2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

- I. Factors and Multiples: If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.
- II. Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.): The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers :

- Factorization Method: Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
- 2. Division Method: Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.

Similarly, the H.C.F. of more than three numbers may be obtained.

- III. Least Common Multiple (L.C.M.): The least number which is exactly divisible by each one of the given numbers is called their L.C.M.
 - Factorization Method of Finding L.C.M.: Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.
 - 2. Common Division Method (Short-cut Method) of Finding L.C.M.: Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.
- IV. Product of two numbers = Product of their H.C.F. and L.C.M.
- V. Co-primes: Two numbers are said to be co-primes if their H.C.F. is 1.
- VI. H.C.F. and L.C.M. of Fractions :

1. H.C.F. =
$$\frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$
 2. L.C.M. = $\frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

- VII. H.C.F. and L.C.M. of Decimal Fractions: In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.
- VIII. Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

New HCW of 83, 105 and 210 is 21

Ex. 16. Two numbers are in the ratio of 15

lengths 4 m 95 cm, 9 m and 16 m 56 cm.

lin, 14, Find the largest number which

Ex. 1. Find the H.C.F. of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^6$, $2^3 \times 5^3$

Sol. The prime numbers common to given numbers are 2, 5 and 7.

$$\therefore$$
 H.C.F. = $2^2 \times 5 \times 7^2 = 980$.

Ex. 2. Find the H.C.F. of 108, 288 and 360.

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$. H.C.F. = $2^2 \times 3^2 = 36$.

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.

H.C.F. of 1134 and 1215 is 81.1 how relyed academic besiteper and 16.1 So, Required H.C.F. = H.C.F. of 513 and 81.

H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

Ex. 5. Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and $11 = 2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$.

Ex. 6. Find the L.C.M. of 72, 108 and 2100.

 $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

L.C.M. = $2^3 \times 3^3 \times 5^2 \times 7 = 37800$.

Ex. 7. Find the L.C.M. of 16, 24, 36 and 54.

 \therefore L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432$.

Ex. 8. Find the H.C.F. and L.C.M. of
$$\frac{2}{3}$$
, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions =
$$\frac{\text{H.C.F. of } 2, 8, 16, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \frac{2}{81}$$

L.C.M. of given fractions =
$$\frac{L.C.M. \text{ of } 2, 8, 16, 10}{H.C.F. \text{ of } 3, 9, 81, 27} = \frac{80}{3}$$

Ex. 9. Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

: H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

: L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15: 11. If their H.C.F. is 13, find the numbers.

Sol. Let the required numbers be 15x and 11x. W ALC BOW BOW IN TO THE

Then, their H.C.F. is x. So, x = 13.

.. The numbers are (15 × 13 and 11 × 13) i.e., 195 and 143.

Ex. 11. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.

Sol. Other number =
$$\left(\frac{11 \times 693}{77}\right)$$
 = 99.

Ex. 12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$$495 = 3^2 \times 5 \times 11,\ 900 = 2^2 \times 3^2 \times 5^2,\ 1665 = 3^2 \times 5 \times 37.$$

H.C.F. = $3^2 \times 5 = 45$.

Hence, required length = 45 cm.

Ex. 13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Sol. Required number = H.C.F. of (1657 - 6) and (2037 - 5) = H.C.F. of 1651 and 2032

Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Directions . Mark (v) against the

Ex. 15. Find the least number exactly divisible by 12, 15, 20 and 27.

Sol. Required number = L.C.M. of 12, 15, 20, 27.

3	12	-	15	-	20	-	27
4	4	_	5	_	20	Ī	. 9
5	1	-	5	171	5	-	9
	1	_	1	_	1	-	9

 \therefore L.C.M. = 3 × 4*× 5 × 9 = 540.

Hence, required number = 540.

Ex. 16. Find the least number which when divided by 6, 7, 8, 9 and 12 leaves the same remainder 1 in each case.

Sol. Required number = (L.C.M. of 6, 7, 8, 9, 12) + 1

 $\therefore \quad \text{L.C.M.} = 3 \times 2 \times 2 \times 7 \times 2 \times 3 = 504.$

Hence, required number = (504 + 1) = 505.

