hours

2. Line and Multiple Integrals

Definition of a line integral and basic properties-examples on evaluation of line integrals.

Definition of a double integral-its conversion to iterated integrals-evaluation of double integrals by change of order of integration and by change of variables-Computation of plane and surface areas, volume underneath a surface and volume of revolution using double integrals.

Definition of a triple integral and evaluation-change of variables-volume as a triple integral. 15 lecture hours

1. Integral Theorems

Line, surface and volume integrals of vector functions-Green's theorem in the plane (with proof) Direct consequences of the theorem.

The divergence theorem (with proof)-Direct consequences of the theorem.

The Stokes theorem (with proof)-Direct consequences of the theorem. 15 lecture hours

2. Calculus of Variations

Variation of a function $f=f(x,y,y)$ -variation of the corresponding functional External of a functional- problem-Euler's equation and its particular forms-Examples-standard-variation problems like geodesics, minimal surface of revolution, hanging chain, Brachistochrone problem- Isoperimetric problems.

12 lecture hours

Book for study/reference

1. D.T. Finkbeiner: An introduction to Matrices and linear Transformations (Freeman and Co.)
2. S. Lipschultz: Linear Algebra (Schaum series)
3. C.Fox: An introduction to the Calculus of variations(Oxford University Press)
4. I.S. Sokoloikoff: Advanced Calculus (Mc Graw Hill)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Linear Algebra: 7 Line and Multiple Integrals: 5 Integral Theorems: 5 Calculus of Variations: 3 Total: 20	15	2	30
II	Linear Algebra: 6	4	5	20
III	Line and Multiple Integrals: 5	3	5	15
IV	Integral Theorems: 5	3	5	15

V	Calculus of Variations:	4	2	5	10
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Note: All questions are to be answered Maximum Marks for the paper Examination Marks: 90 Internal Assessment Marks: 10 Total Marks: 100

MATHEMATICS-VIII

(4 Lecture hours per week+2 hours of problem working classes)

1. Complex Analysis

Complex numbers-The complex plane-conjugate and modulus of a complex number-polar form-geometrical representation-Euler's formula: $\cos \theta + i \sin \theta = e^{i\theta}$

Function of a complex variable: Limit, continuity and differentiability.

Analytic function-Cauchy-Riemann equations in Cartesian and polar forms-Sufficiency conditions for analyticity (in Cartesian form) Standard properties of analytic functions-Construction of analytic functions, given real or imaginary parts Milne-Thomson method

Transformations-definition of a conformal transformation Examples.

Discussion of the transformations: $w=z$, $w=\sin z$, $w=\cos z$, $w=e^z$, $w=\cosh z$, $w=1/2(z+1/z)$

The bilinear transformation-cross ratio property-Bilinear transformation transforms circles into circles or lines-problems thereon.

The complex line integral-Examples and properties.

Cauchy's Integral theorem (proof using Green's theorem) and its direct consequences, The Cauchy's integral formula for the function and the derivatives. Applications to evaluation of simple line integrals Cauchy's inequality Liouville's theorem-Fundamental theorem of algebra. 30
lecture hours

2. Fourier Transforms Integral

The Fourier lateral Complex Fourier transform-Inverse transform-Basic properties Transforms of the derivative and the derivative of the transform-Problems thereon.

15 lecture

hours

3. Numerical Analysis

Solution of Algebraic and transcendental equations. Method of successive bisection Method of false position-secant method-Newton-Raphson method.

Numerical solutions of non-homogeneous system of linear algebraic equations in 3 variables by Jacobi's and Gauss-Seidel methods-Computation of largest eigenvalue of square matrix by power method.

Solution of initial value problem for ordinary linear first order differential equations by Taylor's, Euler's, modified Euler's, Runge-Kutta method. 15 lecture
hours

Books for study/reference

1. L.H. Ahifors Complex Analysis (Mc Graw Hill)
2. R.V. Churchill introduction to Complex Variables and Applications (Mc Graw Hill)
3. I.N. Sengdon: Fourier Transforms (Mc Graw Hill)
4. S.S Shastry: Numerical Analysis (Prentice Hall)
5. M.K. Jain SRK Iyengar and R.K. Jain: Numerical methods for Scientific and Engineering Computation (Wiley Eastern)

Format of Question Paper

Question No.	Topic and No. of subdivisions to be set in the topic	No. of subdivisions to be answered	Marks for each subdivision	Maximum Marks for the Question
I	Complex Analysis (up to transformations:6; remaining Part-4) 10 Fourier Transforms: 5 Numerical Analysis: 5 Total: 20	15	2	30
II	Complex Analysis (up to transformations) 6	4	5	20
III	Complex Analysis (remaining Part) 4	2	5	10
IV	Fourier Transforms: 5	3	5	15
V	Numerical Analysis: 5	3	5	15

I Semester B.Sc/B.A. Examination, Nov/Dec 2005

(Semester Scheme)

MATHEMATICS (Paper-I)

Time: 3 Hours

Max.Marks:90

Instructions: 1) Answer all questions.

2) Answer should be written completely either in English or in Kannada.

1. Answer any fifteen of the following: (2x15=30)

1) Write the negation of $\forall x [P(x) \rightarrow q(x)]$.

2) Find $T [P(x)]$, if $P(x): x^2-3x+2=0$, the replacement set is z .

3) Define an equivalence relation.

4) If $f: Q \rightarrow Q$ is defined by $f(x)=3x+1, \forall x \in Q$, then show that f is onto.

5) Find the n th derivative of $C^{x/2}, \sin 2x$.

- 6) Find the nth derivative of $\sin^3 2x$.
- 7) If $z=xy$, then find $\partial^2 z / \partial x \partial y$
- 8) If $y^2=4ax$, find dy/dx using partial differentiation.
- 9) If $u=x^3-2x^2y+3xy^2+y^3$, prove that $x \partial u / \partial x + y \partial u / \partial y = 3u$.
- 10) If $u=2x-3y$, $v=5x+4y$, show that $\partial(u,v) / \partial(x,y) = 23$.
- 11) Evaluate $\int dx / (1+x^2)^{7/2}$
- 12) Evaluate $\int_0^{\pi/2} \sin^4 x \cos^3 x dx$.
- 13) Find the direction cosines of the joining (2,-3, 6) and (3, -1,-6).
- 14) Show that the line $x-1/-1 = y+1/1 = z/1$ lies on the plane $x-y+2z=2$.
- 15) Find the equation of a plane passing through (2,3, 4) and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$.
- 16) Express the equation of the plane $3x-y+6z=9$ in the normal form.
- 17) Find the distance between the parallel planes $2x-y+3z=-4$ and $4x-2y+6z-6=0$.
- 18) Find the angle between the plane $2x+y+2z=5$ and the line $x-1/2 = y+1/-1 = z-1/2$.
- 19) Find the equation of a sphere concentric with the sphere $2x^2 + 2y^2 + 2z^2 - 4x + 6y - 8z + 1 = 0$, passing through (2,1,-3)
- 20) Find the equation of cone whose vertex is at the origin, the axis is $x/2 = y/1 = z/3$ and the semi vertical angle is 30° .

II. Answer any two of the following: (5x2=10)

- 1) With the usual notation, prove that:

$$T[P(x) \rightarrow q(x)] = [T[P(x)]] \cup T[q(x)].$$

- 2) a) Symbolise and negate 'Some integers are perfect squares or all integers are rational numbers'

- b) Give the direct proof of the statement:

'If $x+y$ is even, then x and y are both odd or both even' where x and y are integers.

- 3) Prove that any partition P of a non empty set A determines an equivalence relation. On A .

- 4) If $f: \mathbb{R} \rightarrow \mathbb{R}$; $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x)=2x+1$ and $g(x)=5-3x$, verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

III. Answer any three of the following: (5x3=15)

- 1) Find the nth derivative of $3x+2 / x^3+x^2$
- 2) If $y = \cos(m \sin^{-1} x)$, show that $\frac{y^{n+1}}{4n^2-m^2}$

$$7n = 4n+2 \quad \text{at } x=0.$$

- 3) State and prove Leibniz's theorem for nth derivative of product of two functions.
- 4) If $u = x/y+z + y/z+x + z/x+y$, show that $x \partial u / \partial x + y \partial u / \partial y + z \partial u / \partial z = 0$.
- 5) If $u = 2xy$, $v = x^2 - y^2$, $x = r \cos \theta$, $y = r \sin \theta$. prove that $\partial(u, v) / \partial(r, \theta) = -4r^3$.

IV. Answer any two of the following: (5x2=10)

- 1) Obtain the reduction formula for $\int \sin^m x \cos^n x \, dx$.
- 2) Using Leibnitz's rule for differentiations under the integral sign, evaluate $\int \frac{x^a - 1}{\log x} \, dx$,
where a is a parameter.

3) Evaluate $\int \frac{x^3 \, dx}{\sqrt{x^2 - 1}}$

V. Answer any three of the following: (5x3=15)

1. Find the volume of the tetrahedron formed by the points. (1, 1, 3), (4, 3, 2), (5, 2, 7) and (6, 4, 8).
2. If a line makes angles α, β, γ and δ with four diagonals of a cube, show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$.
3. Prove that the lines $(x-5)/4 = (y-7)/4 = (z-3)/-5$ and $(x-8)/7 = (y-4)/1 = (z-5)/3$ are coplanar. Find the equation of the plane containing them.
4. Find the length and foot of the perpendicular drawn from (1, 1, 2) to the plane $2x - 2y + 4z + 5 = 0$.
5. Find the shortest distance between the lines $(x-8)/3 = (y+1)/-16 = (z-10)/7$ and $(x-15)/3 = (y-29)/8 = (z-5)/-5$.

VI. Answer any three of the following: (5x2=10)

1. Find the equation of a sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 1), and (2, -1, 1). Find its centre and radius.
2. Derive the equation of a right circular cone in the standard form $x^2 + y^2 = z^2 \tan^2 \alpha$.
3. Find the equation of right circular cylinder of radius 3 units, whose axis passes through the point (1, 2, 3) and has direction ratios (2, -3, 6).

SECOND SEMESTER B.Sc./B.A. EXAMINATION.

APRIL/MAY 2005

(Semester Scheme)

MATHEMATICS (PAPER-II)

Time: 3 Hours

Max. Marks: 90

Instructions: 1. Answer all questions.

2. Answers should be written completely either in English or in Kannada.

(2x15=30)

I. Answer any fifteen of the following:

1. If 0 is an eigen value of square matrix A, then prove that A is singular.
2. For what value of x is the rank of the matrix a equal to 3 given.

$$A = \begin{pmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{pmatrix}$$

$$3 \ 1 \ 2$$

$$1 \ 0 \ x$$

3. Find the value of k such that the following system of equations has non-trivial solutions. $(k-1)x + (3k+1)y + 2kz = 0$ $(k-1)x + (4k+2)y + (k+3)z = 0$ $2x + (3k+1)y + 3(k-1)z = 0$.

4. Find the eigen value of the metric $A = \begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$

$$0 \ c \ c$$

5. For the matrix $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

the characteristic equation is $\lambda^2 - \lambda - 5 = 0$. Using it, find A^{-1} .

6. For an equiangular spiral $r = a e^{c \theta}$ show that the tangent at every point is inclined at a constant angle with the radius vector.

7. Find the pedal equation of the curve $r = a e^{c \theta}$.

8. Show that for the curve $r \theta = a$ the polar subtangent is a constant.

9. In the curve $p^n = r^{n+1}$, show that the radius of curvature varies inversely as the $(n-1)$ th power of the radius vector.

