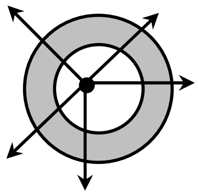
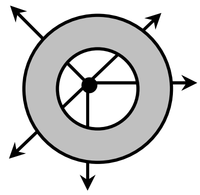
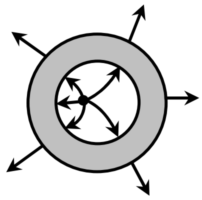
Note: *For the benefit of the students, specially the aspiring ones, the question of IIT-JEE, 2003 Screening are also given in this paper. Keeping the interest of students studying in class XI, the questions based on topics from class XI have been marked with '*', which can be attempted as a test. For this test the time allocated in Physics, Chemistry and Mathematics are **25 minutes, 25 minutes and 25 minutes** respectively.*

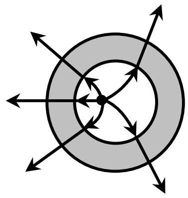
IIT–JEE, 2003 Screening

1. A metallic shell has a point charge 'q' kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces?

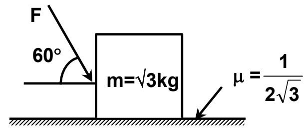
(A)


(B)


(C)


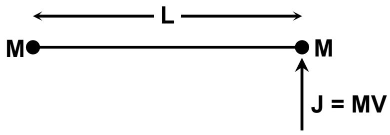
(D)


2*. What is the maximum value of the force F such that the block shown in the arrangement, does not move?



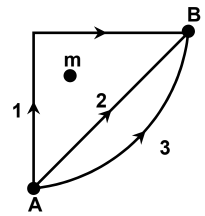
(A) 20 N (B) 10 N
(C) 12 N (D) 15 N

3*. Consider a body, shown in figure, consisting of two identical balls, each of mass M connected by a light rigid rod. If an impulse J = MV is imparted to the body at one of its ends, what would be its angular velocity?



(A) V/L (B) 2V/L
(C) V/3L (D) V/4L

4*. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3.

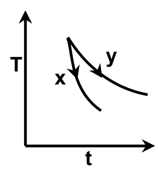


(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$
(C) $W_1 < W_2 < W_3$ (D) $W_2 > W_1 > W_3$

5. A cube has a side of length 1.2×10^{-2}m. Calculate its volume.

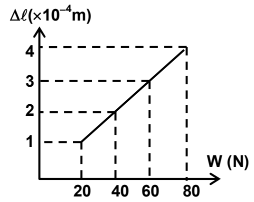
(A) 1.7×10^{-6} m^3. (B) 1.73×10^{-6} m^3.
(C) 1.70×10^{-6} m^3. (D) 1.732×10^{-6} m^3.

6*. The graph, shown in the adjacent diagram, represents the variation of temperature (T) of two bodies, x and y having same surface area, with time (t) due to the emission of radiation. Find the correct relation between the emissivity and absorptivity power of the two bodies.



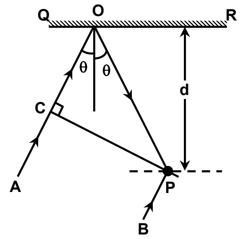
(A) $E_x > E_y$ & $a_x < a_y$ (B) $E_x < E_y$ & $a_x > a_y$
(C) $E_x > E_y$ & $a_x > a_y$ (D) $E_x < E_y$ & $a_x < a_y$

7*. The adjacent graph shows the extension ($\Delta\ell$) of a wire of length 1 m suspended from the top of a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 10^{-6} m^2, calculate the Young's modulus of the material of the wire.



(A) 2×10^{11} N/m (B) 2×10^{-11} N/m
(C) 3×10^{-12} N/m (D) 2×10^{-13} N/m

8. In the adjacent diagram, CP represents a wavefront and AO & BP, the corresponding two rays. Find the condition on θ for constructive interference at P between the ray BP and reflected ray OP.



(A) $\cos\theta = 3\lambda/2d$
(B) $\cos\theta = \lambda/4d$
(C) $\sec\theta - \cos\theta = \lambda/d$
(D) $\sec\theta - \cos\theta = 4\lambda/d$

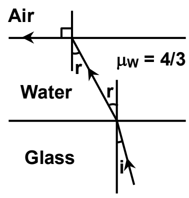
9*. Two rods, one of aluminum and the other made of steel, having initial length ℓ_1 and ℓ_2 are connected together to form a single rod of length $\ell_1 + \ell_2$. The coefficients of linear expansion for aluminum and steel are α_a and α_s respectively. If the length of each rod increases by the same amount when their temperature are raised by t°C, then find the ratio $\ell_1/(\ell_1 + \ell_2)$.

(A) α_s/α_a (B) α_a/α_s
(C) $\alpha_s/(\alpha_a + \alpha_s)$ (D) $\alpha_a/(\alpha_a + \alpha_s)$

10. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.

(A) 1.25 cm (B) 2.5 cm
(C) 1.05 cm (D) 2 cm

11. A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be



(A) (4/3)sini (B) 1/sini
(C) 4/3 (D) 1

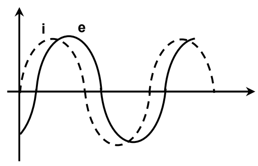
12. The electric potential between a proton and an electron is given by $V = V_0 \ln\frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number?

(A) $r_n \propto n$ (B) $r_n \propto 1/n$
(C) $r_n \propto n^2$ (D) $r_n \propto 1/n^2$

13. If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of $_{100}Fm^{257}$ is n times the Bohr radius, then find n.

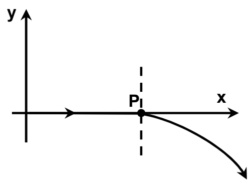
(A) 100 (B) 200
(C) 4 (D) 1/4

14. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference between the emf e and the current i in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of R-C or R-L or L-C in series, find the relationship between the two elements.



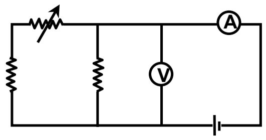
(A) R = 1 kΩ, C = 10 μF (B) R = 1 kΩ, C=1 μF
(C) R = 1 kΩ, L = 10 H (D) R = 1 kΩ, L = 1 H

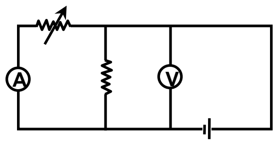
15. For a positively charged particle moving in a x-y plane initially along the x-axis, there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond P. The curved path is shown in the x-y plane and is found to be non-circular.
Which one of the following combinations is possible?