Ex. 17. Find the largest number of four digits exactly divisible by 12, 15, 18 and 27.

Sol. The largest number of four digits is 9999.

Required number must be divisible by L.C.M. of 12, 15, 18, 27 i.e., 540. On dividing 9999 by 540, we get 279 as remainder.

Required number = (9999 - 279) = 9720.

Ex. 18. Find the smallest number of five digits exactly divisible by 16, 24, 36 and 54.

Sol. Smallest number of five digits is 10000.

Required number must be divisible by L.C.M. of 16, 24, 36, 54 i.e., 432.

On dividing 10000 by 432, we get 64 as remainder.

.: Required number = 10000 + (432 - 64) = 10368.

Ex. 19. Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.

Sol. Here,
$$(20-14)=6$$
, $(25-19)=6$, $(35-29)=6$ and $(40-34)=6$.

.. Required number = (L.C.M. of 20, 25, 35, 40) - 6 = 1394.

Ex. 20. Find the least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder.

Sol. L.C.M. of 5, 6, 7, 8 = 840.

∴ Required number is of the form 840k + 3.
Least value of k for which (840k + 3) is divisible by 9 is k = 2.

Required number = $(840 \times 2 + 3) = 1683$.

Ex. 21. The traffic lights at three different road crossings change after every 48 sec., 72 sec. and 108 sec. respectively. If they all change simultaneously at 8:20:00 hours, then at what time will they again change simultaneously?

Sol. Interval of change = (L.C.M. of 48, 72, 108) sec. = 432 sec.

So, the lights will again change simultaneously after every 432 seconds i.e., 7 min. 12 sec.

Hence, next simultaneous change will take place at 8:27:12 hrs.

Ex. 22. Arrange the fractions $\frac{17}{18}$, $\frac{31}{36}$, $\frac{43}{45}$, $\frac{59}{60}$ in the ascending order.

Sol. L.C.M. of 18, 36, 45 and 60 = 180.

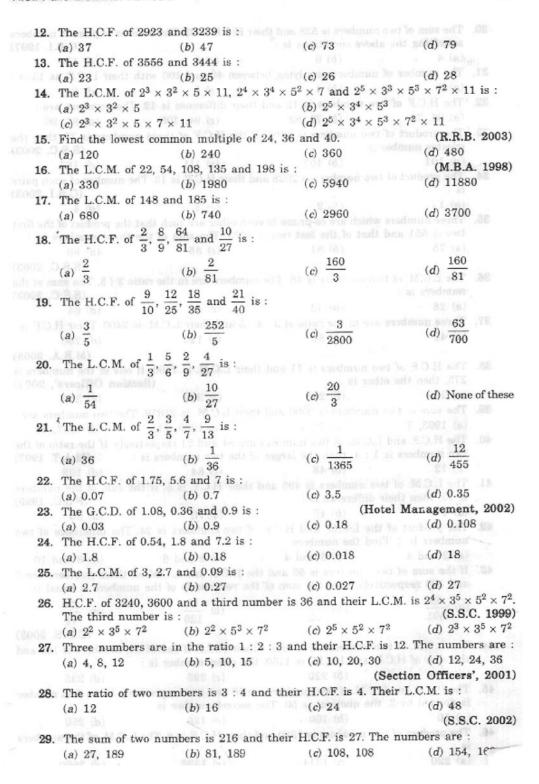
Now,
$$\frac{17}{18} = \frac{17 \times 10}{18 \times 10} = \frac{170}{180};$$
 $\frac{31}{36} = \frac{31 \times 5}{36 \times 5} = \frac{155}{180};$ $\frac{43}{45} = \frac{43 \times 4}{45 \times 4} = \frac{172}{180};$ $\frac{59}{60} = \frac{59 \times 3}{60 \times 3} = \frac{177}{180}.$

Since,
$$155 < 170 < 172 < 177$$
, so, $\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$.

