

KVPY QUESTION PAPER -STREAM SB
November 01, 2009

PART-I (1 Mark)
MATHEMATICS

1. Suppose the sequence a_1, a_2, a_3, \dots is an arithmetic progression of distinct numbers such that the sequence $a_1, a_2, a_4, a_8, \dots$ is a geometric progression. The common ratio of the geometric progression is
 A. 2 B. 4 C. a_1 D. not determinable
2. The positive integer k for which $(101)^{k/2}/k!$ is a maximum is
 A. 9 B. 10 C. 11 D. 101
3. Let $p(x) = a_0 + a_1x + \dots + a_nx^n$ be a non-zero polynomial with integer coefficients. If $p(\sqrt{2} + \sqrt{3} + \sqrt{6}) = 0$, the smallest possible value of n is:
 A. 8 B. 6 C. 4 D. 2
4. Three players play a total of 9 games. In each game, one person wins and the other two lose; the winner gets 2 points and the losers get -1 each. The number of ways in which they can play all the 9 games and finish each with a zero score is
 A. 84 B. 1680 C. 7056 D. 0
5. In a triangle, two vertices are $(2, 3)$ and $(4, 0)$, and its circumcentre is $(2, z)$ for some real number z . The circumradius is
 A. $\frac{6}{2 + \sqrt{13}}$ B. $\sqrt{5}$ C. 2 D. $\frac{13}{6}$
6. Consider an ellipse with foci at $(5, 15)$ and $(21, 15)$. If the x -axis is a tangent to the ellipse, then the length of its major axis equals
 A. 17 B. 34 C. 13 D. $\sqrt{416}$
7. Let the line $2x + 3y = 18$ intersect the y -axis at B . Suppose C ($\neq B$), with coordinates (a, b) , is a point on the line such that $PB = PC$, where $P = (10, 10)$. Then $8a + 2b$ equals
 A. 60 B. 62 C. 66 D. 78
8. If

$$\operatorname{cosec}^2(\alpha + \beta) - \sin^2(\beta - \alpha) + \sin^2(2\alpha - \beta) = \cos^2(\alpha - \beta),$$
 where $\alpha, \beta \in (0, \pi/2)$, then $\sin(\alpha - \beta)$ is equal to
 A. $\frac{-1}{2}$ B. $\frac{1}{2}$ C. $\frac{-\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{2}$

2

9. If $\sin x + \sin y = 7/5$ and $\cos x + \cos y = 1/5$, then $\sin(x + y)$ equals
 A. $7/25$ B. $24/25$ C. $-7/25$ D. $-24/25$
10. The number of solutions to $\sin x = 6/x$ with $0 \leq x \leq 12\pi$ is
 A. 1 B. 6 C. 10 D. 12
11. Define a function $f: \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{\sin x^2}{x} & \text{for } x < 0 \\ x^2 + ax + b & \text{for } x \geq 0. \end{cases}$$
 Suppose $f(x)$ is differentiable on \mathbb{R} . Then
 A. $a = 0, b = 0$ C. $a = 1, b = 0$
 B. $a = 0, b = 1$ D. $a = 1, b = 1$
12. The shortest distance from $(0, 3)$ to the parabola $y^2 = 4x$ is
 A. 2 B. $\sqrt{2}$ C. 5 D. $\sqrt{5}$
13. Ten trucks, numbered 1 to 10, are carrying packets of sugar. Each packet weighs either 999 gms or 1000 gms and each truck carries only the packets equal weights. The combined weight of 1 packet selected from the first truck, 2 packets from the second, 4 packets from the third, and so on, and 2^9 packets from the tenth truck is 1022870 gms. The trucks that have the lighter bags are
 A. 1, 3, 5 B. 2, 4, 5 C. 1, 9 D. 2, 8
14. What is the value of

$$\int_0^1 \cos(\pi x) \cos(2x\pi) dx?$$
 (Here $[t]$ denotes the integral part of the real number t .)
 A. 1 B. -1 C. $\frac{2}{\pi}$ D. $\frac{-2}{\pi}$
15. The value of the limit $\lim_{n \rightarrow \infty} \int_0^1 x^{10} \sin(nx) dx$ equals
 A. 0 B. $1/10!$ C. $\pi/2$ D. 1
16. The area bounded by the parabolas $y = x^2$ and $y = 1 - x^2$ equals
 A. $\sqrt{2}/3$ B. $2\sqrt{2}/3$ C. $1/3$ D. $2/3$

3

17. A vector which bisects the angle between $\vec{a} = 3\vec{i} - 4\vec{k}$ and $\vec{b} = 5\vec{j} + 12\vec{k}$ is

- A. $39\vec{i} - 25\vec{j} + 8\vec{k}$ C. $39\vec{i} + 25\vec{j} + 8\vec{k}$
 B. $3\vec{i} - 5\vec{j} + \frac{8}{5}\vec{k}$ D. $3\vec{i} + 5\vec{j} + \frac{8}{5}\vec{k}$