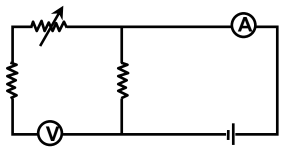


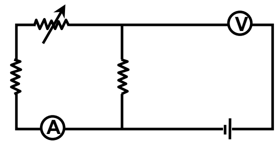
(A) $\vec{E} = 0$; $\vec{B} = b\hat{i} + c\hat{k}$ (B) $\vec{E} = a\hat{i}$; $\vec{B} = c\hat{k} + a\hat{i}$
(C) $\vec{E} = 0$; $\vec{B} = c\hat{j} + b\hat{k}$ (D) $\vec{E} = a\hat{i}$; $\vec{B} = c\hat{k} + b\hat{j}$

16. Express which of the following setups can be used to verify Ohm's law?

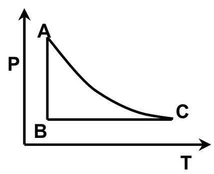
(A)


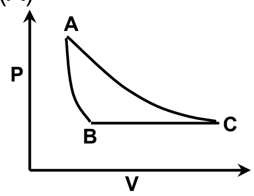
(B)


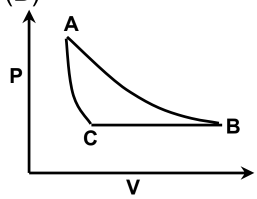
(C)


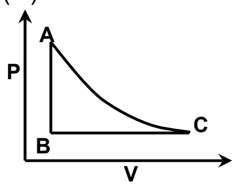
(D)


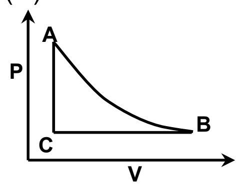
17*. The PT diagram for an ideal gas is shown in the figure, where AC is an adiabatic process, find the corresponding PV diagram.



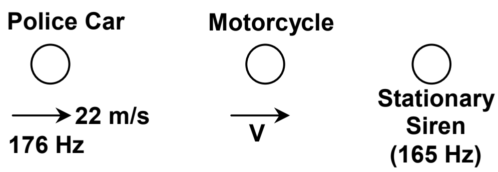
(A)


(B)


(D)


(D)


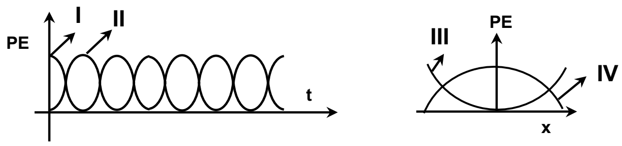
18*. A police car moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observes any beats.



(A) 33 m/s (B) 22 m/s
(C) zero (D) 11 m/s

19. For uranium nucleus how does its mass vary with volume?
(A) $m \propto V$ (B) $m \propto 1/V$
(C) $m \propto \sqrt{V}$ (D) $m \propto V^2$

20. For a particle executing SHM the displacement x is given by $x = A\cos\omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x.

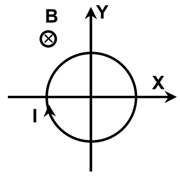


(A) I, III (B) II, IV
(C) II, III (D) I, IV

21*. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.
(A) 0.012 m (B) 0.025 m
(C) 0.05 m (D) 0.024 m

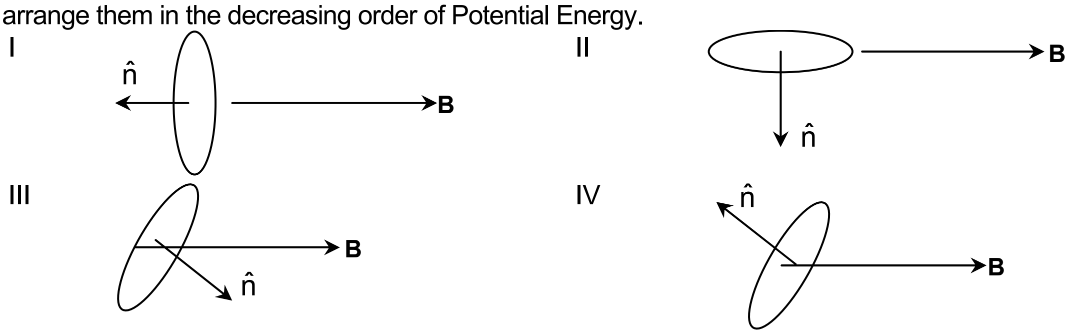
22*. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?
(A) centre of the circle (B) on the circumference of the circle.
(C) inside the circle. (D) outside the circle.

23. A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to



(A) contract (B) expand
(C) move towards +ve x-axis (D) move towards –ve x axis.

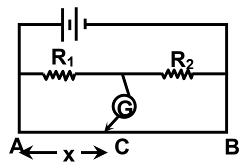
24. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III & IV, arrange them in the decreasing order of Potential Energy.



(A) I > III > II > IV (B) I > II > III > IV
(C) I > IV > II > III (D) III > IV > I > II

25*. 2 kg of ice at –20°C is mixed with 5 kg of water at 20°C in an insulating vessel having a negligible heat capacity. Calculate the final mass of water remaining in the container. It is given that the specific heats of water & ice are 1 kcal/kg/°C & 0.5 kcal/kg/°C while the latent heat of fusion of ice is 80 kcal/kg.
(A) 7 kg (B) 6 kg
(C) 4 kg (D) 2 kg

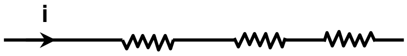
26. In the shown arrangement of the experiment of the meter bridge if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?



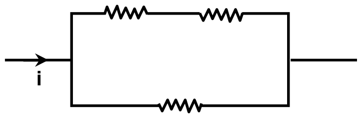
(A) x (B) x/4
(C) 4x (D) 2x

27. The three resistance of equal value are arranged in the different combinations shown below. Arrange them in increasing order of power dissipation.

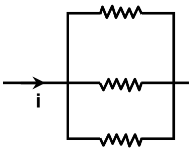
I



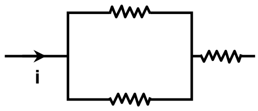
II



III



IV



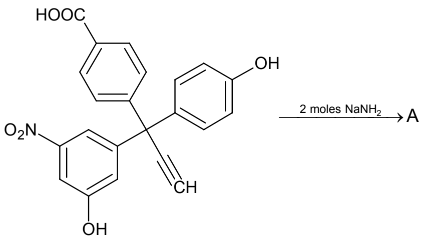
(A) III < II < IV < I (B) II < III < IV < I
(C) I < IV < III < II (D) I < III < II < IV

28. A nucleus with mass number 220 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.
(A) 4.4 MeV (B) 5.4 MeV
(C) 5.6 MeV (D) 6.5 MeV

29*. Among the following, the molecule with the highest dipole moment is:
(A) CH_3Cl (B) CH_2Cl_2
(C) $CHCl_3$ (D) CCl_4

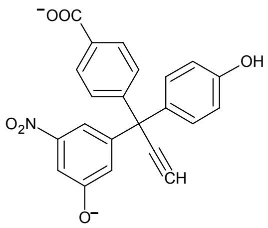
30*. Which of the following are isoelectronic and isostructural?
NO_3^-, CO_3^{2-}, ClO_3^-, SO_3
(A) NO_3^-, CO_3^{2-} (B) SO_3, NO_3^-
(C) ClO_3^-, CO_3^{2-} (D) CO_3^{2-}, SO_3

31.