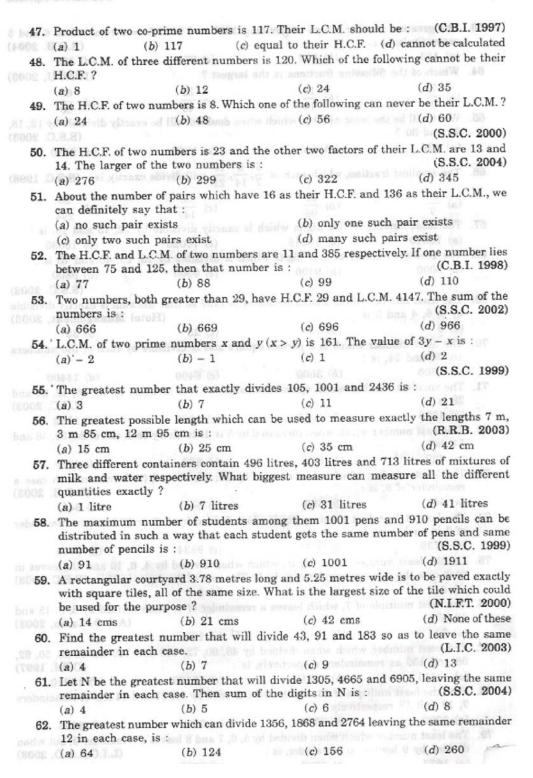
Hence,
$$\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$$