18. An envelope has space for at most 3 stamps. If you are given three stamps of denomination 1, and three stamps of denomination a ($a > 1$), the least positive integer for which there is no stamp value is:

- A. 7 B. 8 C. 9 D. 10

19. If m, n are positive integers such that $m < n$ and $\sum_{d|mn} d = \sum_{d|n} d$ (here $d | k$ means d is a positive divisor of k), then

- A. $\sum_{d|mn} \frac{1}{d} < \sum_{d|n} \frac{1}{d}$
 B. $\sum_{d|mn} \frac{1}{d} = \sum_{d|n} \frac{1}{d}$
 C. $\sum_{d|mn} \frac{1}{d} > \sum_{d|n} \frac{1}{d}$
 D. no relationship can be determined

20. The number of relations R from an m -element set A to an n -element set B satisfying the condition

$$(a, b_1) \in R, (a, b_2) \in R \Rightarrow b_1 = b_2$$

for $a \in A, b_1, b_2 \in B$ is

- A. n^m B. $2^{m+n} - 2^m - 2^n$ C. mn D. $(n+1)^m$

PHYSICS

21. The relation $C_p - C_v = R$ (C_p, C_v : Molar specific heats at constant pressure, volume) is exactly true for

- A. an ideal mono-atomic gas.
 B. any ideal gas, whether mono-, di- or poly-atomic.
 C. any real gas above its critical temperature.
 D. all real gases.

22. The molecules of air in the room that you are sitting are all experiencing the force of gravity tending to bring them down. The molecules are also frequently and randomly undergoing collisions, which tend to oppose the effect of fall under gravity. The density of air is nearly uniform throughout the room because

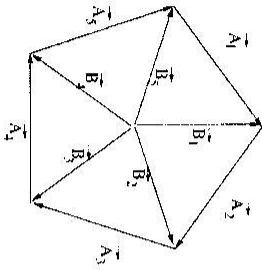
- A. the mass of the molecules is very small.
 B. the gravitational potential energy mgh is much lesser than the average thermal energy KT .
 C. the gravitational potential energy mgh is much greater than the average thermal energy KT .
 D. mgh is nearly of the same magnitude as KT , which results in the cancellation of the two opposing factors.

23. A parallel plate capacitor is charged fully by using a battery. Then without disconnecting the battery, the plates are moved further apart. Then,

- A. the charge on the capacitor increases.
 B. the voltage difference between the plates decreases.
 C. the capacitance increases.
 D. the electrostatic energy stored in the capacitor decreases.

24. The five sides of a regular pentagon are represented by vectors $\vec{A}_1, \vec{A}_2, \vec{A}_3, \vec{A}_4,$ and \vec{A}_5 , in cyclic order as shown.

Corresponding vertices are represented by $\vec{B}_1, \vec{B}_2, \vec{B}_3, \vec{B}_4,$ and \vec{B}_5 , drawn from the centre of the pentagon.



Then, $\vec{B}_2 + \vec{B}_3 + \vec{B}_4 + \vec{B}_5 =$

- A. \vec{A}_1 B. $-\vec{A}_1$ C. \vec{B}_1 D. $-\vec{B}_1$

- 25 Four metallic plates each of surface area (of one side) A , are placed at a distance d apart from each other. The two outer plates are connected to a point P and the two inner plates to another point Q as shown in figure.



Then the capacitance of the system is

- A. $\epsilon_0 \frac{A}{2d}$ B. $\epsilon_0 \frac{A}{d}$ C. $2\epsilon_0 \frac{A}{d}$ D. $3\epsilon_0 \frac{A}{d}$

- 26 A progressive wave travelling in positive x -direction given by $y = a \cos(kx - \omega t)$ meets a denser surface at $x = 0, t = 0$. The reflected wave is then given by

- A. $y = -a \sin(kx - \omega t)$ C. $y = -a \cos(kx + \omega t)$
 B. $y = a \sin(\omega t - kx)$ D. $y = a \cos(kx - \omega t)$

- 27 A charge Q is spread non uniformly on the surface of a hollow sphere of radius R , such that the charge density is given by $\sigma = \sigma_0(1 - \sin \theta)$, where θ is the usual polar angle. The potential at the centre of the sphere is:

- A. $\frac{Q}{2\pi\epsilon_0 R}$ B. $\frac{Q}{\pi\epsilon_0 R}$ C. $\frac{Q}{8\pi\epsilon_0 R}$ D. $\frac{Q}{4\pi\epsilon_0 R}$

- 28 An ideal diatomic gas is heated at constant pressure. The ratio of the work done to the heat supplied is:

- A. $\frac{3}{5}$ B. $\frac{2}{5}$ C. $\frac{2}{7}$ D. $\frac{4}{7}$

- 29 In the hydrogen spectrum, the ratio of the wavelengths for Lyman α -radiation to Balmer- α radiation is