10. Show that the origin is conjugate point of the curve $x^2 + 3y^2 + x^3y = 0$.

11. Find the envelope of the family of lines $y = mx + a/m$, where m is a parameter.

12. Find the asymptotes (if any) of the curve $x^3y^2 + x^2y^3 = x^3 + y^3$ parallel to the y -axis.

13. Prove that $y = e^x$ is everywhere concave upwards.

14. Find the length of the arc of the semi-cubical parabola $ay^2 = x^3$ from the vertex to the point (a, a) .

15. Find the whole area of the circle $r = 2a \cos \theta$.

16. Solve: $dy/dx + y/x = 1/x$

17. Solve: $dy/dx + y = \sin x$,

18. Find the integrating factor of the equation $x dy - y dx + 2x^3 dx = 0$.

19. Solve: $p^2 + p(x+y) + xy = 0$ where $p = dy/dx$.

20. Find the singular solution of $y = xp + p^2$.

II. Answer any three questions:

(3x5=15)

1. Find the rank of the matrix A by reducing to the normal form given: $A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 3 & -3 & 4 & 6 \end{pmatrix}$

$$2 \ 3 \ -1 \ -1$$

$$3 \ 1 \ 3 \ -2$$

$$3 \ -3 \ 4$$

$$6 \ 3 \ 0 \ -7$$

2. If $A = \begin{pmatrix} 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ determine two non-singular matrices P and Q such that $PAQ = I$, Hence find A^{-1} .

$$0 \ -1 \ 1$$

3. For what values of λ and μ the equations: $x+y+z=6$ $x+6y+3z=10$ $x+2y+\lambda z=\mu$ have (1) no solution (2) a unique solution (3) infinite number of solutions.

4. Find the eigen values and eigen sectors of the matrix: $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \end{pmatrix}$

5. State and prove the Cayley-Hamilton theorem.

III. Answer any three questions: (2x5=10)

1. Find the angle of intersection of the parabolas $r = a/1 + \cos\theta$ and $r = b/1 - \cos\theta$
2. Show that the p-r equation of the curves $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$ is $r^2 = a^2 - 3p^2$.
3. For the curve $x = x(t), y = y(t)$ show that the radius of curvature. $P = \frac{[x^2 + y^2]^{3/2}}{x\ddot{y} - \dot{y}\ddot{x}}$
4. Prove that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$

IV. Answer any three questions: (2x5=10)

1. Find the points of inflexion on the curve $x = 10g[y/x]$
2. Determine the position and nature of the double points on the curve $u \setminus y(y-6) = x^2(x-2)^3 - 9$.
3. Find all the asymptotes of the curve $4x^2(y-x) + y(y-2)(x-y) = 4x + 4y - 7$.
4. Trace the curve $y^2(a-x) = x^2(a-x)$

V. Answer any three questions: (2x5=10)

1. Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$.
2. Find the surface area generated by revolving an arch of the cycloid $x = a(0 - \sin \theta), y = a(1 - \cos \theta)$ about the x-axis.
3. Find the volume of the solid generated by revolving the cardioid $r = a(1 + \cos \theta)$ about the initial line.

VI. Answer any three questions: (3x5=15)

1. Solve: $dy/dx + y \cos x = y^n \sin 2x$
2. Solve: $dy/dx = x + 2y - 3 / 2x + y - 3$.
3. Solve: $2y dx + (2x \log x - xy) dy = 0$.
4. Show that the family of parabolas $y^2 = 4a(x+a)$ are self orthogonal.

BANGALORE UNIVERSITY

III SEMESTER BSc. MATHEMATICS

MODEL QUESTION PAPER-III

Time: 3 Hours

Max. Marks: 90

I. Answer any fifteen of the following 15x2=30

1. Show that $O(a) = O(xax^{-1})$ in any group G .
2. Define center of a group.
3. Prove that a cyclic group is abelian.
4. How many elements of the cyclic group of order 6 can be used as generator of the group?
5. Let H be a subgroup of group G . Define $K = \{x \in G : xH = Hx\}$. Prove that K is a subgroup of G .
5. Find the index of $H = \{0, 4\}$ in $G = (Z_8, +_8)$.
6. Define convergence of sequence.
7. Find the limit of the sequence $\sqrt{2}, \sqrt{2} \sqrt{2}, \sqrt{2} \sqrt{2} \sqrt{2}, \dots$
8. Verify Cauchy's criterion for the sequence $\{n/n+1\}$
9. Show that a series of positive terms either converges or diverges.
10. Show that $1/1.2 + 1/2.3 + 1/3.4 + \dots$ is convergent.
11. State Raabe's test for convergence.
12. discuss the absolute convergence of $1 - \frac{x^2}{4!} + \frac{x^4}{6!} - \frac{x^6}{8!} + \dots$. When $x^2 = 4$
13. If a and b belongs to positive reals show that $a-b, a+b, +1/3 [a-b, a+b]^3 + 1/5 [a-b/a+b]^5 + \dots$
14. Name the type of discontinuity of $(x) = \begin{cases} 3x+1, & x > 1 \\ 2s-1, & x \leq 1 \end{cases}$
15. State Rolle's theorem.
16. Verify Cauchy's Mean Value theorem for $f(x) = \log x$ and $g(x) = 1/x$ in $\{1, e\}$
17. Evaluate $\lim_{x \rightarrow 0} (\cot x)^{\sin 2x}$
18. Find the Fourier coefficient a_0 in the function $f(x) = \begin{cases} x, & 0 \leq x < \pi \\ 2\pi - x, & \pi \leq x < 2\pi \end{cases}$
19. Find the half range sine series of $(x) = x$ over the interval $(0, \pi)$.

II. Answer any three of the following 3x5=15

1. Prove that in a cyclic group $\langle a \rangle$ of order d , a^k ($k < d$) is also a generator iff $(k, d) = 1$.
2. Show that $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \right\}$ form a cyclic group w.r.t. matrix multiplication.
3. Prove that if H is a subgroup of G then there exist a one-to-one correspondence between any two right cosets of H in G .
4. Find all the distinct cosets of the subgroup $H = \{1, 3, 9\}$ of a group $G = \{1, 2, \dots, 12\}$ w.r.t multiplication mod 13.
5. If a is any integer p is a prime number then prove that $a^p \equiv (a \text{ mod } p)$.

III. Answer any two of the following 2x5=10

1. Discuss the behavior of the sequence $\{(1 + 1/n)^n\}$
2. If $a_n = 3n - 4/4n + 3$ and $|a_n - 3/4| < 1/100$, $n > m$ find m using the definition of the limit.

3. Discuss the convergence of the following sequences whose nth term are (i) $(n^2-1)/8 - (n+1)^{1/4}$ (ii) $[\log(n+1)-\log n]/\tan(1/n)$

IV. Answer any fifteen of the following 2x5=10

1. State and prove P-series test for convergence.
2. Discuss the convergence of the series $1.2/456 + 3.4/6.7.8 + 5.6/8.9.10 + \dots$
3. Discuss the convergence of the series $x^2 \sqrt{1+x^3} \sqrt{3} \sqrt{2} + x^4/4\sqrt{3} + \dots$
4. Show that $\sum \frac{(-1)^n}{(n+1)^p}$ is absolutely convergent if $p > 1$ and conditionally convergent if $p > 1$

5. Sum the series $\sum (n+1)^3/n!$

V. Answer any two of the following 2x5=10

1. If $\lim_{x \rightarrow a} f(x) = L$, $\lim_{x \rightarrow a} g(x) = m$ prove that $\lim_{x \rightarrow a} [f(x) + g(x)] = L + m$
2. State and prove Lagrange's Mean Value theorem.
3. Obtain Maclaurin's expansion for $\log(1 + \sin x)$
4. Find the values of a, b, c such that $\lim_{x \rightarrow a} \frac{x(2+a \cos x) - b \sin x + c}{x^5} = 1/15$

x5

VI. Answer any two of the following 2x5=10

1. Expand $f(x) = x^2$ as a Fourier series in the interval $(-\pi, \pi)$ and hence Show that $1/1^2 + 1/2^2 + 1/3^2 + \dots = \pi^2/6$
2. Find the cosine series of the function $f(x) = \pi - x$ in $0 < x < \pi$.
3. Find the half-range sine series for the function $f(x) = 2x - 1$ in the interval (0,1)

BANGALORE UNIVERSITY

BSc, IV SEMESTER MATHEMATICS

MODEL QUESTION PAPER-1

Time: 3 Hours

Max. Marks: 90

I. Answer any fifteen of the following

2x15=30

1. Prove that every subgroup of an abelian group is normal.
2. Prove that intersection of two normal subgroups of a group is also a normal subgroup.
3. The center Z of a group G is a normal subgroup of G .
4. Define a homomorphism of groups.
5. If $G = \{x + y\sqrt{2} \mid x, y \in \mathbb{Q}\}$ and $f: G \rightarrow G$ is defined by $f(x + y\sqrt{2}) = x - y\sqrt{2}$, show that f is a homomorphism and find its kernel.
6. Show that $f(x, y) = \sqrt{|xy|}$ is not differentiable at $(0, 0)$.
7. Show that $f(x, y) = \tan^{-1} \left[\frac{y}{x} \right]$ at $(1, 1)$ has limit.
8. Prove that there is a minimum value at $(0, 0)$ for the functions $x^3 + y^3 - 3xy$.
9. Show that $\int_0^{\pi} \cos^{10} \theta d\theta = \frac{11}{2} \frac{1}{2}$

10. Prove that $\Gamma(n+1) = n!$

11. Prove that $\int_0^{\infty} \sqrt{x} e^{-x^2} dx = \frac{1}{2} \Gamma\left(\frac{3}{4}\right)$

12. Find the particular integral of $y'' - 2y' + 4y = e^x \cos x$.

13. Show that $x(2x+3)y'' + 3(2x+1)y' + 2y = (x+1)e^x$ is exact.

14. Verify the integrability condition for $yz \log z dx - zx \log z dy + xy dz = 0$.

15. Reduce $x^2 y'' - 2xy' + 3y = x$ to a differential equation with constant coefficients.

16. Find $\int \sin^3 t dt$.

17. Find $\int \frac{1}{(s+2)(s-1)} ds$

18. Define convolution theorem for the functions $f(t)$ and $g(t)$.

19. Find all basic solutions of the system of equations: $3x + 2y + z = 22$, $x + y + 2z = 9$.

20. Solve graphically $x + y \leq 3$, $x - y \geq -3$, $Y \geq 0$, $x \geq -1$, $x \leq 2$.

II. Answer any two of the following

2x5=10

1. Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is also a right coset of H in G .
2. Prove that the product of two normal subgroups of a group is a normal subgroup of the group.
3. If G and G^1 are groups and $F: G \rightarrow G^1$ is a homomorphism with kernel K , prove that K is a normal subgroup of G .
4. If $G = (\mathbb{Z}_6, +6)$, $G^1 = (\mathbb{Z}_2, +2)$ and the function $f: G \rightarrow G^1$ is defined by $f(x) = r$ where r is the remainder obtained by dividing x by 2, then verify whether f is homomorphism. If so, find its kernel. Is f an isomorphism?

III. Answer any three of the following

3x5=15

1. State and prove Taylor's theorem for a function of two variables.
2. Find Maclaurin's expansion of $\log(1+x-y)$.
3. Find the stationary points of the function $f(x, y) = x^3 y^2 (12 - x - y)$ satisfying the condition $x > 0$ and examine their nature.
4. Show that $\int_0^{\infty} \frac{x^4 (1+x^5)}{(1+x)^{15}} dx = \frac{1}{5005}$

OR

If n is a positive integer, prove that $\Gamma^{1(m+1/2)} = \frac{1.3.5.....(2n-1) \sqrt{\pi}}{2^n}$

5. Show that $\int_0^{\pi/2} \sqrt{\sin \theta} d\theta = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sqrt{\sin \theta} d\theta + \int_0^{\pi/2} \frac{1}{\sqrt{\sin \theta}} d\theta$

OR

Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$

IV. Answer any three of the following 3x5=15

1. Solve $y^{11} - 2y^{11} + 4y = e^x \cos x$.
2. Solve $x^3 y^{11} + 2x^2 y + 2y^{11} + 10(x + 1/x)$.
3. Solve $\frac{d^2 y}{dx^2} - (1 + 4e^x) \frac{dy}{dx} + 3e^{2x} y = e^2(x + e^x)$ using changing the independent variable method.
4. Solve $dx/dt + 3x - y, dy/dt = x + y$
5. Solve $dx/x^2 + y^2 + yz = dy/x^2 + y^2 - zx = dz/z(x + y)$

V. Answer any two of the following 2x5=10

1. Find (i) $\int \{2 \sin st \sin 5t/t\}$ (ii) $\int -1 \log s^2 + 1 s(s+1)$
2. Verify convolution theorem for the functions $(t) = et$ and $g(t) = \cos t$.
3. Solve $y^{11} + 2y^1 = 10 \sin tsy$ given $y(0) = z^0, y(0) = 1$ using Laplace transform method.