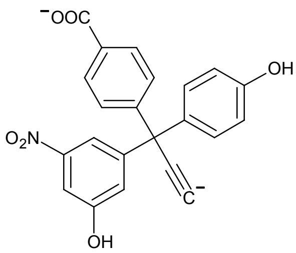


The product A will be

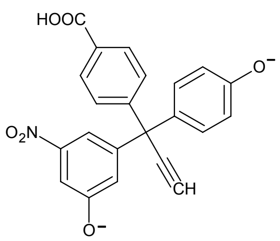
(A)



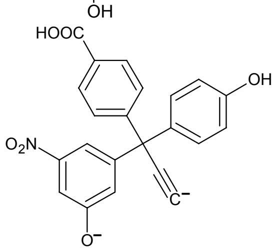
(B)



(C)



(D)

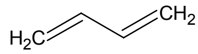


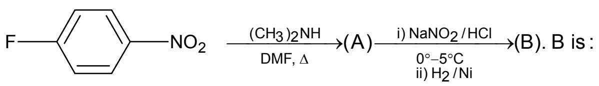
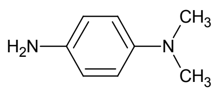
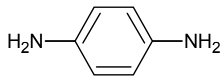
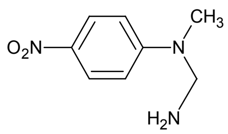
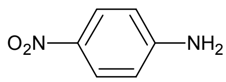
32. [X] + $H_2SO_4 \longrightarrow$ [Y] a colourless gas with irritating smell
[Y] + $K_2Cr_2O_7$ + $H_2SO_4 \longrightarrow$ green solution
[X] and [Y] is:
(A) SO_3^{2-}, SO_2
(B) Cl^-, HCl
(C) S^{2-}, H_2S
(D) CO_3^{2-}, CO_2

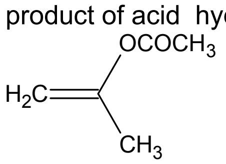
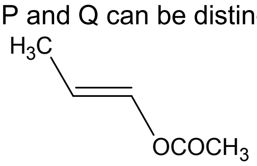
33*. For H_3PO_3 and H_3PO_4 the correct choice is:
(A) H_3PO_3 is dibasic and reducing
(B) H_3PO_3 is dibasic and non–reducing
(C) H_3PO_4 is tribasic and reducing
(D) H_3PO_3 is tribasic and non–reducing

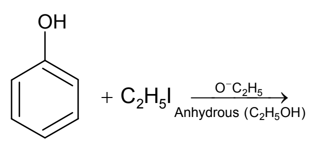
34. When MnO_2 is fused with KOH, a coloured compound is formed, the product and its colour is:
(A) K_2MnO_4, purple green
(B) $KMnO_4$, purple
(C) Mn_2O_3, brown
(D) Mn_3O_4 black

35. Rate of physiorption increases with
(A) decrease in temperature
(B) increase in temperature
(C) decrease in pressure
(D) decrease in surface area

36*. Which of the following represents the given mode of hybridisation $sp^2 - sp^2 - sp - sp$ from left to right?
(A) $H_2C{=}CH{-}C{\equiv}N$
(B) $HC{\equiv}C{-}C{\equiv}CH$
(C) $H_2C{=}C{=}C{=}CH_2$
(D) 

37. 
(A) 
(B) 
(C) 
(D) 

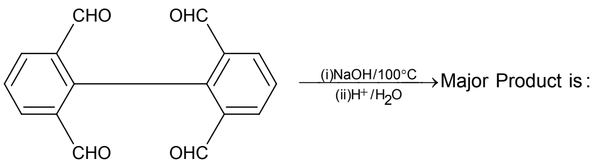
38. The product of acid hydrolysis of P and Q can be distinguished by
P = 
Q = 
(A) Lucas Reagent
(B) 2,4–DNP
(C) Fehling's Solution
(D) $NaHSO_3$

39. 
(A) $C_6H_5OC_2H_5$
(B) $C_2H_5OC_2H_5$
(C) $C_6H_5OC_6H_5$
(D) C_6H_5I

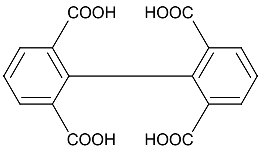
40*. Which has maximum number of atoms?

(A) 24g of C (12)
(B) 56g of Fe (56)
(C) 27g of Al (27)
(D) 108g of Ag (108)

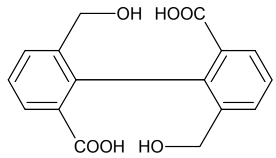
41.



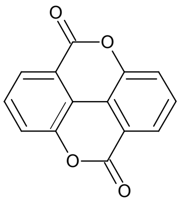
(A)



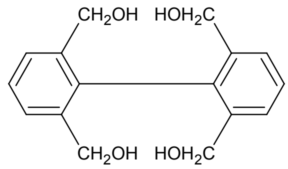
(B)



(C)



(D)



42. In the electrolytic cell, flow of electrons is from

(A) Cathode to anode in solution
(B) Cathode to anode through external supply
(C) Cathode to anode through internal supply
(D) Anode to cathode through internal supply

43*. In a first order reaction the concentration of reactant decreases from 800 mol/dm^3 to 50 mol/dm^3 is 2×10^4 sec. The rate constant of reaction in sec^{-1} is:

(A) 2×10^4
(B) 3.45×10^{-5}
(C) 1.386×10^{-4}
(D) 2×10^{-4}

44. During depression of freezing point in a solution the following are in equilibrium

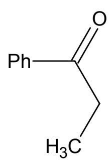
(A) liquid solvent, solid solvent
(B) liquid solvent, solid solute
(C) liquid solute, solid solute
(D) liquid solute solid solvent

45. H_3BO_3 is:

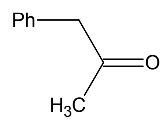
(A) Monobasic and weak Lewis acid
(B) Monobasic and weak Bronsted acid
(C) Monobasic and strong Lewis acid
(D) Tribasic and weak Bronsted acid

46*. $Ph-C\equiv C-CH_3 \xrightarrow{Hg^{2+}/H^+} A$. A is:

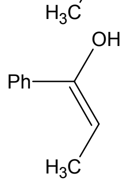
(A)



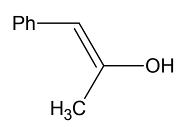
(B)