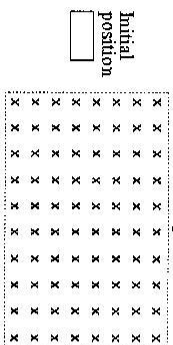
- A. $5/27$ B. $5/48$ C. $27/5$ D. $1/3$

- 30 Two identical conducting spheres carry identical charges. If the spheres are set at a certain distance apart, they repel each other with a force F . A third conducting sphere, identical to the other two, but initially uncharged, is then touched to one sphere, and then to the other before being removed. The force between the original two spheres is now

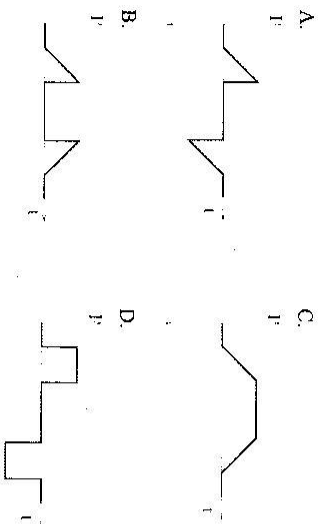
- A. $F/2$ B. $F/4$ C. $3F/4$ D. $3F/8$

- 31 A small rectangular loop of wire in the plane of the paper is moved with uniform speed across a limited region of uniform magnetic field perpendicular to the plane of the paper, as shown.

Uniform magnetic field



Which graph would best represent the variation of the electric current, I , in the wire with time, t ?



- 32 The moment of inertia of a solid disc made of thin metal of radius R and mass M about one of its diameters is given by $\frac{MR^2}{4}$. What will be the moment of inertia about this axis if the disc is folded in half about this diameter?
- A. $\frac{MR^2}{8}$ B. $\frac{MR^2}{2}$ C. $\frac{MR^2}{4}$ D. MR^2
- 33 A plane electromagnetic wave propagating in the direction of the unit vector \hat{n} with a speed c is described by electric and magnetic field vectors \vec{E} and \vec{B} , respectively. Which of the following relations (in SI units) between \vec{E} and \vec{B} can be ruled out on

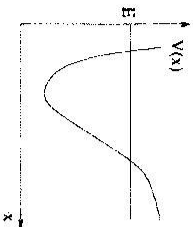
dimensional grounds alone?

- A. $\vec{E} = \frac{\hat{n} \times \vec{B}}{c}$ C. $\vec{B} = \frac{\hat{n} \times \vec{E}}{c}$
 B. $\vec{E} = -c\hat{n} \times \vec{B}$ D. $\hat{n} \times \vec{E} \times \vec{B} = 0$

- 34 A point electric dipole placed at the origin has a potential given by $V(r, \theta) = \frac{p \cos \theta}{4\pi \epsilon_0 r^2}$ where θ is the angle made by the position vector with the direction of the dipole. Then

- A. since the potential vanishes at $\theta = \frac{\pi}{2}$, the electric field is zero everywhere on the $\theta = \frac{\pi}{2}$ plane.
 B. the electric field everywhere on the $\theta = \frac{\pi}{2}$ plane is normal to the plane.
 C. the electric field everywhere on the $\theta = \frac{\pi}{2}$ plane is along the plane.
 D. the electric field vanishes on the $\theta = 0$ line.
- 35 A uniform non-deformable cylinder of mass m and radius R is rolling without slipping on a horizontal rough surface. The force of friction is
- A. μmg , where μ is the coefficient of sliding friction.
 B. zero.
 C. increases with time.
 D. decreases with time.

- 36 Consider a one-dimensional potential $V(x)$ as shown in the figure below. A classical particle of mass m moves under its influence and has total energy E as shown.



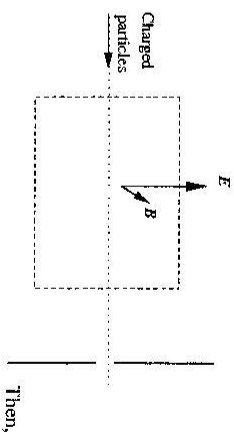
The motion is

- 37 A source of frequency f is emitting sound waves. If temperature of the medium increases, then
- 38 A block of mass m is stationary on a rough plane of mass M inclined at an angle θ to the horizontal while the whole set up is accelerating upwards at an acceleration a . If the coefficient of friction between the block and the plane is μ , then the force that the plane exerts on the block is
- 39 A stream of charged particles enter into a region with crossed electric and magnetic fields as shown in the figure. On the other side is a screen with a hole that is right on the original path of the particles.

- A. non-periodic. C. periodic but not simple harmonic.
 B. stationary. D. simple harmonic.

- A. wavelength of the sound wave increases.
 B. speed of the sound wave decreases.
 C. wavelength of the sound wave decreases.
 D. amplitude of the sound wave increases.