VI. Answer any two of the following: 2x5=10

1. Find all the basic feasible solutions to the LPP:
Maximize: $z = 2x + 3y + 4z + 7t$

Subject to the constraints: $2x + 3y - z + 4t = 8, x - 2y + 6z - 7t = -3, x, y, z, t > 0$.

2. A quality engineer wants to determine the quantity produced per month of products A and B.

Source	Product A	Product B	Available month
Material	60	120	12,000
Working hours	8	5	600
Assembly man hours	3	4	500
Sale price	Rs. 30	Rs. 40	-

Find the product mix that give maximum profit by graphical method.

3. Using Simplex and method to maximize $f=5x+y+4z$ subject to $x+z<8$, $y+z<3$ $x+y+z<5$.

BANGALORE UNIVERSITY

MATHEMATICS MODEL PAPER-3; B.Sc., FIFTH SEMESTRE

PAPER-V

Time: 3 hours

Max Marks:90

I. Answer any fifteen Questions:

2x15=30

1. In a ring $(R, +, \cdot)$ Prove that $a \cdot 0 = 0 = a \cdot 0$, $\forall a \in R$, where 0 is identity in R.
2. Give an example of a non commutative ring, without unity and with zero divisors.
3. show that Z is not an ideal of the ring $(Q, +, \cdot)$
4. If the additive group of a ring R is cyclic then prove that R is Commutative.
5. With an example, show that union of two sub rings of ring need not be a sub ring.
6. Show that $(z, +, \cdot)$ and $(2z, +, \cdot)$ are not isomorphic defined by $f(x) = 2x \forall x \in z$
7. If $\vec{r}(t) = \vec{a} \cos wt + \vec{b} \sin wt$, show that $\frac{d^2 \vec{r}}{dt^2} = -w^2 \vec{r}$ (a and b are constant vectors).
8. Show that the necessary and sufficient condition for the vector $\vec{a}(t)$ to have constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$.
9. For the space curve $\vec{r} = t\vec{i} + t^2\vec{j} + 2/3t^3\vec{k}$, find the unit tangent vector at $t=1$.
10. Write the Serret-Frenet formula for the space curve $\vec{r} = \vec{r}(s)$.
11. Find the Cartesian co ordinates of the point whose cylindrical coordinates are $(3, 3, 5)$

12. Find the unit normal to the surface $4z = x^2 - y^2$ at $(3, 1, 2)$
13. Prove that $\text{div}(\text{curl } \vec{F}) = 0$
14. Show that $\vec{F} = (\sin y + z)\vec{i} + (x \cos y - z)\vec{j} + (x - y)\vec{k}$ is irrotational
15. If \vec{a} is a constant vector, show that $\text{curl}(\vec{r} \times \vec{a}) = -2\vec{a}$
16. Find \vec{a} , so that $\vec{F} = y(ax^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}$ is solenoidal.
17. Using Rodrigue's formula obtain expressions for $P_0(x)$ and $P_1(x)$
18. Evaluate $\int_{-1}^1 x^3 P_4(x) dx$
19. Starting from the expressions of $J_{1/2}(x)$ and $J_{-1/2}(x)$ in the standard form prove that $\int_0^{\pi/2} \sqrt{x} J_{1/2}(2x) dx = 1/\sqrt{\pi}$
20. Using the expansion of $e^{x/2}(t - 1/t)$, show that $-n(x) = J_n(-x)$.

II Answer any four of the following

4x5=20

1. Prove that $R = \{0, 1, 2, 3, 4, 5\}$ is a commutative ring w.r.t. $(+ \text{ mod } 6)$ and $\times \text{ mod } 6$
2. Show that the intersection of two subrings is subring and give an example to show that the union of two subrings need not be a subring.
3. Prove that an ideal S of the ring of integers $(z, +, \cdot)$ is maximal if and only if S is generated by some prime integer.
4. Define kernel of a ring homomorphism. If $f: R \rightarrow R^1$ be a homomorphism or R into R^1 , then show that $\text{Ker } f$ is an ideal of R.
5. If $f: R \rightarrow R^1$ be an isomorphism of rings R into R^1 , then prove that if R is a field then R^1 is also a field.
6. Define Quotient ring. If I is an ideal of the ring, R, then show that the quotient R/I is homomorphism image of R with I as its Kernel.

III Answer any three of the following:

3x5=15

1. Derive the expressions for curvature and torsion in terms of the derivatives of \vec{r} w.r.t parameter u, where $\vec{r} = \vec{r}(u)$ is the equation of the curve.

- For the space curve $x=t, y=t^2, z=2/3t^3$, find (i) \mathbf{t} and (ii) \mathbf{k} at $t=1$.
 - Find the equations of the tangent plane and normal line to the surface $2z=3x^2+4y^2$ at the point $(2, -1, 8)$
 - Find the constants a and b so that the surface $ax^2-byz=(a+x)x$ will be orthogonal to the surface $4x^2+y+z^3=4$ at $(1, -1, 2)$.
 - Express the vector $\mathbf{f}=zi-2xj+yk$ in terms of spherical polar coordinates and find f_r, f_θ and f_ϕ
- IV. Answer any three of the following: 3x5=15

- Find the directional derivative of $f=x^2yz+4xz^2$ at $(1, 2, -1)$ in the direction of the vector $2i-j-2k$. In what direction the directional derivative is maximum. What is the magnitude of the maximum directional derivative.
- Show that $\Delta \left\{ \frac{f(r)}{r} \right\} = \frac{1}{r^2} \frac{d}{dr} \{ r^2 f(r) \}$ where $r = xi+y+z$
- Show that $\mathbf{F}=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ is irrotational. Find \mathbf{A} such that $\mathbf{F}=\Delta \mathbf{A}$
- For any vector field \mathbf{F} , prove that $\text{curl curl } \mathbf{F} = \Delta (\text{div } \mathbf{F}) - \Delta^2 \mathbf{F}$
- Derive an expression for curl \mathbf{F} in orthogonal curvilinear coordinates.

V. Answer any two of the following (2x5=10)

- Prove that $x^4-3x^2+x=8 \frac{P_4(x)}{35} - \frac{10}{7} P_2(x) + P_1(x) - \frac{4}{5} P_0(x)$.
- Show that $(n+1)P_{n+1}(x) = (2n+1)x P_n(x) - n P_{(n-1)}(x)$
- Prove that $J_{1/2}(x) = \frac{\sqrt{2}}{\pi x} \sin x$
- Show that $J_0(x) = \frac{1}{\pi} \int_0^\pi \cos(xs \sin \theta) d\theta = \frac{1}{\pi} \int_0^\pi \cos(x \cos \theta) d\theta$

OR

Prove that $\cos(xs \sin \theta) = J_0(x) + 2 \sum_{n=1}^{\infty} J_{2n}(x) \cos 2n\theta$

BANGALORE UNIVERSITY

MATHEMATICS MODEL PAPER 3:B.Sc, FIFTH SEMESTER

PAPER-VI

Time: 3 hours

Max Marks: 90

I Answer any fifteen questions:

1. Form a partial differential equation by eliminating the arbitrary constants $(x-a)^2+(y-b)^2+z^2=r^2$
2. Solve $pq=k$
3. Solve $p^2=qz$
4. Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 12xy$
5. Use the method of separation of variable solve $x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$

6. Evaluate $\Delta e^{3x} \log 4x$
7. Prove that $\frac{\Delta}{\Delta} - \frac{\Delta}{\Delta} = \Delta + \Delta$

8. Estimate the missing term from the table

X	0	1	2	3	4
y	1	3	9	...	81

9. Write the Lagrange's inverse interpolation formula

10. Evaluate $\int \frac{dx}{1+x}$ using Simpson's 1/3 rule

11. If a particle of mass 2 units moves along the space curve defined by $r \rightarrow (4t^2 - t^3)\mathbf{i} - 5t\mathbf{j} + (t^4 - 2)\mathbf{k}$, find its kinetic energy at $t=1$.
12. In a SHM if $\rightarrow f$ the acceleration, u the velocity at any instant and T is periodic time, show that $f^2 T^2 + 4\pi^2 v^2$ is a constant.
13. If man can throw a stone to a distance of 100m. How long it is in the air.
14. Prove that if the time of flight of a bullet over a horizontal range R is T , the inclination of projection to the horizontal is $\tan^{-1}\left(\frac{gT^2}{2R}\right)$

15. If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.
16. When a particle of mass m moves outside a smooth circle of radius r , mention the equation of motion at any point on it.
17. Find the law of force when the particle describes $r=a \cos\theta$ under the action of central force.
18. Define Apsidal distance and Apsidal angle.
19. A system consists of masses 1,2,3 and 4 units moving with velocities $8\mathbf{i}$, $7\mathbf{i}$, $3\mathbf{k}$ and $2\mathbf{i}+3\mathbf{j}-\mathbf{k}$ respectively. Determine the velocity of the mass centre.
20. Briefly explain the angular momentum of a system of particles.

II. Answer any three of the following:

3x5=15

1. Form the partial differential equation by eliminating the arbitrary function $z=y f(x+xy)$ (y

y)

x)

2. Solve $(x^2+y^2)(p^2+q^2)=1$
3. Solve $x(x^2+3y^2)p-y(3x^2+y^2)q=2z(y^2-x^2)$

OR

Solve by Charpit's method $q=(z+px)^2$

4. Solve $(D^2-DD^1)z=\cos x \cos 2y$
5. Reduce the equation to canonical form $x^2(y-1)r-x(y^2-1)s+y(y-1)t+xy p-q=0$
6. Solve by using the method of separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3(x^2+y^2)u$

OR

An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at a, find the temperature at a distance x from A at time t .

III. Answer any three of the following:

3x5=15

1. Show that the n th differences of a polynomial of the n th degree are constant and all higher order differences are zero.

OR

By separation of symbols prove that $4x - \frac{1}{8} \Delta^2 4_{x-1} + \frac{1.3}{8.16} \Delta^4 4_{x-2} - \frac{1.3.5}{8.16.24} \Delta^6 4_{x-3} = 4x - \frac{1}{8} \Delta^2 4_{x-1/2} + \frac{1}{8} \Delta^4 4_{x-1/2} - \frac{1}{8} \Delta^6 4_{x-1/2}$

$\frac{1}{4} \Delta^4 4_{x-1/2} + \frac{1}{8} \Delta^2 4_{x-1/2} + \frac{1}{8} \Delta^3 4_{x-1/2}$

2. Evaluate $y=e^x$ for $x=0.05$ from the following table

X	0.00	0.10	0.20	0.30	0.40
Y=e ^x	1.000	1.2214	1.4918	1.8221	2.255

3. Use Lagrange's formula find a polynomial to the following data and hence find $f(2)$

X	0	1	3	4
F(x)	-12	0	6	12

4. Find (5) using $f(1)=3$, $f(3)=31$, $f(6)=223$, $f(10)=1011$ and $f(11)=1343$
5. Find the value of $51 \int_1^x \log_{10} x \, dx$ taking 8 sub-intervals correct to four decimal places by Trapezoidal rule.

IV. Answer any four of the following:

4x5=20

1. A particle of mass 2 moves in a force field $\rightarrow F=24t^2 i+(36t-16)j-12tk$. Find (i) the kinetic energy at $t=1$ and $t=2$ (ii) work done in moving the particle from $t=1$ to $t=2$.