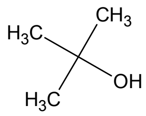
(C)

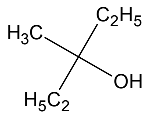


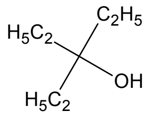
(D)

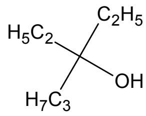


47. Ethyl ester $\xrightarrow[\text{excess}]{CH_3MgBr}$ P. The product P will be

(A) 

(B) 

(C) 

(D) 

48. Mixture X = 0.02 mol of $[Co(NH_3)_5SO_4]Br$ and 0.02 mol of $[Co(NH_3)_5Br]SO_4$ was prepared in 2 litre of solution.
1 litre of mixture X + excess $AgNO_3 \longrightarrow$ Y.
1 litre of mixture X + excess $BaCl_2 \longrightarrow$ Z
No. of moles of Y and Z are
(A) 0.01, 0.01
(B) 0.02, 0.01
(C) 0.01, 0.02
(D) 0.02, 0.02

49. Which of the reaction defines ΔH_f^0
(A) $C_{(diamond)} + O_{2(g)} \longrightarrow CO_{2(g)}$
(B) $\frac{1}{2} H_{2(g)} + \frac{1}{2} F_{2(g)} \longrightarrow HF_{(g)}$
(C) $N_{2(g)} + 3H_{2(g)} \longrightarrow 2NH_{3(g)}$
(D) $CO_{(g)} + \frac{1}{2} O_{2(g)} \longrightarrow CO_{2(g)}$

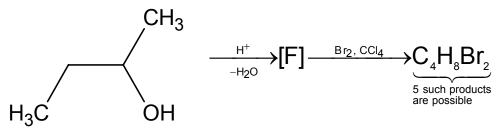
50*. ^{23}Na is the more stable isotope of Na. Find out the process by which $^{24}_{11}Na$ can undergo radioactive decay
(A) β^- emission
(B) α emission
(C) β^+ emission
(D) K electron capture

51. $(Me)_2SiCl_2$ on hydrolysis will produce
(A) $(Me)_2Si(OH)_2$
(B) $(Me)_2Si = O$
(C) $-[-O-(Me)_2Si-O-]_n-$
(D) $Me_2SiCl(OH)$

52*. A solution which is 10^{-3} M each in Mn^{2+}, Fe^{2+}, Zn^{2+} and Hg^{2+} is treated with 10^{-16} M sulphide ion. If K_{sp} of MnS, FeS, ZnS and HgS are 10^{-15}, 10^{-23}, 10^{-20} and 10^{-54} respectively, which one will precipitate first?
(A) FeS
(B) MgS
(C) HgS
(D) ZnS

53. In the process of extraction of gold,
Roasted gold ore + CN^- + $H_2O \xrightarrow{O_2}$ [X] + OH^-
[X] + Zn $\longrightarrow$ [Y] + Au
Identify the complexes [X] and [Y]
(A) X = $[Au(CN)_2]^-$, Y = $[Zn(CN)_4]^{2-}$
(B) X = $[Au(CN)_4]^{3-}$, Y = $[Zn(CN)_4]^{2-}$
(C) X = $[Au(CN)_2]^-$, Y = $Zn(CN)_6]^{4-}$
(D) X = $[Au(CN)_4]^-$, Y = $[Zn(CN)_4]^{2-}$

54*. Positive deviation from ideal behaviour takes place because of
(A) Molecular interaction between atoms and $PV/nRT > 1$
(B) Molecular interaction between atoms and $PV/nRT < 1$
(C) Finite size of atoms and $PV/nRT > 1$
(D) Finite size of atoms and $PV/nRT < 1$

55*.


How many structures of F is possible?

(A) 2 (B) 5
(C) 6 (D) 3

56. An enantiomerically pure acid is treated with racemic mixture of an alcohol having one chiral carbon. The ester formed will be
(A) Optically active mixture (B) Pure enantiomer
(C) Meso compound (D) Racemic mixture

57. If $f : [0, \infty) \rightarrow [0, \infty)$, and $f(x) = \frac{x}{1+x}$ then f is
(A) one–one and onto (B) one–one but not onto
(C) onto but not one–one (D) neither one–one nor onto

58. If $P(B) = \frac{3}{4}$, $P(A \cap B \cap \overline{C}) = \frac{1}{3}$ and $P(\overline{A} \cap B \cap \overline{C}) = \frac{1}{3}$, then $P(B \cap C)$ is
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$
(C) $\frac{1}{15}$ (D) $\frac{1}{9}$

59. In [0, 1] Lagranges Mean Value theorem is NOT applicable to
(A) $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \ge \frac{1}{2} \end{cases}$ (B) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$
(C) $f(x) = x|x|$ (D) $f(x) = |x|$

60*. The area of the quadrilateral formed by the tangents at the end points of latus recta to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is
(A) 27/4 sq. units (B) 9 sq. units
(C) 27/2 sq. units (D) 27 sq. units

61*. The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is
(A) 133 (B) 190
(C) 233 (D) 105

62. If A = $\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which A^2 = B, is
(A) 1 (B) – 1
(C) 4 (D) no real values

63. The value of k such that $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane 2x – 4y + z = 7, is
(A) 7 (B) – 7
(C) no real value (D) 4

64*. If the angles of a triangle are in the ratio 4 : 1 : 1, then the ratio of the longest side to the perimeter is
(A) $\sqrt{3} : (2 + \sqrt{3})$ (B) 1 : 6
(C) $1 : 2 + \sqrt{3}$ (D) 2 : 3

65. If $\lim_{x \to 0} \frac{((a-n)nx - \tan x)\sin nx}{x^2}$ = 0, where n is nonzero real number, then a is equal to
(A) 0 (B) $\frac{n+1}{n}$
(C) n (D) $n + \frac{1}{n}$

66. Two numbers are selected randomly from the set S = {1, 2, 3, 4, 5, 6} without replacement one by one. The probability that minimum of the two numbers is less than 4 is
(A) 1/15 (B) 14/15
(C) 1/5 (D) 4/5

67*. For hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$ which of the following remains constant with change in 'α'
(A) abscissae of vertices (B) abscissae of foci
(C) eccentricity (D) directrix

68. Range of the function f (x) = $\frac{x^2 + x + 2}{x^2 + x + 1}$; $x \in R$ is
(A) $(1, \infty)$ (B) (1, 11/ 7]
(C) (1, 7/ 3] (D) (1, 7/ 5]

69. $\lim_{h \to 0} \frac{f(2h + 2 + h^2) - f(2)}{f(h - h^2 + 1) - f(1)}$, given that $f'(2) = 6$ and $f'(1) = 4$
(A) does not exist (B) is equal to – 3/ 2
(C) is equal to 3/ 2 (D) is equal to 3