- A. $m(g + a)$ upwards.
 B. $mg \cos \theta$ normal to the plane.
 C. resultant of $mg \cos \theta$ normal to the plane and $\mu mg \cos \theta$ along the plane.
 D. resultant of $m(g + a) \cos \theta$ normal to the plane and $\mu mg \cos \theta$ along the plane.

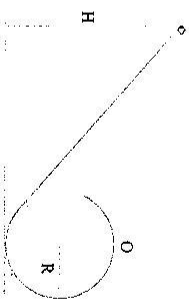


Then,

- 40 A small body is released from a height H of an inclined plane. At the bottom of the plane is a loop of radius R as shown.
- 41 Ignoring friction, the minimum H required for the body to just complete the loop (that is, reach the point O) is

- A. no particle can get through the hole.
 B. all particles can get through the hole.

- C. only positively charged particles with speed $\frac{E}{B}$ can get through the hole.
 D. all particles with speed $\frac{E}{B}$ can get through the hole.



- A. $2R$ B. $\frac{5R}{2}$ C. $3R$ D. $\frac{7R}{2}$

CHEMISTRY

- 41 The gas that has the slowest rate of diffusion among O_2 , H_2 , CO_2 and CH_4 is
- A. O_2 C. CO_2
 B. H_2 D. CH_4
- 42 Assuming ideal behaviour the ratio of kinetic energies of 3 g of H_2 and 4 g of O_2 at any temperature is
- A. 3:4 C. 4:3
 B. 1:16 D. 12:1
- 43 The IUPAC name for the compound is
-
- A. 1-chloro-3-methyl-4-pentanone
 B. 1-chloro-2-methyl-4-pentanone
 C. 5-chloro-3-methyl-2-pentanone
 D. 5-chloro-2-methyl-1-3-pentanone
- 44 The shape of the molecule ClF_3 is
- A. triangular C. T-shape
 B. pyramidal D. linear
- 45 Among CO_3^{2-} , OH^- , NH_3 and HCO_3^- , the species that acts as a Bronsted acid as well as a Bronsted base, is
- A. Na_2CO_3 C. NH_3
 B. OH^- D. HCO_3^-

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- 46 The ratio of the heat capacities C_p/C_v for one mole of a gas is 1.67. The gas is
- A. He C. CO_2
 B. H_2 D. CH_4
- 47 The ion that is isoelectronic with CO is
- A. O_2^+ C. CN^-
 B. O_2^- D. N_2^+
- 48 Among CH_4 , CO_2 , H_2O and SO_2 , the bond angle is the highest in
- A. CH_4 C. H_2O
 B. CO_2 D. SO_2
- 49 The solvent of choice for carrying out a Grignard reaction is
- A. diethyl ether C. ethyl acetate
 B. chloroform D. ethanol
- 50 The reaction of butanal with n-propylmagnesium bromide gives a
- A. chiral secondary alcohol
 B. achiral secondary alcohol
 C. chiral tertiary alcohol
 D. achiral tertiary alcohol
- 51 The hybridization of Ni centre in $Ni[(PPh_3)_2Cl_2]$ and $[NiCl_4]^{2-}$, respectively are
- A. dsp^2 and sp^3 C. sp^3 and sp^3
 B. dsp^2 and sp^2d D. sp^3 and dsp^2
- 52 Oxalic acid when treated with potassium permanganate in the

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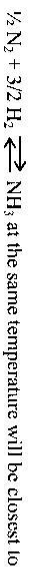
presence of an acid, produces

- A. O₂
- B. C
- C. CO
- D. CO₂

53 The equilibrium constant for the reaction



at 400 K is 4.1. The equilibrium constant for the reaction



- A. 41
- B. 20.5
- C. 6.4
- D. 1.681

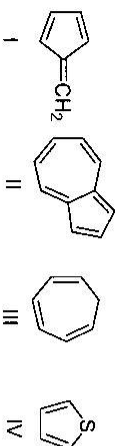
54 In a one component second order reaction, if the concentration of the reactant is reduced to half, the rate

- A. increases two times
- B. increases four times
- C. decreases to one half
- D. decreases to one fourth

55 The conjugate bases for HCO₃⁻ and NH₄⁺, respectively, are

- A. H₂CO₃ and NH₄⁺
- B. CO₃²⁻ and NH₃
- C. H₂CO₃ and NH₂⁻
- D. CO₃²⁻ and NH₄⁺

56 Among the following



the aromatic compounds are

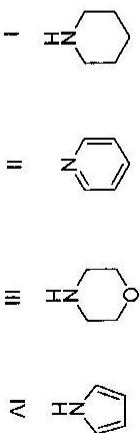
- A. I & II
- C. II & III

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B. I & III

D. II & IV

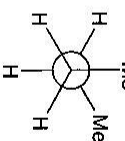
57 Among the compounds



the order of basicity is

- A. I > III > II > IV
- B. III > I > IV > II
- C. II > IV > I > III
- D. II > I > III > IV

58 The Newman projection of



is known as the

- A. eclipsed conformer
- B. staggered conformer
- C. skewed conformer
- D. gauche conformer