- If the displacement of a particle moving in a straight line is expressed by the equation $x = a \cos nt + b \sin nt$. Show that it describes a SHM. Also find its (i) amplitude (ii) periodic time.
- A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and γ the angle of projection, prove that $\tan \gamma = \tan A + \tan B$.
- The angular elevation of an enemy's position on a hill h feet high is β . Show that in order to shell it the initial velocity of the projectile must not be less than $\sqrt{hg(1 + \operatorname{cosec} \beta)}$
- The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr^2 and $\mu \dot{\theta}$. Show that the equation to the path is $\frac{\lambda}{r} = \frac{\mu}{r} + c$ and components of

$$\ddot{\theta} = 2r^2$$

acceleration are $2\lambda r^2 \ddot{r} - \frac{\mu^2 \dot{\theta}^4}{r}$ and $2\lambda \mu \dot{\theta}^2 - \frac{\mu^2 \dot{\theta}^3}{r}$

- A particle projected along the inner side of a smooth circle of radius a , the velocity at the lower point being u . Show that if $2a < u^2 < 5ag$ the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is $\frac{2(u^2 - 2ag)^3}{27a^2 g^3}$

V. Answer any two of the following:

2x5=10

- Derive the differential equation of a central orbit in pedal form $h^2 p^3 (dp/dr) = f$
- A particle describes the cardioid $r = a(1 + \cos \theta)$ under a central force to the pole. Find the law of force.
- A particle moves with a central acceleration $\mu(r + a^4/r^3)$ being projected from an apse at a distance 'a' with velocity $2\sqrt{\mu a}$. Prove that it describes the curve $r^2(2 + \cos \sqrt{3}\theta) = 3a^2$
- Define mass centre of a system of particles and show that the linear momentum of a system of particles relative to its mass centre is zero.

BANGALORE UNIVERSITY

VI Semester B.Sc.,

MATHEMATICS (Paper VII)

Model Paper-I

Time: 3 hours

Max. Marks:

90

I. Answer any fifteen questions:

2x15=30

1. Define vector space over a field.
2. Prove that the subset $W = \{x, y, z \mid x=y=z\}$ is subspace of $V_3(\mathbb{R})$.
3. Prove that the set $S = \{(1,0,0), (0,1,0), (0,0,1)\}$ is linearly independent in $V_3(\mathbb{R})$.
4. Show that the vectors $\{(1,2,1), (2,1,0), (1,-1,2)\}$ form a basis of $V_3(\mathbb{R})$.
5. Prove that $T: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x,y,z) = (x,y)$ is a linear transformation.
6. Find the matrix of the linear transformation $T: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x,y,z) = (x+y, y+z)$ w.r.t. standard basis.
7. Define Rank and Nullity of the linear transformation.
8. Evaluate $\int (x+y) dx + (y-x) dy$ along the parabola $y^2 = x$ from $(1,1)$ to $(4,2)$.
9. Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dy dx}{1+x^2+y^2}$.
10. Find the area bounded by one loop of the lemniscates $r^2 = a^2 \cos 2\theta$ by double integration.
11. If A is the region representing the projection of a surface S on the zx plane, write the formula for the surface area of S .
12. Prove that $\int_0^2 \int_{-1}^1 \int_1^2 (x^2 + y^2 + z^2) dz dy dx = 6$.
13. Find the total work done by a force $F = 2xyi - 4zj + 5xk$ along the curve $x = t^2, y = 2t + 1, z = t^3$ from $t = 1$ to $t = 2$.
14. $F = e^x \sin y i + e^x \cos y j$, evaluate $\int_{\rightarrow} F \cdot d\rightarrow r$ along the line joining from $(0,0)$ to $(1,0)$.
15. State Gauss Divergence theorem.
16. Evaluate using Green's theorem $\int_C (x^2 - 2xy) dx + (x^2 + 3) dy$ around the boundary of the region defined by $y^2 = 8x$ and $x = 2$.
17. Using Stokes theorem Prove that $\int_C \rightarrow r \cdot d\rightarrow r = 0$.
18. Write the Euler's equation when f does not contain y explicitly.
19. Show that the Euler's equation for the extremism of $\int_{x_1}^{x_2} (y^2 + y^{12} + 2ye^x) dx$ reduces to $y^{11} - y = e^x$.
20. Prove that the shortest distance between two points in a plane is along a straight line.

II. Answer any four of the following:

4x5=20

1. Prove that the set $V = \{z + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$, \mathbb{Q} the field of rationals forms a vector space w.r.t addition and multiplication of rational numbers.
2. Prove that:
 - i) the subset $W = \{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 + x_3 = 0\}$ of the vector space $V_3(\mathbb{R})$ is a subspace of $V_3(\mathbb{R})$.
 - ii) the subset $W = \{(x_1, x_2, x_3) \mid x_1^2 + x_2^2 = x_3 \leq 1\}$ of $V_3(\mathbb{R})$ is not subspace of $V_3(\mathbb{R})$.
3. Find the dimension and basis of the subspace spanned by $(1,2,3,4), (1,5-2,4), (1,3,2,4)$ and $(1,6,-3,4)$ in $V_4(\mathbb{R})$.
4. If matrix of T w.r.t basis B_1 and B_2 is $\begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ where $B_1 = \{(1,2,0), (0,-1,0), (1,-1,1)\}$,

$B_2 = \{(1,0), (2,-1)\}$ then find $T(x,y,z)$.

- Let $T:u \rightarrow v$ be a linear transformation. Then prove that (i) $R(T)$ is a subspace of V . (ii) $N(T)$ is a subspace of u .
- Find all eigen values and a basis for each eigen space of the linear transformation $T:R^3 \rightarrow R^3$ defined by $T(x,y,z) = (x+y+z, 2y+z, 2y+3z)$.

III. Answer any three of the following: 3x5=15.

1. Prove that $\oint_C (x^2 - y^2) dx + x^3 y dy = 56\pi$ where C is the semicircle with centre $(0,4)$ and radius $\frac{2}{2}$ units.

2. Evaluate $\int_R xy(x+y) dx dy$ over the region R bounded between parabola $y=x^2$ and the line $y=x$.

3. Evaluate $\int_0^a \int_0^a \frac{xy dx dy}{x^2 + y^2}$ by changing the order of integration.

4. Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

5. Find the volume common to the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinders $x^2 + y^2 = ax$.

IV. Answer any three of the following: 3x5=15

1. Evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where S denotes the part of the plane $2x+y+2z=6$ which lies in the positive octant and $\vec{F} = 4xi + yj + z^2k$.

2. State and prove Green's theorem in the plane.

3. Evaluate $\iiint_V \text{div } F dv$ where $F = 2x^2yi - y^2j + 4xz^2k$ and V is region in the first octant bounded by the cylinder $y^2 + z^2 = 9$ and $x=2$.

4. Using divergence theorem evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where $F = 4xi - 2y^2j + z^2k$ and S is the surface enclosing the region for which $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$.

5. Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)i + 2xyj$ over the rectangular box bounded by the planes $x=0, x=a, y=0, y=b, z=0, z=c$ with the face $z=0$ removed.

V. Answer any two of the following: 2x5=10

7. Prove the necessary condition for the integral $I = \int_{x_1}^{x_2} f(x, y, y') dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is that $\frac{\partial f}{\partial y} + \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$.

1. Find the extremal of the functional $I = \int_0^4 \sqrt{y(1+y'^2)} dx; y(0)=1, y(4)=5$.

2. Find the geodesics on a right circular cone.

3. Show that the extremal of the functional $\int_0^2 \sqrt{1+(y')^2} dx$ subject to the constraint $\int_0^2 y dx = \pi/2$ and end conditions $y(0)=0, y(2)=0$ is a circular arc.

BANGALORE UNIVERSITY

VI Semester B.Sc.,

MATHEMATICS (Paper VIII)

Model Paper-I

Time: 3 hours

Max. Marks: 90

I. Answer any fifteen questions:

2x15=30

1. Find the locus of the point z , satisfying $|z-i/z+i| > 2$.
2. Evaluate $\lim_{z \rightarrow e^{i\pi/4}} \frac{z^2}{z^4+z^2+1}$.
3. Show that $f(z)=e^y(\cos x+isinx)$ is not analytic.
4. Find the harmonic conjugate of $u=e^x \sin y+x^2-y^2$.
4. Show that $w=e^z$ is a conformal transformation.
5. Find the fixed points of the transformation $w = \frac{1-z}{1+z}$.
6. Evaluate $\int_{(0,1)}^{(2,5)} (3x+y)dx+(xy-x)dy$ along the curve $y=x^2+1$.
7. State Cauchy's integral formula.
8. Evaluate $\oint_C \frac{e^{2z} dz}{cz+2i}$ where C is the unit circle with centre at origin.
9. State Lovell's theorem.
10. Prove that $F\{f(t-a)\} = e^{i\lambda a} f(\lambda)$.
11. Write (i) cosine form of Fourier integral, (ii) sine form of Fourier Integral.
12. Find the Fourier cosine transform of $f(x) = \begin{cases} x_0 & 0 < x < 2 \\ \text{otherwise} & \end{cases}$
13. Find the Fourier sine transform of $f(x) = 1/x, x > 0$.
14. Prove that $F_s[f'(x)] = -\lambda F_c[f(x)]$.
15. Using Regula Falsi method, find the fifth root of 10 using $x_0=0, x_1=1$ in two steps.
16. Write the general formula for secant method.
17. Find the greatest value $\frac{12}{23}$ using power method. Do two steps only.
18. Using Taylor's series method find y at $x=1.1$ considering terms up to second degree, given that $dy/dx=2+y$ and $y(1)=0$.
19. Using Eulers method solve $dy/dx=x+y$ with the initial value $y(0)=1$ for $x=0.1$ in two steps.

II. Answer any four of the following:

4x5=20

- a. State and prove the necessary condition for the function $f(z)$ to be analytic.
- b. If $f(z)$ is analytic function of z , prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} |f(z)|^2 = 2|f'(z)|^2$
- c. If $f(z)=u+iv$ and $u-v=e^x(\cos y-siny)$, find $f(z)$ in terms of z .
- d. Show that the transformation $W=zw$ transforms the circle $|z-a|=r$ on to a lima on or cardioids.
- e. Prove that a bilinear transformation transforms circles into circles or straight lines.
- f. Find the bilinear transformation which maps $z=\infty, 1, 0$ on to $w=0, i, \infty$

III. Answer any two of the following:

2x5=10

1. Evaluate $\int_C [x(x+y)dx + x^2ydy]$ along:
 - i) the Straight line $y=3x$ from $(0,0)$ to $(3,9)$
 - ii) the parabola $y=x^2$ between $(0,0)$ to $(3,9)$.
2. State and prove Cauchy's integral formula.
3. Evaluate $\oint_C \frac{z}{(z^2+1)(z^2-9)} dz$ where $C:|z|=2$.
4. State and prove Cauchy's inequality.

IV Answer any three of the following

3x5=15

1. Using fourier integral formula, show that $f(x) = \frac{2}{\pi} \int_0^\infty \frac{1 - \cos \pi s x}{s} ds$ Where $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq \pi \\ 0 & \text{for } x > \pi \end{cases}$
2. Find the Fourier transform of $f(x) = \begin{cases} 1 - |x| & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ and hence deduce that $\int_0^\infty \frac{\sin^2 t}{t^2} dt = \pi/2$
3. Find the Fourier sine transform of e^{-ax}/x , ($a > 0$).
4. Find the Fourier cosine transform of $f(x) = x^{n-1} \cos x$ ($n \geq 2$).
5. Assuming $f(x) \rightarrow 0$ as $x \rightarrow \infty$, Prove that $F_c\{f'(x)\} = -\sqrt{2/\pi} f(0) + \sqrt{2/\pi} F_s\{f(x)\}$ and $F_c\{f''(x)\} = -\sqrt{2/\pi} f'(0) + \sqrt{2/\pi} F_s\{f'(x)\}$

V. Answer any three of the following:

3x5=15

1. Find a real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0,1)$ perform 4 iterations of the secant method.
2. Find a real root of the equation $x e^x - 2 = 0$ correct to three decimal place
3. Solve the system of equation by Gauss-Seidel method. $10x + y + z = 12, x + 10y + z = 12, x + y + 10z = 12$
4. Use power method to find the largest eigen value of the matrix $A = \begin{bmatrix} -4 & -5 \\ 1 & 2 \end{bmatrix}$
5. By using Runge-Kutta method. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $Y(0)=1$. Compute $y(0.2)$ by taking

$h=0.2$

B.Sc. Course in Statistics

First Semester: Theory Paper

ST P1. BASIC STATISTICS AND PROBABILITY

Unit-1

Unvaried Data Analysis: Basic Statistical concepts-Population, Sample, Variable and Attribute. Types of Data: Qualitative and Quantitative, Nominal and Ordinal; Cross sectional and Time series; discrete and continuous. Types of scales: ratio and interval. Frequency Distribution, Graphical presentation-Histogram, frequency polygon, frequency curve and ogives. Stem and Leaf chart, Turkey's Box plot. Central tendency or location, partition values-Quartiles, Deciles and Percentiles, Dispersion and relative dispersion, moments, Skewness and Kurtosis and their measures and properties. (20h)