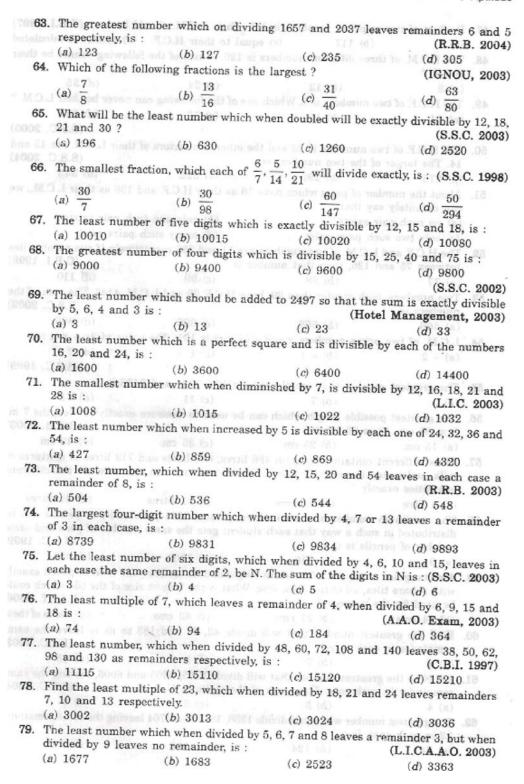
EXERCISE 2

	(OBJECTIVE TY	PE QUESTIONS)	
rections : Mark	() against the corr	ect answer ·	
252 can be expr	essed as a product of	primes as :	(IGNOU, 2002)
(a) $2 \times 2 \times 3 \times$	3×7	(b) $2 \times 2 \times 2 \times$	3 × 7
	COLDS a	(u) 2 x 3 x 3 x	Sol. The level
			(M.B.A. 2002)
A number n is s	said to be perfect if th	e sum of all its divisor	
(a) 6	(b) 9	(c) 15	(d) 21
$\frac{1095}{1168}$ when exp		m is:	(M.B.A. 1998)
(a) $\frac{13}{16}$	(b) 16	(c) 26	$(d) \frac{1}{26}$
Reduce 238368	to its lowest terms.		(IGNOU, 2003)
(a) 4	(b) 13	(c) 13	(d) 13
The H.C.F. of 22	\times 3 ³ \times 5 ⁵ , 2 ³ \times 3 ² \times 5	$^2 \times 7$ and $2^4 \times 3^4 \times 5 \times$	$7^2 \times 11$ is:
(a) $2^2 \times 3^2 \times 5$			7 × 11 and m bud 2.
(c) $2^4 \times 3^4 \times 5^5$			
The H.C.F. of 24	\times 3 ² \times 5 ³ \times 7, 2 ³ \times 3 ³	\times $5^2 \times 7^2$ and $3 \times 5 \times$	7 × 11 is :
(a) 105	(b) 1155	(c) 2310	(d) 27720
H.C.F. of 4 × 27	× 3125, 8 × 9 × 25 ×	7 & 16 × 81 × 5 × 11 ×	49 is : (C.B.I. 1997)
(a) 180	(b) 360	(c) 540	(d) 1260
Find the highest	common factor of 36	and 84.	(R.R.B. 2003)
(a) 4	(b) 6	ON A SHORT OF YEAR OF SHARE BY TA	(d) 18
The H.C.F. of 20-	4, 1190 and 1445 is :	He in All (F.T.) = electricis	
(a) 17	(b) 18	(c) 19	(d) 21
Which of the foll	owing is a pair of co-r		(4/ 21
(a) (16, 62)	(b) (18, 25)	(c) (21, 35)	(d) (23, 92)
	252 can be expr (a) $2 \times 2 \times 3 \times$ (c) $3 \times 3 \times 3 \times$ Which of the foll (a) 99 A number n is a equal to n. An e (a) 6 $\frac{1095}{1168}$ when exp (a) $\frac{13}{16}$ Reduce $\frac{128352}{238368}$ (a) $\frac{3}{4}$ The H.C.F. of 2^2 (a) $2^2 \times 3^2 \times 5$ (c) $2^4 \times 3^4 \times 5^5$ The H.C.F. of 2^4 (a) 105 H.C.F. of 4×27 (a) 180 Find the highest (a) 4 The H.C.F. of 20- (a) 17 Which of the foll	rections: Mark (\checkmark) against the corrections: Mark (\checkmark) against the corrections: 252 can be expressed as a product of (a) $2 \times 2 \times 3 \times 3 \times 7$ (c) $3 \times 3 \times 3 \times 3 \times 7$ Which of the following has most num (a) 99 (b) 101 A number n is said to be perfect if the equal to n . An example of perfect num (a) 6 (b) 9 1095 1168 when expressed in simplest form (a) $\frac{13}{16}$ (b) $\frac{15}{16}$ Reduce $\frac{128352}{238368}$ to its lowest terms. (a) $\frac{3}{4}$ (b) $\frac{5}{13}$ The H.C.F. of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^5$ (a) $2^2 \times 3^2 \times 5$ (b) $2^4 \times 3^4 \times 5^5$ The H.C.F. of $2^4 \times 3^2 \times 5^3 \times 7$, $2^3 \times 3^3$ (a) 105 (b) 1155 H.C.F. of $4 \times 27 \times 3125$, $8 \times 9 \times 25 \times 9^3$ (a) 180 (b) 360 Find the highest common factor of 36 (a) 4 (b) 6 The H.C.F. of 204, 1190 and 1445 is: (a) 17 (b) 18 Which of the following is a pair of co-particle of the following is a pair	(c) $3 \times 3 \times 3 \times 3 \times 7$