59 The half-life of a first order reaction is 30 min. The time required for 75% completion of the same reaction will be

- A. 45 min
- B. 60 min
- C. 75 min
- D. 90 min

60 The hydrogen ion concentration in a mixture of 10 ml of 0.1 M H₂SO₄ and 10 ml of 0.1 M KOH solution in water, is

- A. 0.1 M
- B. 0.05 M
- C. 0.2 M
- D. 0.02 M

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BIOLOGY

- 61 During photosynthesis the chemical conversion of water is termed,
A. photolysis C. hydration
B. hydrolysis D. condensation
- 62 In the organism mussel, oxygen is carried by,
A. albumin C. myoglobin
B. myosin D. hemoglobin
- 63 Enzymes do the following:
A. make products and reactants of equal energy
B. help the chemical processes by lowering the energy of products
C. reduce the activation barrier and speed up chemical processes
D. hydrolyze all the biopolymers indiscriminately
- 64 Glycolysis is,
A. biosynthesis of glucose C. degradation of glucose
B. biosynthesis of glycine D. reaction of glucose with proteins
- 65 Plants are attracted to light through the hormonal action of,
A. Gibberellic acid C. Chlorophyll
B. Auxin D. Thiamine
- 66 During development, unspecified cells become cells having unique functions. This process is called,
A. evolution C. translation
B. differentiation D. replication
- 67 The chromosomal attachment site of the spindle microtubule is,
A. centrosome C. centromere
B. liposome D. telomere
- 68 Which of the following diseases is NOT sexually transmitted ?
A. Syphilis C. AIDS
B. Gonorrhoea D. Tuberculosis
- 69 This cell organelle consists of two granule-like centrioles and is found in animal cells only. It helps in cell division. What is it called?
A. centrosome C. centromere
B. chromosome D. chromatids
- 70 Nucleotides are monomers of DNA. What does each nucleotide consist of ?
A. A nitrogenous base and a pentose sugar.
B. A nitrogenous base and a phosphate group.
C. A pentose sugar and a phosphate group.
D. A nitrogenous base, a pentose sugar and a phosphate group.
- 71 Fertilization in humans usually takes place in,
A. uterus C. ovary
B. Graafian follicle D. Fallopian tube
- 72 ELISA, the standard screening test for HIV, detects which of the following ?
A. HIV DNA C. HIV proteins
B. HIV RNA D. Antibodies to HIV proteins
- 73 Sickle cell anemia is caused by,

- 73 complete absence of the haemoglobin gene
 B. point mutation of the haemoglobin gene
 C. increased affinity of haemoglobin for oxygen
 D. truncation of the haemoglobin protein
- 74 The natural source of Ti plasmid is
 A. bacteria
 B. virus
 C. plants
 D. animals
- 75 Earthworms are bisexual but still cross-fertilization is common. This is because,
 A. spermatozoa of different earthworms are different
 B. spermatozoa and ova mature at different times in the same earthworm
 C. ova from other earthworms may be larger
 D. sperm and ova from the same earth worm cannot fertilize
- 76 One difference between blood and lymph is that,
 A. blood contains WBC and lymph contains RBC
 B. blood contains RBC and WBC and lymph contains only WBC
 C. blood contains RBC and lymph contains WBC
 D. blood is liquid while lymph is solid
- 77 The abnormal development of which of the following lymphoid organs would result in the most severe immunodeficiency?
 A. Spleen
 B. Thymus
 C. Tonsil
 D. Lymph node
- 78 Mitochondria are associated with all of the following functions, EXCEPT.
- 79 The probability of having a girl child with blood group O when the parents have blood group A and B is
 A. 0%
 B. at least 50%
 C. at most 25%
 D. exactly 75%
- 80 Wooden doors and windows swell up in the rainy season by,
 A. a special type of diffusion called imbibition
 B. evaporation of stored water in wood
 C. conduction of water from walls
 D. transpiration

PART-II (2 Marks)
MATHEMATICS

81. Let $p(x) = a_0 + a_1x + \dots + a_nx^n$. If $p(-2) = -15$, $p(-1) = 1$, $p(0) = 7$, $p(1) = 9$, $p(2) = 13$ and $p(3) = 25$, then the smallest possible value of n is
A. 5 B. 4 C. 3 D. 2
82. Let a, b, c be the sides of a triangle. If t denotes the expression $(a^2 + b^2 + c^2)/(ab + bc + ca)$, the set of all possible values of t is
**A. $\{x \in \mathbb{R} \mid x > 1\}$ C. $\{x \in \mathbb{R} \mid 1 \leq x < 2\}$
B. $\{x \in \mathbb{R} \mid 1 < x < 2\}$ D. $\{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$**
83. The area of the region bounded by $y = ||x - 3| - 4| - 5$ and the x -axis is
A. 24.5 B. 37 C. 49 D. 35 $\sqrt{2}$
84. The lengths of the sides and the diagonal of an isosceles trapezium form a two-element set $\{a, b\}$. If $a > b$, then a/b equals
A. $\frac{1}{2}(\sqrt{6} + \sqrt{2})$ B. $\frac{1}{2}(\sqrt{5} + 1)$ C. $\sqrt{3}$ D. $\sqrt{2}$
85. Define a sequence $\{a_n\}_{n \geq 0}$ by