Unit-2

Bivariate Data Analysis: Related variables, Scatter diagram, Principle of least squares. Fitting of linear, quadratic, exponential and geometric curves. Product moment correlation coefficient and its properties. Coefficient of determination and its interpretation. Regression equations, Regression coefficients and their properties. Rank correlation-Spearman's and Kendall's measures. (12h)

Unit-3

Time Series Analysis: Components of Time series. Additive and multiplicative models. Measurement of trend by moving averages and by least squares. Construction of seasonal indices by simple averages and ratio to moving averages. (06h)

Unit-4

Index Numbers: Meaning and Applications. Price and quantity relatives, link and chain relatives. Construction of Index numbers: their computation and interpretation. Simple, aggregative and weighted average methods Laspeyres, Paasche's, Marshall-Edge worth's, Drobish-Bowley and Fisher's Index numbers. Time and factor reversal tests, Consumer price Index. (06h)

Unit-5

Probability: Basic concepts-trial, sample point and sample space, simple, composite and compound events, mutually exclusive events and exhaustive events. Classical, empirical and axiomatic approaches to probability. Elementary properties of probability. Additive law, conditional probability, independence of events, multiplicative law. Bayes's theorem and its applications. (16h)

Total 60hrs

Practical Paper I

1. Construction of frequency distribution and graphical presentation:
2. Measures of central tendency: (Arithmetic, Geometric, Harmonic and Weighted means).
3. Measures of location: (Median and Mode)

4. Measures of dispersion-1: (Range, Mean deviation and Quartile deviation).
5. Measures of dispersion-2 (Standard deviation and coefficient of variation).
6. Moments, measures of skewness and kurtosis.
7. Fitting first and second degree curve by the method of least squares.
8. Fitting exponential and geometric curves by the method of least squares.
9. Correlation and regression for ungrouped data.
10. Correlation and regression for grouped data.
11. Rank correlation-Spearman's and Kendall's measures.
12. Determination of secular trend by moving averages and least squares methods.
13. Measurement of seasonal variation by simple averages and ratio to moving averages.
14. Construction and tests for index numbers.
15. Construction of Consumer Price Index-interpretation.
16. Computation of probabilities using combinatorial methods and Bayes' formula.

Books for Study

1. Bhat B.R. Srivenkataramana T and Rao Madhava K.S. (1996): Statistics: A Beginner's Text Vols I and II, New Age International (P) Ltd.
2. Croxton F.E, Cowden D.J and Kelin S (1973): Applied General Statistics. PHI.
3. Freund JE and Walpole RE (1987) Mathematical Statistics (4th edition)PHI.
4. Goon A.M. Gupta M.K. Das Gupta B. (1991): Fundamentals of Statistics Vol.I, World Press, Calcutta.
5. Gupts, S.C. and V.K. Kapoor (2001): Fundamentals of Mathematical Statistics: Sultan Chand & Sons.
6. Medhi J (1992): Statistical Methods: An introductory text. New Age.

References:

1. Anderson T.W. and Sclove S.L. (1978) An Introduction to the Statistical Analysis of Data, Houghton Mifflin & Co.
2. Cooke, Cramer and Clarke: Basic Statistical Computing, Chapman and Hall.
3. Mood A.M. Graybill F.A. and Boes D.C. (1974): Introduction to the Theory of Statistics, McGraw Hill.
4. Snedecor G.W. and Cochran W.G. (1967): Statistical Methods. Iowa State University Press.
5. Spiegel, M.R. (1967): Theory & Problems of Statistics, Schaum's Publishing Series.

Second Semester: Theory Paper

ST P2. PROBABILITY DISTRIBUTIONS AND MULTIPLE REGRESSION

Unit-1:

Random variables: Distribution function and its properties. Discrete and continuous random variables. Probability mass function and probability density function. Expectation, variance and moments. Moment generating function (m.g.f) and its properties. (10h)

Unit-2

Standard discrete distributions: Uniform, Bernoulli. Binomial, Poisson, negative binomial, geometric and hyper geometric distributions –mean, variance, moments and m.g.f. Recursive relations for probabilities and moments for binomial, Poisson and negative binomial distributions. Additive property. (16h)

Unit-3:

Standard continuous distributions: Uniform, exponential, gamma and beta distributions- definition through p.d.f.s, Mean, variance, moments and m.g.f. Additive property of exponential and gamma variates. Normal distribution and its properties. Cauchy distribution. Transformation of univariate random variables-discrete and continuous. (14h)

Unit-4:

Bivariate distributions: Joint, marginal, conditional distributions for discrete and continuous variates. Conditional expectation. Covariance and correlation coefficient. Independence of random variables. Addition and multiplication theorems of expectation. Mean and variance of linear combination of random variables. (10h)

Unit-5:

Multiple Regression: Trivariate data Equation to the plane of regression. Properties of residuals and residual variance. Multiple correlation and partial correlation coefficients. Derivation and their properties. Coefficient of multiple determination. (10h)

Total 60hrs.

Practical Paper II:

1. Expectation, moments, skewness and kurtosis for a probability distribution.
2. Bivariate distribution-marginal and conditional distributions.
3. Computation of probabilities based on hypergeometric, geometric and negative binomial distributions.
4. Computation of probabilities based on binomial and Poisson distributions.
5. Fitting of binomial distribution and computing expected frequencies-testing goodness of fit.
6. Fitting of Poisson distribution and computing expected frequencies-testing goodness of fit.
7. Fitting of negative binomial distribution and computation of the expected frequencies-testing goodness of fit.
8. Computation of probabilities based on normal distribution.
9. Fitting of normal distribution by area method and computing expected frequencies-goodness of fit test.
10. Fitting of multiple regression plane and prediction.
11. Computation of multiple and partial correlation coefficients. Coefficient of determination-interpretation.
12. Computation of Residual variance-interpretation.

Book for Study:

- a. Chandra. T K. and Chatterjee. D (2001) A First course in Probability. Narosa
- b. Hogg. R.V. and Craig.A.T. (1978) Introduction to Mathematical Statistics-4/e, Macmillan.
- c. Mood. A.M. Graybill F and Boes (1974): Introduction to the Theory of Statistics. McGrawHill.
- d. Mukhopadhyay P. (196): Mathematical Statistics. Calcutta Publishing House.
- e. Gupta S.C, and V.K. Kapoor (2001): Fundamentals of Mathematical Statistics. Sultan Chand & Co.
- f. Walpoel, R.E and Myers, R.H and Myers S.L (198): Probability and Statistics for Engineers and Scientists. 6th Edition, Prentice Hall, New Jersey.

References:

- i. Bhattacharya and N.L. Johnson (1986): Statistical concepts. John Wiley.
- ii. Dudewicz E.J. and Mishra S.N. (1980). Modern Mathematical Statistics. John Wiley..
- iii. Lindgren, B.W. (1996): Statistical Theory Collier Macmillan Int. Ed, 3rd Ed.,
- iv. Rohatgi, V.K. and A.K. Md. Ehsanes Saleh (2002). An Introduction to Probability theory and Mathematical Statistics. John Wiley. (WSE)
- v. Schaum Series:Probability and Statistics.

Third Semester: Theory Paper

ST P3, SAMPLING DISTRIBUTIONS AND ESTIMATION

Unit-1

Sampling Distributions: Sampling from a distribution, Definition of a random sample. Basic concepts of Statistic, Sampling distribution and Standard error. Definition of Chi-square, t and F distributions through pdf-their properties and uses. Sampling distributions of sample mean, sample variance, Student's and F-statistics under normality assumption. Statement of interrelations between Chi square, t and F statistics. Independence of sample mean & variance in random sampling from a Normal distribution. (18h)

Unit-2

Limit Thorems: Chebyshev's inequality-proof and its use in approximating probabilities. Convergence in probability. Weak law of large numbers. Central limit theorem. DeMoivre-Laplace and very-Lindbergh theorems. Proof and applications. (10h)

Unit-3

Point Estimation: Concepts of parameter, estimator, estimate and standard error of an estimator. Unbiasedness. Mean squared error as a criterion for comparing estimators. Relative efficiency. Minimum variance unbiased estimator. Consistency-definition and criteria for consistency Sufficient Statistic. Fisher-Neyman criterion and Neyman-Factorization theorem. Measure of information-Fisher information function. Cramer-Rao inequality (without proof) and its application in the construction of minimum variance unbiased estimators. (16h)

Unit-4

Methods of Estimation:- Maximum likelihood and moment methods. Standard examples. Illustration for no uniqueness of MLEs. Properties of MLE and MME. (04h)

Unit-5

Interval Estimation: Meaning of confidence interval and pivotal quantity. Confidence interval based on pivotal quantity. Confidence coefficient. Confidence intervals for mean, difference between two means, variance and ratio of variances under normality. Large sample confidence intervals for a proportion and correlation coefficient. (08h)

Practical Paper III

- a. Drawing random samples from binomial and Poisson distributions.
- b. Drawing random samples from uniform and normal distributions.

- c. Drawing random samples from Cauchy and exponential distributions.
- d. Construction of sampling distribution of sample mean and sample variance
- e. Applications of Chebychev's inequality.
- f. Operation of Central Limit Theorem: Bowl drawing experiments.
- g. Applications of Central Limit theorem.
- h. Comparison of estimators by plotting mean square error.
- i. Estimation of parameters by maximum Likelihood method-Set I.
- j. Estimation of parameters by Maximum Likelihood method-Set II.
- k. Estimation of parameters by method of moments.

Books for Study:

1. Freund J.E. (2001): Mathematical Statistics, Prentice Hall of India.
2. Goon A.M. Gupts M.K. Das Gupta B. (1991): Fundamentals of Statistics, Vol.I, World Press, Calcutta.
3. Hogg R.V. and Tannis E.A. (1988): Probability and Statistical Inference, Collier MacMillan.
4. Hodges J.L and Lehman E.L (1974): Basic Concepts of Probability and Statistics, Holden Day.
5. Mood A.M., Graybill F. A and Boes D.C. (1974): Introduction to the Theory of Statistics, McGraw Hill.

References

- i. Bhattacharya and Johnson (1986): Statistical Concepts, Wiley Int. Ed.
- ii. Rohatgi. V.K. and A.K. Md. Ehasnes Saleh (2002). An Introduction to Probability theory and Mathematical Statistics. John Wiley (WSE).
- iii. Ross. S.M: Introduction to Probability and Statistics. John Wiley & Sons

Fourth Semester: Theory Paper

ST P4. STATISTICAL INFERENCE

Unit-1:

Testing of Hypotheses: Statistical hypotheses-null and alternative, simple and composite hypotheses. Critical region and critical function. Type-I and Type-2 errors, level of significance, size and power of a test. Most Powerful (MP) test. Statement of Neyman-Pearson Lemma and its use in the construction of MP tests. (12h)

Unit-2:

UMP and Likelihood ratio tests: Monotone likelihood ratio (MLR) property. Uniformly most powerful (UMP) test. Statement of the theorem on UMP tests for testing one sided hypotheses for distributions with MLR property. Likelihood ratio tests (LRT). Large sample approximation to the distribution of the likelihood ratio statistics (without proof). LRT for single mean for normal case (large and small samples). (12h)

Unit-3:

Tests of Significance: Tests for the mean, equality of two means, variance and equality of two variances (for large and small samples). Large sample tests for proportions. Test for correlation coefficients-simple, multiple and partial. Test for regression coefficient-Fisher's Z-transform and its applications. Chi-square test for goodness of fit and for independence of attributes in contingency tables. (18h)

Unit-5

Non-Parametric tests: Need Sign test for quintiles. Sign test based on paired observations. Wilcoxon signed rank test. Kolmogorov-Smirnov one sample test. Runs test, median test and Mann-Whitney-Wilcoxon-test for two sample problems. Runs test for randomness. Test for independence based on Spearman's rank correlation coefficient (Large sample approximation to the distribution of the statistics is to be used in all (cases). Tests for normality Q-Q plot and some simple tests. (10h)

Total 60hrs

Practical Paper IV:

1. Evaluation of Probabilities of Type-I and Type-II errors and Power of tests. (Based on binomial, Poisson, uniform and normal distributions). Power curve for testing the mean of normal distribution (with known variance).
2. Construction of M.P. tests and computation of power of tests based on binomial, Poisson and Normal distributions.
3. UMP test for the mean of exponential distribution and the power curve.
4. UMP test for the mean of normal distribution (with known variance) and power curve.
5. Tests for single mean, equality of means when variance is (i) known (ii) unknown, under Normality (both for small and large samples)
6. Tests for single proportion and equality of two proportions.
7. Tests for single variance and equality of two variances under normality
8. Tests for correlation coefficient.
9. Tests for independence of attributes in contingency tables.
10. SPRT for proportions –OC and ASN.
11. SPRT for mean of normal distribution –OC and ASN.
12. Nonparametric tests-I
13. Nonparametric tests-II.