30	 The sum of two satisfying the 	numbers is 528 and the above conditions is :	ir H.C.F. is 33. The nu	mber of pairs of numbers (C.B.I. 1997)
	(a) 4	(b) 6	(c) 8	(d) 12
31	. The number of	number-pairs lying bet	ween 40 and 100 mis	th their H.C.F. as 15 is:
	(a) 3	(b) 4	(c) 5	an their H.C.F. as 15 is:
32	The H.C.F. of t	two numbers is 12 and	their difference is 10	mi (a) bar at
	(a) 66 78	(b) 70 92	their difference is 12	. The numbers are :
22	The product of	(b) 70, 82	(c) 94, 106	(d) 84, 96
00.	greater number	r 15 : 008 (e)		
(800 F.	(a) 101	(b) 107	(c) 111	(d) 185
34.	The product of is:	two numbers is 2028 an	d their H.C.F. is 13. T	he number of such pairs
	(a) 1	(b) 2	(c) 3	(d) 4
35.	Three numbers two is 551 and	which are co-prime to ea that of the last two is	ach other are such tha 1073. The sum of th	at the product of the first
	(a) 75	(b) 81	(c) 85	(d) 89
				(S.S.C. 2003)
36.	The L.C.M. of t numbers is:	wo numbers is 48. The r	numbers are in the ra	tio 2:3. The sum of the (S.S.C. 2003)
	(a) 28	(b) 32	(c) 40	(d) 64
37.	Three numbers	are in the ratio of 3:4		is 2400. Their H.C.F. is:
	(a) 40	(b) 80	(c) 120	(d) 200
90	The HOR of			(M.B.A. 2003)
00.	275, then the o	ther is :	(Se	f one of the numbers is ection Officers', 2001)
00	(a) 279	(b) 283	(c) 308	(d) 318
	(a) 1993, 7	numbers is 2000 and to (b) 1991, 9	(c) 1989, 11	(d) 1987, 13
40.	The H.C.F. and two numbers is	L.C.M. of two numbers : 1:4, then the larger of	are 84 and 21 respect	tively. If the ratio of the
	(a) 12	(b) 48	(c) 84	(d) 108
41.	The L.C.M. of the is 10, then their	wo numbers is 495 and r difference is :	their H.C.F. is 5. If t	he sum of the numbers (S.S.C. 1999)
	(a) 10		(c) 70	(5.5.0. 1999)
42.	The product of	the L.C.M. and H.C.F.	of two numbers is 24	. The difference of two
	(a) 2 and 4	(b) 6 and 4	(c) 8 and 6	(d) 8 and 10
43.	If the sum of tw and 120 respect	yo numbers is 55 and the ively, then the sum of t	e H.C.F. and L.C.M.	of these numbers are 5
	(a) $\frac{55}{601}$	(b) 601	(c) 11 0088	(d) 120
	001	55	120	11
44				(C.D.S. 2003)
	the sum of H.C.	vo numbers is 45 times t F. and L.C.M. is 1150, t	heir H.C.F. If one of the other number is :	the numbers is 125 and
	(a) 215	(b) 220	(c) 225	(d) 235
45.	The H.C.F. and I is divided by 2,	.C.M. of two numbers ar the quotient is 50. The	e 50 and 250 respecti second number is	vely. If the first number
	(a) 50	(b) 100	(c) 125	(d) 250
	The product of to	wo numbers is 1320 and	their H.C.F. is 6. The	L.C.M. of the numbers
	(a) 220	(b) 1314	(c) 1326	(d) 7920





	Find the least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder
	in each case, but when divided by 7 leaves no remainder.

(a) 17004 (b) 18000

(c) 18002

(d) 18004

81. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together?

(a) 4

(b) 10

(c) 15

82. Four different electronic devices make a beep after every 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the devices beeped together at 12 noon. They will again beep together at:

(a) 12 midnight

(b) 3 a.m.

(c) 6 a.m.

83. A, B and C start at the same time in the same direction to run around a circular stadium. A completes a round in 252 seconds, B in 308 seconds and C in 198 seconds, all starting at the same point. After what time will they meet again at the starting (S.S.C. 2003) point ?

(a) 26 minutes 18 seconds

(b) 42 minutes 36 seconds

(c) 45 minutes

(d) 46 minutes 12 seconds

(c)
(b)
(d)
(c)
(c)
(a)
(b)
(b)
(d)

SOLUTIONS

1. Clearly, $252 = 2 \times 2 \times 3 \times 3 \times 7$.

2. $99 = 1 \times 3 \times 3 \times 11$; $101 = 1 \times 101$;

 $176 = 1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 11$; $182 = 1 \times 2 \times 7 \times 13$.

So, divisors of 99 are 1, 3, 9, 11, 33 and 99;

divisors of 101 are 1 and 101;

divisors of 176 are 1, 2, 4, 8, 16, 22, 44, 88 and 176;

divisors of 182 are 1, 2, 7, 13, 14, 26, 91 and 182.

Hence, 176 has the most number of divisors.

3.	n	Divisors excluding n	Sum of divisors
	6	1, 2, 3	6
	9	1, 3	4 740
	15	1, 3, 5	9
	21	1, 3, 7	11

Clearly, 6 is a perfect number.