$$a_n = \sqrt{\frac{1 + a_{n-1}}{2}} \text{ for } n \geq 1, \quad a_0 = \cos \theta \neq \pm 1.$$
Then $\lim_{n \rightarrow \infty} 4^n(1 - a_n)$ equals
A. θ^2 B. $\theta^2/2$ C. $\theta/2$ D. θ
86. The range of the function $f(x) = (\sin x)^{\sin x}$ defined on $(0, \pi)$ is
A. $(0, 1)$ B. $(e^{-1/e}, 1)$ C. $[e^{-1/e}, 1)$ D. $[e^{-1/e}, 1]$
87. Let A denote the area bounded by the curve $y = 1/x$ and the lines $y = 0$, $x = 1$, $x = 10$, let $B = 1 + \frac{1}{2} + \dots + \frac{1}{10}$, and let

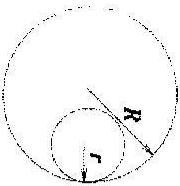
$$C = \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{10}.$$
Then
**A. $C < B < A$
**B. $A < C < B$
**C. $C < A < B$ and $A - C < B - A$
D. $C < A < B$ and $B - A < A - C$******

88. Two points are randomly chosen on the circumference of a circle of radius r . The probability that the distance between the two points is at least r is equal to
A. $2/\pi$ B. $\sin r$ C. $1/2$ D. $2/3$
89. Consider all natural numbers whose decimal expansion has only the even digits 0, 2, 4, 6, 8. Suppose these are arranged in increasing order. If a_n denotes the n -th number in this sequence, then $\lim_{n \rightarrow \infty} \log a_n / \log n$ equals
A. 0 B. $\log_5 10$ C. $\log_2 10$ D. 2
90. The sum of all absolute values of the differences of the numbers 1, 2, 3, ..., n , taken two at a time, i.e.,

$$\sum_{1 \leq i < j \leq n} |i - j|$$
equals
A. $\binom{n-1}{3}$ B. $\binom{n}{3}$ C. $\binom{n+1}{3}$ D. $\binom{n+2}{3}$

PHYSICS

- 91 A spherical cavity of radius r is carved out of a uniform solid sphere of radius R as shown in the figure.



The distance of the center of mass of the resulting body from that of the solid sphere is given by

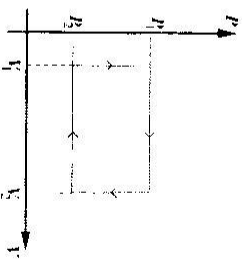
- A. $\frac{R-r}{2}$ C. 0
 B. $\frac{R+r}{2}$ D. $\frac{r^3}{R^2 + Rr + r^2}$

- 92 A plano-convex lens made of material of refractive index μ with radius of curvature R is silvered on the curved side. How far away from the lens-mirror must you place a point object so that the image coincides with the object?

- A. $\frac{R}{\mu}$ B. R C. $\frac{R}{\mu-1}$ D. μR

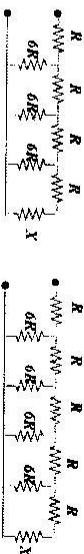
- 93 n moles of a van der Waals gas obeying the equation of state $(P + \frac{n^2 a}{V^2})(V - nb) = nRT$, where a and b are gas dependent constants, is made to undergo a cyclic process that is depicted by a rectangle in the PV diagram as shown in the figure. What is the heat absorbed by the gas in one cycle?

22



- A. $n(P_1 - P_2)(V_2 - V_1)$
 B. $(P_1 - P_2)(V_2 - V_1)$
 C. $(P_1 + \frac{n^2 a}{V_1^2} - P_2 - \frac{n^2 a}{V_2^2})(V_1 - V_2)$
 D. $P_1 + \frac{n^2 a}{V_1^2} - P_2 - \frac{n^2 a}{V_2^2})(V_2 - V_1)$

- 94 For what value of the resistor X will the equivalent resistance of the two circuits shown be the same?