Books for Study:

- a. Hogg R.V. and Craig, A.T. (1978). Introduction to Mathematical Statistics-4/e, Macmillan, New York.
- b. Goon, A.M. Gupta, M.K. and Das Gupta B. (1986) Fundamentals of Statistics. Vo.. I 6/e. World Press Calcutta.
- c. Mood A.M. Graybill F and Boes (1974): Introduction to the theory of Statistics. McGraw Hill.
- d. Mukhopadhyay. P (1996). Mathematical Statistics. Calcutta Publishing House.
- e. Gupta. S.C. and V.K.Kapoor (2001): Fundamentals of Mathematical Statistics Sultan Chand & co.

References:

1. Bhattacharya and N.L. Johnson: Statistical concepts. John Wiley.
2. Dudwicz E.J. and Mishra S.N. (1980) Modern Mathematical Statistics. John Wiley.
3. Kale B.K (199) A First Course on Parametric Inference, Narosa
4. Randles R.H. and Wolfe DA (1979): Introduction to the Theory of nonparametric Statistics, John Wiley.
5. Rohatgi. V.K. and A.K Md. Ehsanes Saieh (2002). An Introduction to Probability theory and Mathematical Statistics. John Wiley. (WSE)

Fifth Semester Theory Paper-1

ST P5: SAMPLING THEORY AND APPLICATIONS

Unit-1

Basics: Concepts of population and sample. Need for sampling-complete enumeration vs sample surveys. Non-probability and probability sampling-meaning, need and illustrations. Methods of drawing random samples-Lottery system, Use of random numbers. Bias, accuracy and precision of the estimates. 10 hours

Unit-2

Simple random sampling: Sampling with and without replacement. Unbiased estimators of population mean and total. Derivation of sampling variances. Standard errors of the estimators. Confidence limits. Sampling for proportions Derivation of the variances of the estimators and their estimation. Determination of sample size. Formulas for sample size in sampling for proportions and means. 10 hours.

Unit-3

Stratified random sampling: Need for stratification. Unbiased estimator of mean and total in stratified random sampling. Derivation of the SE's and their estimation. Allocation of sample size under proportional, optimum and Neyman allocations. Comparison of $V(\text{ran})$, $V(\text{prop})$ and $V(\text{opt})$ ignoring fpc. Estimation of gain in precision due to stratification. 10hours

Unit-4

Linear systematic sampling: Advantages and limitations. Estimation of mean and standard error of the estimate. Comparison with sample random and stratified random sampling.

Unit-5

Survey methods: Principal steps in a sample survey-Planning, execution, analysis and reporting stages. Requisites of a good questionnaire. Drafting of questionnaires and schedules and their pre-test. Pilot surveys. Non-sampling errors and simple methods of controlling them.

Working of national sample survey organization. Applications of environmental studies. Ecological measurements-density, frequency, biomass, coverage. Ecological sampling-plot sampling transect sampling. Point-quarter sampling capture-recapture sampling. 12 hours

(50 Hours)

Practical Paper_V

Part A:

1. Drawing random samples using random number tables (grouped and ungrouped cases).
2. Listing of all possible SRSWR and SRSWOR from a given population and verifying that the estimators of the mean, total and the sampling variance of the estimator are unbiased.
3. Drawing of random sample under SRSWR and SRSWOR design from a given population and estimation of the mean and total and the standard error of the estimators.
4. Estimation of the proportion and the standard error of the estimator under SRSWR and SRSWOR designs.

5. Estimation of the mean, total and the standard error of the estimators under stratified random sampling.
6. Allocation of sample size under stratified random sampling. Comparison of the precisions of the estimators under stratified random sampling with proportional and optimum allocations and that under SRSWOR.
7. Estimation of gain in precision due to stratification.
8. Listing of possible systematic samples from a given population and computation of variance of the estimator and its comparison with that of SRSWOR.
9. Design of questionnaires and their pretest.

Part-B:

Project Work: Survey Proposal, data collection, Analysis and Report.

(50% of the record marks in Practical Paper V are assigned to Project work)

Books for Study

- 1) Cochran, W.G. (1984): Sampling Techniques. (3rd ed.) (Wiley Eastern)
- 2) Singh, D and Chaudhary, F.S (1986): Theory and Analysis of sample survey design. (Wiley Eastern).
- 3) Goon, A.M. et.al: Fundamentals of Statistics Vol II (World Press, Calcutta).
- 4) Gupta, S.C and V.K Kapur: Fundamentals of Applied Statistics. (Sultan Chand and Co.)

Reference Books:

- 1) Murthy M.N. (1967): Sampling theory and methods. (Statistical Society, ISI, Kolkata)
- 2) Des Raj and Chandok (1998): Sampling Theory, Narosa, New Delhi.
- 3) Sukhatme, P.V. et. Al (1984): Sampling theory of surveys with applications (Indian Society of Agricultural Statistics, New Delhi)
- 4) Sampath: Sampling Theory Narosa Pub.

Fifth Semester Theory Paper-2

(Elective Paper 10

ST P 6.1: STATISTICAL METHODS FOR QUALITY MANAGEMENT

Unit-1

Basics: Quality assurance and management. Quality Pioneers. Quality costs. Aims and objectives of statistical process control. Chance and assignable causes of variation. Statistical quality control. Process control, product control. Importance of statistical quality control in Industry. 10 hours

Unit-2

Chart for variables: Theoretical basis and practical background of control charts for variables. 3 sigma limits, warning limits and probability limits. Criteria for detecting lack of control. Derivation of limits and construction of X-R charts and interpretation. 8hours.

Unit-3

Control charts for attributes: Rational subgroups. Group control charts and sloping control charts. Natural tolerance limits and specification limits. Process capability studies.

6hours

Unit-5

Product Control: Sampling inspection and 100 percent inspection. AQL, LTPD, Producer's risk and consumer's risk. Acceptance sampling. Sampling plans-single and double sampling plans by attributes. Derivation of O.C A.T.I. A.O.Q and A.S.N functions. Construction of single sampling plans by attributes given (i) AQL, LTPD, producer's risk, consumer's risk (ii) a point on the O.C. curve and either sample size or acceptance number. 12 hours

Unit-6

Reliability: Reliability concepts. Reliability of components and systems. Life distributions, reliability function, hazard rate, common life distributions-exponential, gamma, Weibull. System reliability-; Series and parallel systems. Examples. 8hours

(50 hours)

Practical Paper-VI

1. X-R charts. (Standard values known and unknown).
2. X-s chars (Standard values known and unknown)
3. Group control chart.
4. Sloping control chart.
5. np and p charts. (Standard values known and unknown).
6. c and u charts. (Standard values known and unknown).
7. OC and ARL curves for X and R charts.
8. Drawing O.C. A.S.N, A.T.I and A.O.Q curves for single sampling plans by attributes.
9. Drawing O.C. A.S.N A.T.I and A.O.Q curves for double sampling plans by attributes.
10. Construction of single sampling plans by attributes.
11. System reliability evaluation
12. Sketching reliability function
13. Sketching hazard function.

Books for Study

- 1) Grant, E.L and Leavenworth, R.S (1988): Statistical Quality control. 6th edition, McGrawHill.
- 2) Gupta, R.C. Statistical Quality control. (Khanna Pub. Co.)
- 3) Montgomery, D.C. (1985): Introduction to Statistical Quality control. (Wiley Int. Edn).
- 4) Jerry Banks: Quality Control. John Wiley
- 5) Sivazlian and Stenfel (1975): Optimizatoin techniques in Operations Research. Prentice-Hall.

Reference Books

- 1) John, S. Oakland and Followell, R.F. (190): Statistical Process Control. (East West Press, India)
- 2) Wetherill, G.B. and D.W. Brown: Statistical Process Control: Theory and practice. (Chapman and Hall)
- 3) Mahajan, M (2001): Statistical Quality Control. Dhanpat Rai & Co. (P) Ltd.
- 4) Donne, C.S. (1997): Quality, Prentice Hall.
- 5) Medhi. J. (2001); Stochastic Processes. New Age Pub.

Sixth Semester Theory Paper-1

ST P7:EXPERIMENTAL DESIGNS AND DEMOGRAPHY

Unit-1

Analysis of variance: Meaning and assumptions. Gauss Markov model and Gauss Markov theorem (statement only). Analysis of variance (fixed effects model)- Analysis of one-way, two-way and three-way classified data-expected mean squares, ANOVA tables. Least significant difference. Case of multiple but equal number of observations per cell in two-way classification (with interaction). 12 hours

Unit-2

Design of experiments: Principles of randomization, replication and local control. Completely randomized, randomized block and Latin square designs-layout, models. Least squares estimates of parameters, hypotheses, test procedures and ANOVA tables. Efficiency of a design. Missing plot technique for RBD and LSD- Estimation of single missing observation. 15hours

Unit-3

Factorial experiments: 22 and 23 factorials. Main effects and interactions, their best estimates and orthogonal contrasts. Yates method of computing factorial effects. Total and partial confounding in a 23 experiment with RBD layout. 9hours

Unit-4

Demography: Sources of demographic data. Measurement of mortality. Crude, specific and standardized death rates, infant mortality rate. Maternal mortality rate. Fecundity and fertility, measurement of fertility: crude, age specific general and total fertility rates. Reproduction rates. 8 hours

Unit-5

Life Table: Components of a life table, force of mortality and expectation of life, construction of life table. Abridged life table. Uses of life table. 6 hours

(50 hours)

Practical Paper-VII

- i. Analysis of one way classified data.
- ii. ANOVA for two way classified data: Single observation per cell
- iii. ANOVA for two way classified data: multiple but equal number of observations per cell (assuming interaction)
- iv. Analysis of CRD, RBD and LSD.
- v. Missing plot technique for RBD and LSD with single observation missing.
- vi. Analysis of 22 factorial experiment.
- vii. Analysis of 23 factorial experiment (With and without confounding total and partial)
- viii. Computation of mortality rates.
- ix. Computation of fertility rates.
- x. Computation of reproduction rates.
- xi. Construction of life table and computation of expectation of life.
- xii. Computation of force of mortality

Book for study:

1. Cochran, W.G. and G.M Cox: Experimental Designs. John Wiley.
2. Goon, A.M et al.: Fundamentals of Statistics Vol II (World Press, Calcutta).
3. Gupta, S.C. and V.K. Kapur: Fundamentals of Applied Statistics. (Sultan Chand and Co.)
4. Montgomery, D.C. Design and analysis of experiments. John Wiley.
5. Cox. P.R. (1970): Demography. Cambridge University Press.
6. Srivastava, O.S (1983): A Textbook of Demography. Vikas Publishing.

Reference Books:

- a. Das, M.N. and Giri, N (1979): Design of experiments: theory and applications. Wiley Eastern.
- b. Joshi. D.D. (1987): Linear estimation and design of experiments. New Age International.

Sixth Semester Theory Paper-2

(Elective Paper-2)

ST P8.1: OPERATIONS RESEARCH AND PROGRAMMING IN 'C'

Unit-1

Introduction: Definition and scope of operations research (OR). Phases of OR Modeling and solution. 2 hours.