70*. If f (x) = $x^2 + 2bx + 2c^2$ and g (x) = $-x^2 - 2cx + b^2$ such that min f (x) > max g (x), then the relation between b and c, is
(A) no real value of b and c (B) $0 < c < b\sqrt{2}$
(C) $|c| < |b|\sqrt{2}$ (D) $|c| > |b|\sqrt{2}$

71*. The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is
(A) (4, 7) (B) (7, 4)
(C) (9, 4) (D) (4, 9)

72*. The focal chord to $y^2 = 16x$ is tangent to $(x - 6)^2 + y^2 = 2$, then the possible values of the slope of this chord, are

(A) {–1, 1} (B) {–2, 2}
(C) {–2, 1/2} (D) {2, –1/2}

73. Domain of definition of the function f(x) = $\sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued x, is

(A) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ (B) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(C) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ (D) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

74*. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then Re(ω) is

(A) 0 (B) $-\frac{1}{|z+1|^2}$
(C) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$

75*. If $\alpha \in \left(0, \frac{\pi}{2}\right)$ then $\sqrt{x^2 + x} + \frac{\tan^2 \alpha}{\sqrt{x^2 + x}}$ is always greater than or equal to

(A) 2 $\tan\alpha$ (B) 1
(C) 2 (D) $\sec^2\alpha$

76. If I(m, n) = $\int_0^1 t^m (1+t)^n dt$, then the expression for I(m, n) in terms of I(m + 1, n - 1) is

(A) $\frac{2^n}{m+1} - \frac{n}{m+1} I(m+1, n-1)$ (B) $\frac{n}{m+1} I(m+1, n-1)$
(C) $\frac{2^n}{m+1} + \frac{n}{m+1} I(m+1, n-1)$ (D) $\frac{m}{n+1} I(m+1, n-1)$

77. If f (x) = $\int_{x^2}^{x^2+1} e^{-t^2} dt$, then f (x) increases in

(A) (2, 2) (B) no value of x
(C) $(0, \infty)$ (D) $(-\infty, 0)$

78. The area bounded by the curves $y = \sqrt{x}$, 2y + 3 = x and x–axis in the I^{st} quadrant is

(A) 9 (B) 27/4
(C) 36 (D) 18

79*. Coefficient of t^{24} in $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is

(A) ${}^{12}C_6 + 3$ (B) ${}^{12}C_6 + 1$
(C) ${}^{12}C_6$ (D) ${}^{12}C_6 + 2$

80. The value of 'a' so that the volume of parallelopiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is
(A) – 3 (B) 3
(C) $1/\sqrt{3}$ (D) $\sqrt{3}$

81. If the system of equations x + ay = 0, az + y = 0 and ax + z = 0 has infinite solutions, then the value of a is
(A) – 1 (B) 1
(C) 0 (D) no real values

82. If y(t) is a solution of $(1+t)\frac{dy}{dt}-ty=1$ and y(0) = -1, then y(1) is equal to
(A) – 1/2 (B) e + 1/2
(C) e – 1/2 (D) 1/2

83*. Tangent is drawn to ellipse $\frac{x^2}{27}+y^2=1$ at $(3\sqrt{3}\cos\theta,\ \sin\theta)$ (where $\theta \in (0, \pi/2)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is
(A) $\pi/3$ (B) $\pi/6$
(C) $\pi/8$ (D) $\pi/4$

84*. Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is
(A) $\left(3, \frac{5}{4}\right)$ (B) (3, 12)
(C) $\left(3, \frac{3}{4}\right)$ (D) (3, 9)

*Note: **FIITJEE** solutions to IIT–JEE, 2003 Screening Test is based on Screening Test paper created using memory retention of select **FIITJEE** students appeared in this test. Every effort has been made to reproduce the original paper in the interest of the aspirant students, yet **FIITJEE** does not guarantee its authenticity.*

Answers

1.	**C**	2.	**A**	3.	**A**
4.	**B**	5.	**A**	6.	**C**
7.	**A**	8.	**B**	9.	**C**
10.	**B**	11.	**B**	12.	**A**
13.	**D**	14.	**A**	15.	**B**
16.	**A**	17.		18.	**B**
19.	**A**	20.	**A**	21.	**B**
22.	**A**	23.	**B**	24.	**C**
25.	**B**	26.	**A**	27.	**A**
28.	**B**	29.	**A**	30.	**A**
31.	**A**	32.	**A**	33.	**A**
34.	**A**	35.	**A**	36.	**A**
37.	**A**	38.	**C**	39.	**A**
40.	**A**	41.	**B**	42.	**C**
43.	**C**	44.	**A**	45.	**A**
46.	**A**	47.	**A**	48.	**A**
49.	**B**	50.	**A**	51.	**C**
52.	**C**	53.	**A**	54.	**A**
55.	**D**	56.	**A**	57.	**B**
58.	**A**	59.	**A**	60.	**D**
61.	**B**	62.	**D**	63.	**A**
64.	**A**	65.	**D**	66.	**D**
67.	**B**	68.	**C**	69.	**D**
70.	**D**	71.	**A**	72.	**A**
73.	**A**	74.	**A**	75.	**A**
76.	**A**	77.	**D**	78.	**A**
79.	**D**	80.	**C**	81.	**A**
82.	**A**	83.	**B**	84.	**C**

Hints & Solution

1. Electric field is perpendicular to the equipotential surfaces and is zero everywhere inside the metal.

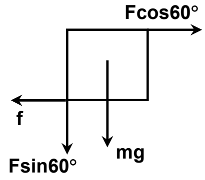
2. For no motion F.B.D.

$F\cos 60° \le \mu(mg + F\sin 60°)$

$\Rightarrow \quad F/2 \le \frac{1}{2\sqrt{3}}\left(\sqrt{3}\times g + \frac{F\sqrt{3}}{2}\right)$

$\Rightarrow \quad F/2 \le g$

$\Rightarrow \quad F_{max} = 20$ N



3. Applying conservation of angular momentum about centre of the rod

$$I_{cm}\omega = J\frac{L}{2} \Rightarrow 2\times M\left(\frac{L}{2}\right)^2 = MV\times\frac{L}{2} \Rightarrow \omega = V/L$$

4. In a conservative field work done does not depend on the path.

$\therefore W_1 = W_2 = W_3$

5. $V = \ell^3 = (1.2 \times 10^{-2}\text{ m})^3 = 1.728 \times 10^{-6}\text{ m}^3$

$\because$ ℓ has two significant figure. Hence V will also have two significant figure.