Quantitative Aptitude

So, H.C.F. of 1095 and 1168 = 73.

$$\therefore \frac{1095}{1168} = \frac{1095 + 73}{1168 + 73} = \frac{15}{16}.$$

5. 128352) 238368 (1 128352 110016) 128352 (1 110016 18336) 110016 (6 110016 ×

So, H.C.F. of 128352 and 238368 = 18336.

$$\frac{128352}{238368} = \frac{128352 + 18336}{238368 + 18336} = \frac{7}{13}$$

- 6. H.C.F. = Product of lowest powers of common factors = $2^2 \times 3^2 \times 5$.
- 7. H.C.F. = Product of lowest powers of common factors = $3 \times 5 \times 7 = 105$.
- 8. $4 \times 27 \times 3125 = 2^2 \times 3^3 \times 5^5$; $8 \times 9 \times 25 \times 7 = 2^3 \times 3^2 \times 5^2 \times 7$; $16 \times 81 \times 5 \times 11 \times 49 = 2^4 \times 3^4 \times 5 \times 7^2 \times 11$.
 - \therefore H.C.F. = $2^2 \times 3^2 \times 5 = 180$.
 - 9. $36 = 2^2 \times 3^2$; $84 = 2^2 \times 3 \times 7$.
 - \therefore H.C.F. = $2^2 \times 3 = 12$.
 - 10. $204 = 2^2 \times 3 \times 17$; $1190 = 2 \times 5 \times 7 \times 17$; $1445 = 5 \times 17^2$.
 - ∴ H.C.F. = 17.
 - 11. H.C.F. of 18 and 25 is 1. So, they are co-primes.

	 29, (a)	
40. (d) 40. (d)	33444 3360	(30
) ·112 (1 84
		28) 84 (3 84
		×

14. L.C.M. = Product of highest powers of prime factors = $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$.

L.C.M. = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

L.C.M. = $2 \times 3 \times 3 \times 3 \times 11 \times 2 \times 5 = 5940$.

17. H.C.F. of 148 and 185 is 37.

$$\therefore$$
 L.C.M. = $\left(\frac{148 \times 185}{37}\right) = 740$.

18. Required H.C.F. =
$$\frac{\text{H.C.F. of } 2, 8, 64, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \frac{2}{81}$$
.

19. Required H.C.F. = $\frac{\text{H.C.F. of } 9, 12, 18, 21}{\text{L.C.M. of } 10, 25, 35, 40} = \frac{3}{2800}$.

20. Required L.C.M. = $\frac{\text{L.C.M. of } 1, 5, 2, 4}{\text{H.C.F. of } 3, 6, 9, 27} = \frac{20}{3}$.

- 21. Required L.C.M. = $\frac{\text{L.C.M. of } 2, 3, 4, 9}{\text{H.C.F. of } 3, 5, 7, 13} = \frac{36}{1} = 36$
- Given numbers with two decimal places are: 1.75, 5.60 and 7.00. Without decimal places, these numbers are: 175, 560 and 700, whose H.C.F. is 35.
 - :. H.C.F. of given numbers = 0.35.
- 23. Given numbers are 1.08, 0.36 and 0.90. H.C.F. of 108, 36 and 90 is 18.
 - .. H.C.F. of given numbers = 0.18.
- 24. Given numbers are 0.54, 1.80 and 7.20. H.C.F. of 54, 180 and 720 is 18.
 - :. H.C.F. of given numbers = 0.18.
- 25. Given numbers are 3.00, 2.70 and 0.09. L.C.M. of 300, 270 and 9 is 2700.
 - :. L.C.M. of given numbers = 27.00 = 27.
- 26. $3240 = 2^3 \times 3^4 \times 5$; $3600 = 2^4 \times 3^2 \times 5^2$; H.C.F. = $36 = 2^2 \times 3^2$.

Since H.C.F. is the product of lowest powers of common factors, so the third number must have $(2^2 \times 3^2)$ as its factor.

Since L.C.M. is the product of highest powers of common prime factors, so the third number must have 3⁵ and 7² as its factors.

