[This question paper contains 4 printed pages.]

1463-A *Your Roll No.* ....

B.A./B.Sc. (Hons.)/I **A**

MATHEMATICS – Unit – IV

(Analysis – II)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt any **two** *parts from each question.*

SECTION – I

1. (a) If f be continuous on [a, b] and $f(a) < 0 < f(b)$ then show that there exists a real number x_0 in]a, b[such that $f(x_0) = 0$. (5)

(b) Show that the function f defined on $\underset{\sim}{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is discontinuous at every point of $\underset{\sim}{R}$. (5)

(c) Define uniform continuity of a function f defined on an interval. Discuss the uniform continuity of the function.

$$f(x) = \sin\frac{1}{x} \quad \text{on }]0, \infty[\qquad (5)$$

SECTION – II

2. (a) Let f be the function defined on $\underset{\sim}{R}$ as

$$f(x) = |x-1| + |x+1|, \quad \forall\ x \in \underset{\sim}{R}$$

Discuss the derivability of f at $x = -1$ and $x = 1$. (5)

(b) If $f''(x) > 0$ for all $x \in R$, then show that:

$$f\left(\frac{1}{2}(x_1 + x_2)\right) \le \frac{1}{2}\left[f(x_1) + f(x_2)\right]$$

for every pair of real numbers x_1 and x_2. (5)

(c) State the conditions under which a function can be expanded as a Maclaurin's series and hence obtain series expansion of

$$f(x) = \cos x, \quad x \in \underset{\sim}{R} \qquad (5)$$

SECTION – III

3. (a) Show that sin x lies between

$$x - \frac{x^3}{6} \quad \text{and} \quad x - \frac{x^3}{6} + \frac{x^5}{120} \qquad (5)$$

(b) Find the values of a and b in order that

$$\lim_{x\to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3}$$ may be equal to 1/3. (5)

(c) Show that the function f, defined by

$$f(x) = x^5 - 5x^4 + 5x^3 - 1, \quad \forall\, x \in \mathbb{R}$$

has a maximum value when $x = 1$, a minimum value $x = 3$ and neither where $x = 0$. (5)

SECTION – IV

4. (a) Integrate any **two** of the following :

(i) $\int \frac{dx}{(1+x)\sqrt{x^2 - 1}}$

(ii) $\int \cos^{-\frac{3}{7}} x \sin^{-\frac{11}{7}} x \, dx$

(iii) $\int_0^{2a} x^2 \left(2ax - x^2\right)^{5/2} dx$ (4)

(b) If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta$, show that :

$$I_n + I_{n-2} = \frac{1}{n-1}, \quad n > 1$$

Deduce the value of I_5. (4)

(c) Evaluate the surface area of the solid generated by revolving the cycloid

$$x = a(\theta - \sin\theta), \ y = a(1 - \cos\theta),$$

about the line $y = 0$. (4)