6. $E_x > E_y$ & $a_x > a_y$

$\because$ E/a = constant

7. $Y = \frac{F}{A}\Big/\frac{\Delta\ell}{\ell} = \frac{20\times 1}{10^{-6}\times 10^{-4}} = 2 \times 10^{11}\text{ N/m}^2$

8. For constructive interference $\Delta\phi = (d\sec\theta + d\sec\theta\cos 2\theta)\frac{2\pi}{\lambda} + \pi = 2n\pi$

$\Rightarrow \quad \cos\theta = \frac{(2n-1)\lambda}{4d}$

9. $\ell_1\alpha_a t = \ell_2\alpha_s t$

$\Rightarrow \frac{\ell_2}{\ell_1} = \frac{\alpha_a}{\alpha_s} \Rightarrow \frac{\ell_2+\ell_1}{\ell_1} = \frac{\alpha_a+\alpha_s}{\alpha_s}$

$\Rightarrow \frac{\ell_1}{\ell_1+\ell_2} = \frac{\alpha_s}{\alpha_a+\alpha_s}$

10. For concave lens

$\frac{1}{v} - \frac{1}{4} = \frac{1}{-20} \quad \Rightarrow \quad v = 5$ cm

$\frac{h_2}{h_1} = \frac{|v|}{|u|} \Rightarrow h_2 = 2\times\frac{5}{4} = 2.5$ cm

11. Using Snell's law

(1) $\sin 90° = \mu_{glass}\sin i$

$\mu_{glass} = 1/\sin i$

12. $|F| = \left|-\frac{dU}{dr}\right| = \frac{mv^2}{r}$

$\Rightarrow v = \sqrt{\frac{v_0 r}{m}}$ which is a constant

$mv_n r_n = \frac{nh}{2\pi}$

$r_n \propto n$

13. $r_n = \frac{m^2}{Z}(0.53)A^\circ$

$(r_n)_{Fm} = \frac{m^2(0.53)A^\circ}{100} = 0.53\ A^\circ \times n$

m is 5 for Z = 100

$\Rightarrow n = 1/4$

14. Circuit is R-C circuit

$\tan\frac{\pi}{4} = \frac{1}{RC\omega}$

$\Rightarrow \quad R = \frac{1}{\omega C}$

15. Plane of motion must be perpendicular to at least one of the component of the magnetic field.

16. Ammeter is always connected in series and voltmeter is always connected in parallel

17. Out of the alternatives provided, none appears completely correct.

18. f_1 = frequency of the police car heard by motorcyclist.
f_2 = frequency of the siren heard by motorcyclist.

$f_1 = \frac{330 - v}{330 - 22} \times 176$;

$f_2 = \frac{330 + v}{330} \times 165$

$\because \quad f_1 - f_2 = 0$

$\Rightarrow \quad v = 22$ m/s

19. Since nuclear density is constant hence mass α volume.

$\Rightarrow \quad m \propto V$

21. $\frac{v}{4(\ell_1 + x)} = \frac{3v}{4(\ell_2 + x)}$

x = 2.5 cm

22. About the centre of circle the torque of the force causing the circular motion is zero.

24. $U = -\vec{\mu}\cdot\vec{B}$

25. $m_{ice}C_{ice}[0 - (-20)] + m'L + m_{water}C_{water}[0-20] = 0$

$m' = 1$ kg

mass of water = 6 kg

27. Power dissipated $\propto R_{equivalent}$

28. Applying conservation of linear momentum and COE
$\therefore KE_{\alpha} = 5.4$ MeV

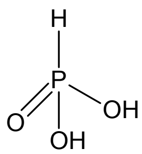
29. Methane molecule being symmetrical, has zero dipole moment. Replacement of one of the H– atoms by Cl atom increases the dipole moment. The increase in dipole moment is rather more than what can be expected because of the fact that the bond dipole moment of C – H bond and that of C – Cl bond reinforce one another. Replacement of another H atom of Cl increases the bond angle due to lone pair – lone pair repulsion between two Cl–atoms thereby reducing the dipole moment of the molecule. Increase in angle is again caused by the introduction of third Cl–atom. When the fourth Cl–atom is introduced, the molecule (CCl_4) again becomes symmetrical and dipole moment reduces to zero. So, CH_3Cl will have the maximum dipole moment.

30. NO_3^- and CO_3^{2-} both have same number of electrons (equal to 32) and central atom in each being sp^2 hybridised, are isostructural too

31. Two moles of $\overset{-}{N}H_2$ ions will abstract two moles of most acidic hydrogen out of the four moles of hydrogen present per mole of the acidic compound. The acidic strength is in the order:
– COOH > – OH (phenolic deactivated by NO_2 group) > – OH (phenolic) > alkynic H.

32. SO_2 and H_2S both being reducing agents, can turn acidified dichromate solution green. SO_2 can be obtained by the action of acid upon sulphite while H_2S is evolved by the action of acid upon sulphide. However, SO_2 has a burning sulphur smell which is irritating. H_2S has rotten egg like smell.

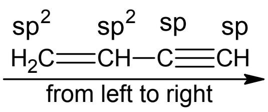
33. The structure of H_3PO_3 is as follows: There are only two –OH groups and hence dibasic. The oxidation number of P in this acid is +3. Whereas P may have +5 oxidation state also. Therefore, H_3PO_3 can be oxidised which means H_3PO_3 is a reducing agent.



34. Stable oxidation state of Mn in alkaline medium is +6. So, MnO_2 is oxidised to K_2MnO_4 by atomospheric oxygen in KOH medium.
$2MnO_2 + 4KOH + O_2 \longrightarrow 2K_2MnO_4 + 2H_2O$

35. Rate of physiorsption increases with decrease of temperature

36.



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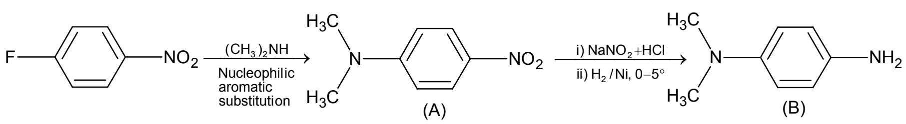

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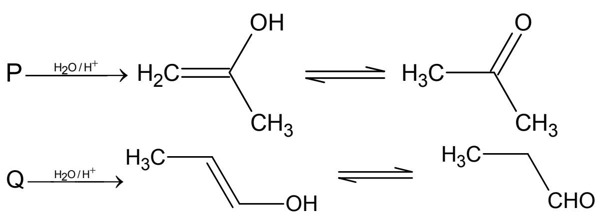
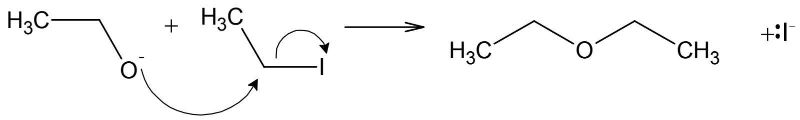


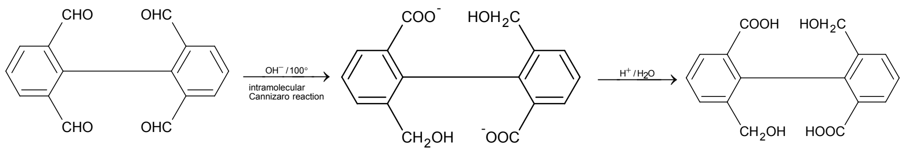
Ketone (non–reducing) and aldehyde (reducing) can be distinguished by Fehling solution.