- A. R B. $6R$ C. $2R$ D. $\frac{\sqrt{5}-1}{2}R$

- 95 A solid uniform sphere having a mass M , radius R , and moment of inertia of $\frac{2}{5}MR^2$ rolls down a plane inclined at an angle θ to the horizontal starting from rest. The coefficient of static friction between the sphere and the plane is μ . Then,

23

- A. the sphere will always roll without slipping.
 B. the sphere will always slide.
 C. the sphere will roll without slipping only if $\theta \leq \sin^{-1} \frac{7\mu}{2}$.
 D. the sphere will roll without slipping only if $\theta \leq \tan^{-1} \frac{7\mu}{2}$.

96 A cubical box of side a sitting on a rough table-top is pushed horizontally with a gradually increasing force until the box moves. If the force is applied at a height from the table top which is greater than a critical height H , the box topples first. If it is applied at a height less than H , the box starts sliding first. Then the coefficient of friction between the box and the table top is

- A. $\frac{a}{2H}$ B. $\frac{2H}{a}$ C. $\frac{a}{H}$ D. $\frac{H}{a}$

97 A vehicle is moving with speed v on a curved road of radius r . The coefficient of friction between the vehicle and the road is μ . The angle θ of banking needed is given by

- A. $\tan \theta = \frac{v^2 - \mu r g}{v^2 - r g}$ C. $\tan \theta = \frac{v^2 - \mu r g}{r g + \mu v^2}$
 B. $\tan \theta = \frac{v^2 - \mu r g}{v^2 + \mu r g}$ D. $\tan \theta = \frac{\mu r g - v^2}{r g + \mu v^2}$

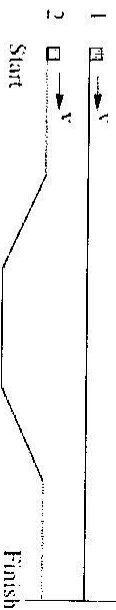
98 Two small identical speakers are connected in phase to the same source. The speakers are 3m apart and at ear level. An observer stands at P, 4m in front of one speaker as shown. The sound she hears is least intense when the wavelength is λ_1 and most intense when the wavelength is λ_2 .



Then possible values of λ_1 and λ_2 are:

- A. $\lambda_1 = 1\text{m}$ and $\lambda_2 = 2\text{m}$
 B. $\lambda_1 = 4\text{m}$ and $\lambda_2 = 3\text{m}$
 C. $\lambda_1 = 2\text{m}$ and $\lambda_2 = 1\text{m}$
 D. $\lambda_1 = 0.5\text{m}$ and $\lambda_2 = 0.25\text{m}$

99 Two small blocks slide without losing contact with the surface along two frictionless tracks 1 and 2, starting at the same time with same initial speed v . Track 1 is perfectly horizontal, while track 2 has a dip in the middle, as shown.



Which block reaches the finish line first?

[Hint: Use velocity-time graph to solve.]

- A. Block on track 1 reaches the finish line first.
 B. Block on track 2 reaches the finish line first.
 C. Both blocks reach the finish line at the same time.
 D. It depends on the length of the dip in the second track, relative to the total length of the tracks.

100 Consider 1 kg of liquid water undergoing change in phase to water vapour at 100°C . At 100°C , the vapour pressure is $1.01 \times 10^5 \text{ Nm}^{-2}$ and

the latent heat of vaporization is $22.6 \times 10^5 \text{ J kg}^{-1}$. The density of liquid water is 10^3 kg m^{-3} and that of vapour is $\frac{1}{1.8} \text{ kg m}^{-3}$. The change in internal energy in this phase change is nearly

- A. $1.8 \times 10^7 \text{ J kg}^{-1}$
 B. $20.8 \times 10^5 \text{ J kg}^{-1}$
 C. $22.6 \times 10^5 \text{ J kg}^{-1}$
 D. $11.3 \times 10^5 \text{ J kg}^{-1}$

CHEMISTRY

101 If the pH of a mixture of 10 ml of 0.1 M NH_4OH and 10 ml of 1 M NH_4Cl solution is 8, the $\text{p}K_a$ value of NH_4OH is then closest to

- A. 3 B. 5 C. 7 D. 9

102 A cylinder of cooking gas in a household contains 11.6 kg of butane. The thermochemical reaction for the combustion of butane is



If the household needs 15000 KJ of energy per day, the cooking gas cylinder will last for about

- A. 64 days B. 45 days C. 20 days D. 35 days

103 The addition of 0.643 g of a compound to 50 ml of benzene (density = 0.879 g ml^{-1}) lowers the freezing point from 5.51°C to 5.03°C . If the freezing point constant, K_f for benzene is $5.12 \text{ K kg mol}^{-1}$, the molar mass of the compound is approximately

- A. 156 g mol^{-1} C. 60 g mol^{-1}
 B. 88 g mol^{-1} D. 312 g mol^{-1}

104 Consider the following electrochemical cell.



If $E_{\text{cell}}^0 = 2.57 \text{ V}$, then e.m.f. of the cell at 298 K, is

- A. 2.5 V B. 1.5 V C. 0.5 V D. -0.5 V

105 When Co(II) chloride is dissolved in concentrated HCl a blue solution is obtained. Upon dilution with water, the color changes to pink because