Unit-2

Linear Programming: Linear programming problem (L.P.P)- Graphical solution. Simplex algorithm (without derivation). Examples. 5 hours

Unit-3

Statistical Decision Theory and Game theory: Statistical decision problem. Maximin, Laplace and expected payoff criteria. Regret function. Expected value of perfect information. Sampling and posterior distributions. Decision tree analysis. 5 hours

Unit-4

Inventory Theory: Description of an inventory system. Inventory costs. Demand and lead time. EOQ model with and without shortages. EOQ model with finite replenishment. Probabilistic demand. News paper boy problem. 5 hours

Unit-5

Queuing Models: Queuing models-specifications and effectiveness measures. Steady-state solutions of M|M|1 models with associated distributions of queue-length and waiting time (no derivation of results). Examples. 6 hours

Unit-6

CPM and PERT: Project planning with CPM and PERT. Drawing of project network. Critical path calculation. Critical path, slack time and float. PERT three estimate approach. Calculation of probabilities of completing a project within a specified period. 5 hours

Unit-7

Simulation: Introduction to simulation. Monte Carlo method. Generation of random observations from discrete and continuous distributions. Simple illustrations. Numerical

integration by simulation. Monte Carlo estimation of Simulation of inventory and queuing systems. 6 hours

Unit-8

Computer Programming 'C' Introduction of programming in C language. Structure of a C program. Data types. Variable declaration. Numeric, Character, real and string constants. Arithmetic, relational and logical operator. The decision and control structure. The loop control structure. Input/output operations. Header files. Use of functions. Rules of functions, recursion. One and two dimensional arrays.

Illustrative programs in Statistics and numerical analysis.

Use of statistical software. Statistical analysis using MS.EXCEL and MINITAB.. 12 hours

(50 hours)

Practical Paper-VIII

1. Solution of L.P.P by graphical method.
2. Solution of L.P.P involving slack variables only-using simplex algorithm.
3. Decision theory problems
4. Game theory problems.
5. Inventory problems.
6. Queuing problems
7. CPM and PERT
8. Generation of observations from discrete exponential, gamma and normal distributions.
9. Programming in 'C'-5 programs in statistical data analysis.
10. Statistical analysis using MS. EXCEL and MINITAB.

(Every student is required to write computer programs and execute these programs on a computer and enclose the relevant computer outputs in the practical record)

Books for Study:

1. Kanthiswaroop, Manmohan and P.K. Gupta (2003): Operations Research, Sultan Chand & co.
2. Churchman, C.W. Ackoff, R.L and Amoff, E.L (1957): Introduction to Opeartions Research, John Wiley.
3. Shenoy, G.V., Srivastava, U.K. and Sharma, S.C: Operations Research for Management, New Age International.
4. Yashawant Kanetkar (1999): Let us C. BPB Pub.
5. Balaguruswamy, E. (2001): Computer Programming in C Tata McMrawhill. Pub.
6. Sarma, K.V.S (2000): Statistics made simple with MS. EXCEL. Tata McGrawhill.
7. Minitab manuals.

Reference Books:

1. Mustafi, C.K. Operations Research methods and practice. New Age Pub.
2. Mital, K.V. Optimization method. New Age Pub.
3. Narag. A.S. Linear Programming and Decision making Sultan Chand & Co.
4. Kapoor, V.K. Operations Research Sultan Chand & co.
5. Kernighan, B.W. and D.M. Ritchir (1988): The C-programming language. PHI

BANGALORE UNIVERSITY

B.Sc. Statistics Model Question Paper

First Semester: Theory Paper

STP1: Basic Statistics and Probability

Duration: 3 Hrs.

Max Marks: 60

Instructions: 1. Answer Section A, Section B and any four question from Section C

2. Answer sub-divisions from Section A in the first two pages only.

SECTION 'A'

Answer any Six Sub-division: (2 Marks each)

1. a. Distinguish between (i) Nomial and ordinal data (ii) Simple and weighted averages
- b. State the propertics of arithmet- Mean
- c. What is curve Fitting?
- d. If two regression lines are $3x+xy-26=0$ and $6x+y-31=0$ find correlation coefficient.
- e. Prove the Laspeyrc's price index number is the weighted A.M. of the price relatives.
- f. What is time series? State the models used in time series analysis.
- g. Define conditional probability.
- h. Given $P(A)=0.3$ $P(A \cup B)=0.6$ $P(B)=p$ find p if (i) A & B are mutually exclusive (ii) A & B are independent.

SECTION 'B'

Answer any three Sub-division: (4 Marks each)

2. a. What are partition values? How are they determined graphically? Explain
- b. What is skew ness? Describe its various measures.
- c. What is scatter diagram? Interpret various types of correlation coefficient using scatter diagram.
- d. Explain the various steps involved in the construction of consumer price index numbers
- e. Define "sample space" and "event". Write the sample space of the random experiment of throwing two dice.

SECTION 'C'

Answer any four Sub-division: (9 Marks each)

3. a) Define weighted Arithmetic mean, Find the weighted arithmetic mean of first 'n' natural numbers, with weights being corresponding numbers.
b) Derive the formula for mode in case of a continuous frequency distribution (4+5)
4. a) Define mean deviation and prove that is least when it is measured from the median
b) Define Kurtosis and prove that moment co-efficient of Kurtosis is independent of change of origin and scale. (5+4)
5. a) Describe the principle of least squares in curve fitting. Obtain the normal equations-for fitting curve of the type $y=a+bx$
b) Derive the expression for spearman's Rank correlation co -efficient. (4+5)
6. a) Explain the method of obtaining seasonal indices by the method of ratio to moving average.
b) Explain time and factor reversal tests and show that Fisher's index number satisfies both the tests. (4+5)
7. a) Define mutually exclusive events.
Two cards are drawn from a pack of playing cards randomly. Find the probability of getting i) Two kings, (ii) One spade and one club. iii) An ace and a knave.
b) If A and B are two events then prove that i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (4+5)
8. a) What are independent events? Obtain the total number of conditions required for mutual independence of 'n' events. (4+5)
b) State Bayes theorem. (4+5)

Three urns contain 2 white and 3 black balls. 3 White & 2 black balls and 4 white and 1 black ball respectively. One ball is drawn from an urn chosen at random and it was found to be white. Find the probability that it was drawn from the first urn.

BANGALORE UNIVERSITY

Second Semester B.Sc. Statistics Examination

Model Question Paper

STP2: Probability Distributions and Multiple Regression

Duration: 3 Hrs.

Max Marks: 60

Instructions: 1. Answer Section A, Section B and any four question from Section C

2. Answer sub-divisions from Section A in the first two pages only.

SECTION 'A'

Answer any Six Sub-division: (2 Marks each)

- 1 a. Define i) Distribution function ii) Probability Density function
b. Suppose that X has the p.d.f. $f(x)=2x, 0 < x < 1$ find the p.d.f of $Y=3X+1$
c. State the p.m.f. of a negative binomial distribution. Show that geometric distribution is a particular case of negative binomial distribution.

- d. State the 'Lack of Memory' property of exponential distribution.
- e. State the properties of a Normal distribution
- f. If X and Y are independent random variables, show that they are uncorrelated. Comment on the converse.
- g. Distinguish between multiple correlation and partial correlation
- h. Define and explain how can one interprets the coefficient of multiple determination.?

SECTION 'B'

Answer any three Sub-division: (4 Marks each)

- 2. a. Define moment generating function of a.r.v State its properties and prove any one of them.
- b. What is convergence in distribution? State the conditions for the convergences of (i) Binomial distribution to normal distribution and (ii) Hyper geometric distribution to Binomial distribution.
- c. State and prove additive property of Gamma variates.
- d. Define: i) Joint and marginal probability density functions ii) Conditional expectation
- e. i) State the properties of least squares residuls.
- ii) Define residual variance in a trivariate set up.

SECTION 'C'

Answer any four Sub-division: (9 Marks each)

- 3. a) Suppose X is a Continuous random variable with p.d.f: $f(x)=c(1-x^2)$ for $0 < x < 1$. Determine c and find the distribution function.
- b) Find the first four moments about the mean of the distribution having the p.d.f $f(x)=f(x)=a/2; 0 < x < a$ (4+5)
- 4 a) Obtain the recurrence relation for central moment of a binomial distribution
- b) If x is a Poisson variate and $P(0)=P(1)=c$, then show that $c=1/e$ (6+3)
- 5 a) Find m.g.f. of negative binomial distribution
- b) Find mean and variance of beta distribution of second kind (3+6)
- 6. a) Show that a linear combination of independent normal variates is a normal variate and hence establish the additive property.
- b) Define Cauchy distribution and show that it is symmetric. (6+3)
- 7. a) State and prove addition theorem expectation
- b) If the joint p.m.f of X and Y is $p(x,y)=xy/9$ $x=1,2$ and $y=1,2$, obtain the marginal p.m.f s and show that: X and Y are independent (5+4)

8.a) Define multiple correlation coefficient and show that in the usual notation $R_{21.23}^2 = 1 -$

b) If $r_{12} = r_{13} = r_{23} = r$, prove That $r_{12.3} = r/(1+r)$ (7+2)

BANGALORE UNIVERSITY

B.Sc. Statistics Model Question Paper

Third semester: Theory Paper

STP3: Sampling Distributions and Estimation

Duration: 3 Hrs.

Max Marks: 60

Instructions: 1. Answer Section A, Section B and any four question from Section C

2. Answer sub-divisions from Section A in the first two pages only.

SECTION 'A'

Answer any Six Sub-division: (2x6=12)

1.a) What is meant by a "random sample"?

b) If X has a chi-square distribution with 10 degrees of freedom find its mean and variance.

c) State the inter-relationship between, X^2 , t and F distributions.

d) If \bar{x} is the sample mean of a random sample from $N(\mu, \sigma^2)$ show that \bar{x} is unbiased estimator of μ in probability.

e) Determine the relative efficiency of sample mean over sample median for $N(\mu, \sigma^2)$ distribution.

f) Define i) Mean square error, ii) Fisher's information function

g) State Cramer-Rao, Inequality.

h) Given a random sample of size n from Poisson population find MLE of $P[X=0]$. Where X follows Poisson distribution with parameter

SECTION 'B'

Answer any Three Sub-division: (4x3=12)

2. a) i) State the Features of F -distribution.

ii) Mention any two applications of t -distribution

b) State Levy Linderberg Central limit theorem and using it find the large sample distribution of the sample mean \bar{X} from a sampling from Poisson distribution.

c) If $E(T_n) \rightarrow 0$ and $V(T_n) \rightarrow 0$ as $n \rightarrow \infty$ then show that T_n is consistent for 0.

d) In a random sampling from a population with P.D.F: $f(x, \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$ show $T = \sum_{i=1}^n x_i$ is sufficient for θ

e) Obtain large sample (1- α) 100% confidence interval for the mean of a normal population when variance is known.

SECTION 'C'

Answer any four Questions: (9x4=36)

3. a) State and prove additive property of K independent chi-square variates
b) Obtain mean and variance of t-distributions. [4+5]
4. a) Obtain the distribution of reciprocal of F-variate
b) In random sampling from the population $N(u, 2)$, show that \bar{X} and S^2 are independent [4+5]
5. a) State Chebyshev's inequality. Let r.v. X have the pdf: $f(x) = \frac{1}{2} - \frac{3}{2}x + 3x^2$, $0 < x < 1$, Find the actual probability $P(u - 1.5 \leq X \leq u + 1.5)$ and compare it with the upper bound obtained by Chebyshev's inequality.
b) Show that a sequence of i.i.d. random variables satisfies the W.L.L.N. [5+4]
6. a) If T is an unbiased estimator of θ . then show that T^2 is also a consistent estimator of θ^2
b) Obtain minimum variance bound estimator for u in a sampling from the population $N(m, \sigma^2)$ where σ^2 is known. [5+4]
7. a) Define a "M.L.E" and State some large sample properties of a M.L.E
b) Obtain moment estimators of the parameters of uniform distribution $U(a, b)$. [4+5]
8. a) Obtain M.L.E. of θ in exponential distribution with mean $1/\theta$ based on random sample of size n .
b) Explain the pivotal quantity method of finding confidence interval for a parameter and obtain $(1-\alpha)$ 100% confidence interval for the ratio of two variances of normal populations. [3+6]

BANGALORE UNIVERSITY

B.Sc. Statistics Model Question Paper

Fourth semester: Theory Paper

STP4: Sampling Distributions and Estimation

Duration: 3 Hrs.