39. $C_2H_5O^-$ will abstract proton from phenol converting the latter into phenoxide ion. This would then make nucleophilic attack on the methylene carbon of alkyl iodide. But $C_2H_5O^-$ is in excess. $C_2H_5O^-$ is better nucleophile than $C_6H_5O^-$ (phenoxide) ion since while in the former the negative charge is localised over oxygen and in the latter it is delocalised over the whole molecular framework. So, it is $C_2H_5O^-$ ion that would make nucleophilic attack at ethyl iodide to give diethyl ether (Williamson's synthesis).



40. 24g carbon amounts to 2 mole i.e. $2 \times 6.023 \times 10^{23}$ atoms whereas all the rest amount to 1 mole each only.

41.

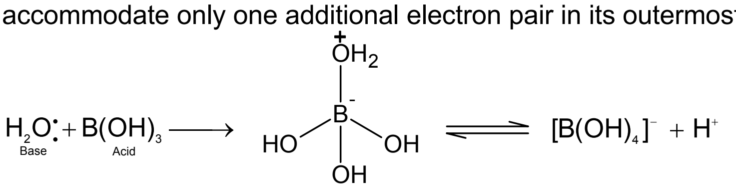


42. In an electrolytic cell electrons do not flow themselves. It is the migration of ions towards oppositely charged electrons that indirectly constitutes the flow of electrons from cathode to anode through internal supply.

43. $K = \dfrac{2.303}{t}\log\dfrac{[A]_0}{[A]_t} = \dfrac{2.303}{2\times10^4}\log\dfrac{800}{50} = 1.386 \times 10^4\ s^{-1}$

44. At the freezing point liquid and solid remain in equilibrium. If a solution of a non–volatile solute is cooled to a temperature below the freezing point of solution, some of liquid solvent will separate as a solid solvent and thus the concentration of solution will increase.

45. The central boron atom in boric acid, H_3BO_3 is electron–deficient i.e. boric acid is a Lewis acid with one p–orbital vacant. There is no d–orbital of suitable energy in boron atom. So, it can accommodate only one additional electron pair in its outermost shell.



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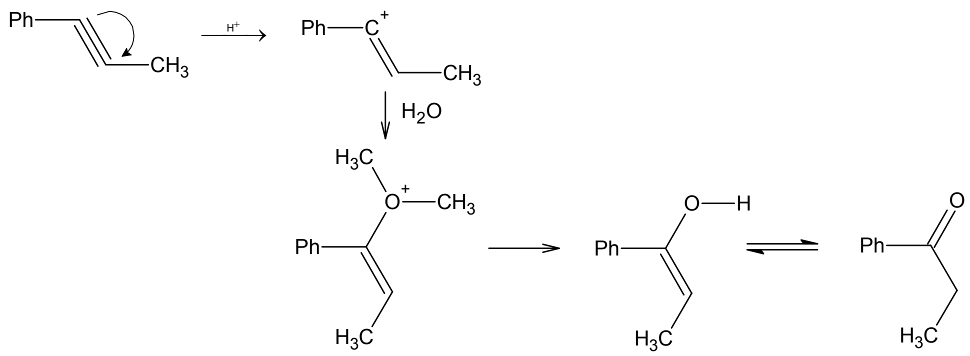
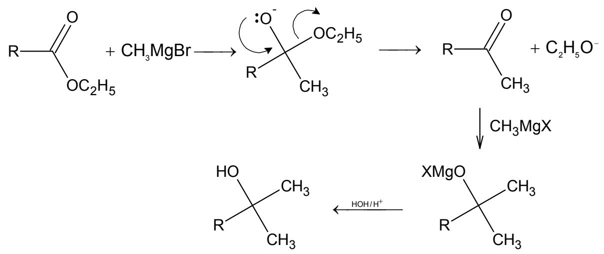


47. 

Thus, the choice with at least two methyl groups at the carbon linked with –OH group will be the correct choice.

48. $\underset{\text{Excess}}{Ag^+} + \underset{0.01\text{ mole}}{Br^-} \longrightarrow \underset{0.01\text{ mole}}{AgBr}$

$\underset{\text{Excess}}{Ba^{2+}} + \underset{0.01\text{ mole}}{SO_4^{2-}} \longrightarrow \underset{0.01\text{ mole}}{BaSO_4}$

50. $\frac{n}{p}$ ratio of ^{24}Na nuclide is $\frac{13}{11}$ i.e. greater than unity and hence radioactive. To achieve stability, it would tend to adjust its $\frac{n}{p}$ ratio to the proper value of unity. This can be done by breaking a neutron into proton and electron.

$${}_0n^1 \longrightarrow {}_{+1}p^1 + {}_{-1}e^0 \text{ or } \beta^-$$

The proton will stay inside the nucleus whereas electron which cannot exist in the nucleus, will be emitted out as β–ray.

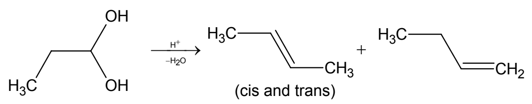
51. It appears at the first sight that Me_2SiCl_2 on hydrolysis will produce $Me_2Si(OH)_2$ which ultimately upon loss of water, will form $Me_2Si = O$. But silicon atom, because of its very large size in comparison to oxygen, is unable to form π–bond. Thus, the product of hydrolysis is polymeric in nature.

52. HgS having the lowest K_{sp} among the lot will precipitate first.

53. $2Au + 4CN^- + H_2O + \frac{1}{2}O_2 \longrightarrow 2[Au(CN)_2]^- + 2OH^-$

$2[Au(CN)_2]^- + Zn \longrightarrow [Zn(CN)_4]^{2-} + 2Au$

54. For positive deviation: PV = nRT + nPb
Thus, the factor nPb is responsible for increasing the PV value above ideal value. b is actually the effective volume of molecule. So, it is the finite size of molecules that leads to the origin of b and hence positive deviation at high pressure.

55*. 

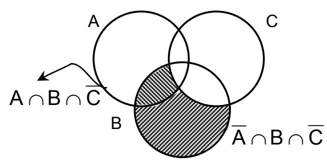
56. The optically active acid will react with d and l forms of alcohol present in the racemic mixture to form two types of isomeric esters. In each the configuration of the chiral centre of acid will remain the same. So the mixture will be optically active.