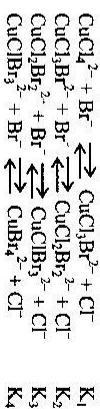
- A. CoCl_4^{2-} is converted to CoCl_6^{3-}
 B. CoCl_4^{2-} is converted to Co(OH)_2^{2+}
 C. Co(OH)_2^{2+} is converted to Co(OH)_2^{3+}
 D. CoCl_4^{2-} is converted to Co(OH)_2^{3+}

106 The rate constant for the reaction $\text{COCl}_2(\text{g}) \rightarrow \text{CO}(\text{g}) + \text{Cl}_2(\text{g})$ is given by

$\ln [k/(\text{min}^{-1})] = -11067/T + 31.33$. The temperature at which the rate of this reaction will be doubled from that at 25°C is

- A. 75°C C. 31°C
 B. 100°C D. 50°C

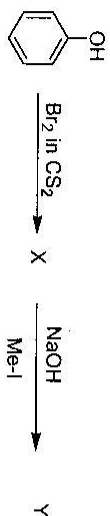
107 For the reactions and their equilibrium constants given below,



the equilibrium constant, K for the reaction $\text{CuCl}_4^{2-} + 3\text{Br}^- \rightleftharpoons \text{CuClBr}_3^{2-} + 3\text{Cl}^-$, is

- A. $K_1K_2K_3$ C. $K_1 + K_2 + K_3$
 B. $K_1K_2K_3K_4$ D. $1/(K_1K_2K_3)$

108



In the above sequence of reactions, the major products X and Y are

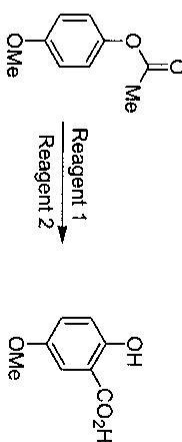
- A. X = Y =
- B. X = Y =
- C. X = Y =
- D. X = Y =

109 $^{234}\text{Th}_{90}$ gets converted to $^{206}\text{Pb}_{82}$ through a series of radioactive decay processes. The number of alpha and beta particles lost in this transformation respectively, are

- A. 6 and 6
B. 4 and 2
C. 6 and 7
D. 7 and 6

28

110 In the following transformation



reagents 1 and 2 are:

- A. H_2SO_4 ; alkaline KMnO_4
B. AlCl_3 ; I_2 / NaOH
C. H_3PO_4 ; CHCl_3 / KOH
D. KOH ; CHCl_3 / KOH

BIOLOGY

111 The mode of action of penicillin is as follows:

- A. it inhibits viral replication
B. it enhances immunity
C. it inhibits bacterial cell wall synthesis
D. it inhibits transcription

112 Which of the following statements is true for meiosis ?

- A. One round of chromosome duplication and one round of cell division
B. One round of chromosome duplication and two rounds of cell division
C. Two rounds of chromosome duplication and one round of cell division
D. Two rounds of chromosome duplication and two rounds of cell division

29

- 113 Instead of 3, if 2 bases code for an amino acid, the degeneracy of codons coding for the same amino acid would have
- increased
 - remained the same
 - decreased
 - been uncertain
- 114 Gregor Mendel showed that unit factors exist in pairs and exhibit a dominant-recessive relationship. These unit factors, in modern terminology, are called
- genes
 - alleles
 - loci
 - determinants
- 115 *E. coli* has optimal growth temperature of 37°C. Which of the following is an INCORRECT explanation for this?
- The membrane is most permeable at this temperature
 - DNA synthesis makes the least mistakes at this temperature
 - Most enzymes in the cell have the highest activity at this temperature
 - Protein synthesis is most efficient at this temperature
- 116 Male offspring of which of the following couples have the highest chance of haemophilia?
- Haemophilic father and normal, non-carrier mother
 - Haemophilic father and normal, carrier mother
 - Normal father and normal, carrier mother
 - Normal father and haemophilic mother
- 117 The effect of consumption of excess protein by normal individuals would result in,
- excretion of excess protein in urine
 - increase in the amount of adipose tissue
 - increase in the synthesis of muscle protein
 - increase in the circulatory plasma proteins
- 118 The condition varicose veins is swelling of veins, that occurs due to,
- loss of elasticity of the muscular layer
 - condition of high blood pressure
 - condition of low blood pressure
 - condition of anoxia
- 119 Greatest proportion of photosynthesis in the world is carried out by,
- trees in the rain forests of the world
 - trees in the temperate forests of the world
 - algae in oceans
 - irrigated crop fields
- 120 Energetically unfavourable reactions occur in human cells through,
- heat energy supplied by the body
 - heat energy released through exercise
 - coupling of energetically favourable reactions with unfavourable ones
 - photosynthesis