Max Marks: 60

Instructions: 1. Answer Section A, Section B and any four question from Section C

2. Answer sub-divisions from Section A in the first two pages only.

SECTION 'A'

Answer any Six Sub-division: (2x6=12)

1. a) Distinguish between level of significance and size of a test.

- b) State Neyman-Pearson fundamental lemma
- c) A test function based on a sample (x_1, x_2) for testing certain null hypothesis is given by $(x_1 x_2) = 1$, if $x_1 + x_2 < 4$. What is your decision if the sample is [i] $(x_1 = x_2, x_2 = 5)$ and [ii] $(x_1 = 3, x_2 = 1)$?
- d) define a Uniformly Most Powerful [UMP] test
- e) State properties of likelihood ratio tests
- f) State the assumptions required for applying t-test
- g) Describe Sequential Probability Ratio Test [SPRT]
- h) State some of the advantages and disadvantages of non-parametric tests.

SECTION-B

Answer any three Questions: (4x3=12)

2. a) If $C = \{x: x < c\}$ is the critical region of a test based on single observations X for testing $H: \theta = 2$ against $K: \theta = 1$ being the parameter of $f(x, \theta) = \exp(-\theta x)$, $0 < x$, compute size and power of the test.
- b) State the MLR property. Examine whether the family of Poisson distributions with mean $\theta < \infty$ has this property
- c) Describe chi-square test of goodness of fit.
- d) In an SPRT, describe how stopping bounds are obtained using its strength
- e) Define a run with examples. Describe run test for randomness of a sample.

SECTION -C

Answer any Four Questions : (9x4=36)

3. a) Explain [i] simple and composite hypotheses, [ii] Power of a test
 - b) Construct the most powerful [MP] test of size α for $H: \mu = \mu_0$. Against $K: \mu = \mu_1$. based on a random sample of size n from $N(\mu, \sigma^2)$ -distribution With σ^2 known. [4+5]
4. a) Describe likelihood ratio test
 - b) Given a random sample of size n from $N(\mu, \sigma^2)$, derive the likelihood ratio test for testing $H: \mu = \mu_0$ against $K: \mu = \mu_1$, when σ^2 is unknown. [3+6]
5. a) Describe a large sample test for the equality of means of two normal Distributions with unknown variances.
 - b) Describe chi-square for testing $H: \sigma^2 = \sigma_0^2$ against. One sided and two-sided alternatives. σ_0^2 being the variance of a normal distribution. [4+5]

6. a) Explain F-test for testing the equality of variances of two normal distributions.
 b) Explain Fisher's Z-transformation. Describe a test for $H_0: \rho = \rho_0$ Where ρ is the population correlation coefficient. [4+5]
7. Construct SPRT for testing $H_0: p = p_0$ against $K: p = p_1, p_1 > p_0$, being the Parameter the Bernoulli distribution. Give approximate expressions for the O.C. and A.S.N. functions. [9]
8. a) Describe sign test for paired observations
 b) Describe Mann-Whitney-Wilcoxon test. [4+5]

BANGALORE UNIVERSITY

B.Sc. Statistics Model Question Paper

Fifth Semester: Theory Paper

STP5: Sampling Theory And Applications

Duration: 3 Hrs.

Max Marks: 60

Instructions: Answer five subdivisions from section A and five question from Section B

SECTION 'A'

1. Answer any FIVE Sub-division: (5x3=15)

- Distinguish between SRSWOR and SRSWR and compare their efficiencies
- Prove that the probability of selecting a specified unit of the population at any given draw is equal to the probability of its being selected at the first draw.
- Prove that the sample proportion is an unbiased estimator of population proportion.
- What is Stratified Random Sampling? Mention its advantages.
- Distinguish between linear systematic sampling and circular systematic sampling
- Explain the factors causing Non-sampling errors.
- Distinguish between plot sampling and transect sampling.

SECTION-B

Answer any FIVE questions (5x9=45)

- Explain the use of random number tables in the selection of random sample.
 b. Describe the advantages of sampling over complete enumeration. (4+5)

3. What is Simple Random Sampling? Derive the expression for the variance of sample mean under SRSWOR and Set up the Confidence Interval for population mean and total. [9]
4. a. With usual notations show that $V(\bar{A}) = \frac{N-m}{N} P Q \frac{1}{n}$
- b. Obtain an expression for sample size while estimating population mean. (4+5)
5. Suggest an unbiased estimator of the population mean in case of stratified random sampling.
Derive its variance and deduce it under proportional and Neyman allocations (9)
6. a. In Stratified random sampling, ignoring f.p.c prove that variance of the estimated mean under proportional allocation is not less than the variance of the estimated mean under neyman allocation.
- b. Prove that the mean of a systematic sample is more precise than the mean of a simple random sample iff $S^2_{wsy} > S^2$
(5+4)
7. Describe systematic sampling and obtain an expression for variance of mean under systematic sampling in terms of intra class correlation co-efficient and compare the efficiency of this estimator with SRSWOR. (9)
8. a. State the principal steps involved in conducting a sample survey.
- b. Explain the working of N.S.S.O.
- c. Define the ecological measurements [i] density, [ii] frequency, [iii] Biomass (3+3+3)

B.Sc. Statistics Model Question Paper

Fifth Semester: Theory Paper

STP6.1: Statistical Methods For Quality Management

Duration: 3 Hrs.

Max Marks: 60

Instructions: Answer five subdivisions from section A and five question from Section B

SECTION 'A'

1. Answer any FIVE Sub-division: (5x3=15)

- a) Explain the meaning of 'quality' and 'quality assurance'.
- b) Mention the various types of 'quality costs'.

- c) Distinguish between action limits and warning limits and state their uses in a control chart.
- d) Define process capability ratio and indicate its use.
- e) Obtain the control limits for C-chart.
- f) Define the terms AQL, AOQL and LTPD.
- g) Define the following terms
 - i] Reliability of a component
 - ii] Hazard rate
 - iii] I.F.R. distribution

SECTION-B

Answer any FIVE questions

(5x9=45)

- 2. a) What is statistical quality control? Discuss its need and utility in industry
 - b) Distinguish between chance and assignable causes. What are the advantages of statistically controlled process?
 (5+4)
- 3. a) Derive the control limits for X-S chart. When process standards are unknown.
 - b) Discuss the criteria for detecting the lack of control in a control chart.
 (5+4)
- 4. a) What is rational sub grouping? Explain the methods for this purpose
 - b) Describe the construction and interpretation of sloping control chart for mean.
 (5+4)
- 5. a) Explain the basis and construction of p-chart. Give instances of its applicability.
 - b) Define OC and ARL functions of a control chart. Derive their expressions for X-chart.
 (4+5)
- 6. a) What are acceptance sampling plans? And explain the terms
 - i] Indifference quality and ii] Acceptance number
 - b) Describe double sampling plan
 - c) Explain the method of designing a SSP such that OC curve will pass through two designated points.
 (3+3+3)
- 7. a) Distinguish between ASN and ATI

- b) Obtain an expression for consumer's risk, producer's risk and AOQ function for double sampling plan
(2+7)
8. a) Show that failure rate is constant if the time of a component has exponential distribution.
- b) If R_1 and R_2 are the reliabilities of two components functioning independently and connected in series, then prove that the reliability of the system is given by $R_s = R_1 \cdot R_2$
- c) Write the block diagram of a system whose structure function is $(x) = x_1 \cdot x_2 \cdot \max(x_3, x_4)$ and find the reliability of the system. (3+3+3)

B.Sc. Statistics Model Question Paper

Sixth Semester: Theory Paper-I

STP7: Experimental Designs And Demography

Duration: 3 Hrs.

Max Marks: 60

Instructions: Answer five subdivisions from section A and five question from Section B

SECTION 'A'

1. Answer any FIVE Sub-division:

(5x3=15)

- a) State Gauss-Markov theorem.
- b) Explain randomisation with reference to CRD and give its layout.
- c) What is a contrast? When are two contrasts said to be orthogonal?
- d) Give a layout of RBD with three replications where in AB, BC and AC are confounded.
- e) Distinguish between crude and standardized death rates.
- f) Explain the terms 'Fertility' and 'fecundity' And 'Morbidity'
- g) Mention the uses of life tables.

SECTION -B

Answer any FIVE Sub-division:

(5x9=45)

2. a) For the linear statistical model.

$$Y_{ij} = \mu + \alpha_i + \epsilon_{ij} \quad j=1,2,\dots,n, \quad i=1, 2,\dots,k \text{ and}$$

Show that mean error sum of square is an unbiased estimator of the population variance.

b), Explain the role of 'critical difference' in Analysis of variance (5+4)

3. Describe the analysis of a Two-way classified data with single observation per cell. (9)

4. a) Explain the basic principles of Design of experiments.
 b) Obtain the relative efficiency of RBD over CRD. (5+4)
5. a). Obtain the least square estimates of the parameters of a fixed effect model of Latin Square Design.
 b) Derive an expression for estimating a single missing value in a LSD. (5+4)
6. a) Define 'Main' and 'Interaction effects' Derive expressions for the effects and AB in 22 factorial experiment.
 b) Explain the analysis of 22 factorial experiment in RBD with 'r' replications. (4+5)
7. a) Explain the different methods of collecting vital statistics.
 b) Define Gross reproduction rate and net reproduction rate show that $GRR > NRR$. (3+6)
8. a) Explain the different components of a life table.
 b) With usual notations show that: $q_x = 2m_x/2 + m_x$ (4+5)

BANGALORE UNIVERSITY

B.Sc. Statistics Model Question Paper

Sixth Semester: Theory Paper

STP8.1: Operations Research And Programming In 'C'

Duration: 3 Hrs.

Max Marks: 60

Instructions: Answer five subdivisions from section A and five question from Section B

SECTION-A

1. Answer any FIVE Sub-division: (5x3=15)

- a. Explain the following models
 (i) Analog (ii) Iconic (iii) Symbolic
- b. Explain the Laplace criterion for arriving at a decision under uncertainty.
- c. Define a rectangular game and write the condition for the existence of a saddle point.
- d. Define the following
 (i) Demand (ii) Lead time (iii) Shortage cost.
- e. Describe the queuing model (M/M/I: FIFO/N/)
- f. Explain the following with reference to a PERT problem:

(i) Total float (ii) Free float (iii) Independent float

g. Describe the method of Inverse transformation for generating observations from exponential distributions.

SECTION-B

Answer any FIVE Sub-division:

(5x9=45)

2. a. Obtain all basic solutions for the following system of equations and classify them on the basis of feasibility and degeneracy. $X_1+2x_2+3x_3=12$ $3X_1+2X_3+X_3=12$

b. Outline the steps under simplex algorithm for solving a Linear Programming problem.

(3+6)

3. a. Explain EMV criterion of arriving at a decision under uncertainty. How do you calculate expected value of perfect information?

b. For the (2x2) rectangular game without saddle point.

(i) Obtain the pay-off function

(ii) Determine the optimum mixed strategies for both the players.

(iii) What will be the value of the game?

(4+5)

4. a. Define inventory and mention the advantages of maintaining inventory.

b. Obtain an expression for Optimum Quantity to be ordered under EOQ model when shortages are not permitted.

(3+6)

5. a. Explain the following with respect to a queuing system

(i) Balking and Reneging

(ii) Series and parallel service channels.

b. Derive the expression for Average Queue Length [Ls] in (M/M/1: FIFO/N/o) queuing system.

(4+5)

6.a. Construct the network for the following project.

Activity: (1,2) (1,3) (1,4) (2,5) (2,3) (3,5) (4,5)

Duration: 5,14 8 6 3 5 4

b. Describe the method of estimating using Monte Carlo method

(4+5)

7. a. Define the basic data types in 'C' with examples

b. List out the loop statements available in C, Explain any two of them with examples.
(4+5)

8. a. Bring out the difference between the conditional operation and if else statements. Write the output of the following program.

```
Main() {int x=5; switch (5-x) {"case 0:x=x+5 case 4: x++; break; case 10: x=10; printf ("%d" x);
```

b. Explain the rules for defining a function in C. Also explain the use of continue statement in a function.