57. $f'(x) = \frac{1}{(1+x)^2} > 0 \ \forall \ x \in [0, \infty)$ and range $\in [0, 1) \Rightarrow$ function is one–one but not onto

58. From venn diagram, we can see that

$P(B \cap C) = P(B) - P(A \cap B \cap \overline{C}) - P(\overline{A} \cap B \cap \overline{C})$

$= \frac{3}{4} - \frac{1}{3} - \frac{1}{3} = \frac{1}{12}$.



59. The function defined in (A) is not differentiable at x = 1/2.

60. By symmetry the quadrilateral is a rhombus. So area is four times the area of the right angled triangle formed by the tangent and axes in the I[st] quadrant.

Now, $ae = \sqrt{a^2 - b^2} \Rightarrow ae = 2$

$\Rightarrow$ tangent (in first quadrant) at end of latus rectum $\left(2, \frac{5}{3}\right)$ is $\frac{2}{9}x + \frac{5}{3}\frac{y}{5} = 1$ i.e. $\frac{x}{9/2} + \frac{y}{3} = 1$

Area $= 4.\frac{1}{2}\cdot\frac{9}{2}.3 = 27$ sq units.

61. Number of points exactly in the interior of the triangle $= \frac{20 \times 20 - 20}{2} = 190$.

62. $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

clearly, no real value of α.

63. Point (4, 2, k) should lie in the given plane $\Rightarrow 2(4) - 4(2) + 1(k) = 7 \Rightarrow k = 7$.

64. Angles are 30°, 30° and 120°. Hence sides are x, x and 2x cos30° ratio is $\frac{2x\cos 30^0}{2x + 2x\cos 30^0}$

$\Rightarrow \sqrt{3} : (2 + \sqrt{3})$.

65. $\lim_{x\to 0} n\frac{\sin nx}{nx}\cdot \lim_{x\to 0}\left((a-n)n - \frac{\tan x}{x}\right) = 0 \Rightarrow n((a-n)n - 1) = 0 \Rightarrow (a-n)n = 1 \Rightarrow a = n + \frac{1}{n}$.

66. Total ways $2!\,{}^6C_2 = 30$. Favourable cases = 30 − 6 = 24. Probability $= \frac{24}{30} = \frac{4}{5}$

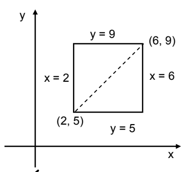
67. $ae = \sqrt{a^2 + b^2} = \sqrt{\cos^2\alpha + \sin^2\alpha} = 1$.

68. $f(x) = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} \Rightarrow$ Range = (1, 7/ 3].

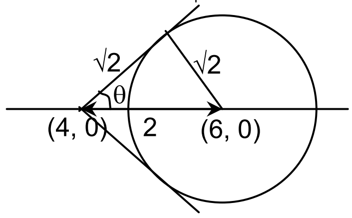
69. $\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h-h^2+1)-f(1)} = \lim_{h\to 0}\frac{f'(2h+2+h^2)(2+2h)}{f'(h-h^2+1)(1-2h)} = \frac{6\times 2}{4\times 1} = 3$.

70. $f(x) = (x+b)^2 + 2c^2 - b^2$ and $g(x) = b^2 + c^2 - (x+c)^2 \Rightarrow 2c^2 - b^2 > b^2 + c^2 \Rightarrow |c| > \sqrt{2}\,|b|$.

71. Centre is (4, 7).

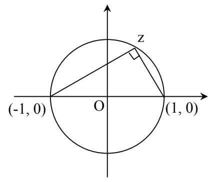


72. From diagram $\theta = 45° \Rightarrow$ slope $= \pm 1$



73. $-\frac{\pi}{6} \le \sin^{-1}(2x) \le \frac{\pi}{2} \Rightarrow -\frac{1}{2} \le 2x \le 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

74. arg $(\omega) = \pm \frac{\pi}{2}$
$\Rightarrow \omega$ is purely imaginary.
$Re(\omega) = 0$

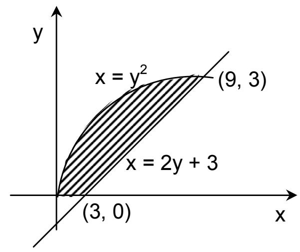


75. $\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \ge 2\tan\alpha$ (AM ≥ GM)

76. $I(m, n) = \int_0^1 t^m(1+t)^n dt = (1+t)^n \left.\frac{t^{m+1}}{m+1}\right|_0^1 - \int_0^1 n(1+t)^{n-1}\frac{t^{m+1}}{m+1}dt = \frac{2^n}{m+1} - \frac{n}{m+1}I(m+1, n-1)$.

77. $f'(x) = e^{-(x^2+1)^2} . 2x - e^{-(x^2)^2} . 2x = 2x e^{-(x^4+1+2x^2)}\left(1 - e^{2x^2+1}\right) \Rightarrow f'(x) > 0 \quad \forall x \in (-\infty, 0)$.

78. $y^2 = 2y + 3 \Rightarrow y = 3$
The required area
$= \int_0^3 (2y + 3 - y^2)\, dy = y^2 + 3y - \left.\frac{y^3}{3}\right|_0^3$
$= 9 + 9 - 9 = 9$.



79. $(1 + t^2)^{12}(1 + t^{12})(1 + t^{24}) = (1 + t^2)^{12}(1 + t^{12} + t^{24} + t^{36})$
$\Rightarrow$ coefficient of t^{24} is $^{12}C_{12} + {}^{12}C_6 + 1 = {}^{12}C_6 + 2$.

80. $V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 1 + a^3 - a \Rightarrow \frac{dV}{da} = 3a^2 - 1 = 3\left(a + \frac{1}{\sqrt{3}}\right)\left(a - \frac{1}{\sqrt{3}}\right) \Rightarrow$ Minimum at $\frac{1}{\sqrt{3}}$.

81. $\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0 \Rightarrow 1 + a(a^2) = 0 \Rightarrow a^3 = -1 \Rightarrow a = -1$.

82. By multiplying e^{-t} and rearranging the terms, we get
$e^{-t}(1 + t)dy + y(e^{-t} - (1 + t)e^{-t})dt = e^{-t}dt$
$\Rightarrow d(e^{-t}(1 + t)y) = d(-e^{-t}) \Rightarrow ye^{-t}(1 + t) = -e^{-t} + c$. Also $y_0 = -1 \Rightarrow c = 0 \Rightarrow y(1) = -1/2$.

83. $\frac{x\cos\theta}{3\sqrt{3}} + y\sin\theta = 1$. Sum of intercepts $3\sqrt{3}\sec\theta + \text{cosec}\theta = f(\theta)$ (say)

$f'(\theta) = \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta} \Rightarrow$ At $\theta = \pi/6$, $f(\theta)$ is minimum.

84. From figure $\tan\theta = \frac{1}{4} = \frac{k}{3} \Rightarrow k = \frac{3}{4}$