- ∴ Third number = 2² × 3⁵ × 7².
 a = (11 4) (2511 2)
- 27. Let the required numbers be x, 2x and 3x. Then, their H.C.F. = x. So, x = 12.
 - : The numbers are 12, 24 and 36.
- 28. Let the numbers be 3x and 4x. Then, their H.C.F. = x. So, x = 4.
 So, the numbers are 12 and 16.

L.C.M. of 12 and 16 = 48.

- 29. Let the required numbers be 27a and 27b. Then, 27a + 27b = 216 ⇒ a + b = 8. Now, co-primes with sum 8 are (1, 7) and (3, 5).
 - ∴ Required numbers are (27 × 1, 27 × 7) and (27 × 3, 27 × 5) i.e., (27, 189) and (81, 135).
 Out of these, the given one in the answer is the pair (27, 189).
- 30. Let the required numbers be 33a and 33b. Then, $33a + 33b = 528 \implies a + b = 16$. Now, co-primes with sum 16 are (1, 15), (3, 13), (5, 11) and (7, 9).
 - ∴ Required numbers are (33 × 1, 33 × 15), (33 × 3, 33 × 13), (33 × 5, 33 × 11), (33 × 7, 33 × 9).

The number of such pairs is 4.

31. Numbers with H.C.F. 15 must contain 15 as a factor.

Now, multiples of 15 between 40 and 100 are 45, 60, 75 and 90.

.. Number-pairs with H.C.F. 15 are (45, 60), (45, 75), (60, 75) and (75, 90).

[: H.C.F. of (60, 90) is 30 and that of (45, 90) is 45]

Clearly, there are 4 such pairs.

- 32. Out of the given numbers, the two with H.C.F. 12 and difference 12 are 84 and 96.
- 33. Let the numbers be 37a and 37b. Then, 37a × 37b = 4107 ⇒ ab = 3.
 Now, co-primes with product 3 are (1, 3).

So, the required numbers are $(37 \times 1, 37 \times 3)$ i.e., (1, 111).

.. Greater number = 111. - M.O.I. of it is residuon surregion to T.O.H. 51

Quantitative Aptitude

42

- 34. Let the numbers be 13a and 13b. Then, 13a × 13b = 2028 ⇒ ab = 12. Now, co-primes with product 12 are (1, 12) and (3, 4). So, the required numbers are $(13 \times 1, 13 \times 12)$ and $(13 \times 3, 13 \times 4)$. Clearly, there are 2 such pairs.
- 35. Since the numbers are co-prime, they contain only 1 as the common factor. Also, the given two products have the middle number in common. So, middle number = H.C.F. of 551 and 1073 = 29;

First number = $\left(\frac{551}{29}\right)$ = 19; Third number = $\left(\frac{1073}{29}\right)$ = 37.

- \therefore Required sum = (19 + 29 + 37) = 85.
- 36. Let the numbers be 2x and 3x. Then, their L.C.M. = 6x. So, 6x = 48 or x = 8.

.. The numbers are 16 and 24. Hence, required sum = (16 + 24) = 40.

- Let the numbers be 3x, 4x and 5x. Then, their L.C.M. = 60x. So, 60x = 2400 or x = 40. :. The numbers are (3×40) , (4×40) and (5×40) . Hence, required H.C.F. = 40.
- 38. Other number = $\left(\frac{11 \times 7700}{275}\right) = 308$.
- 39. Let the numbers be x and (2000 x). Then, their L.C.M. = x (2000 x). $\Leftrightarrow x^2 - 2000x + 21879 = 0$ So, x(2000 - x) = 21879 \Leftrightarrow $(x - 1989) (x - 11) = 0 <math>\Leftrightarrow$ x = 1989 or x = 11.Hence, the numbers are 1989 and 11.
 - **40.** Let the numbers be x and 4x. Then, $x \times 4x = 84 \times 21 \iff x^2 = \left(\frac{84 \times 21}{4}\right) \iff x = 21.$ Hence, larger number = 4x = 84.
- 41. Let the numbers be x and (100 x). Then, $x(100 - x) = 5 \times 495 \iff x^2 - 100x + 2475 = 0$ $(x-55)(x-45)=0 \Leftrightarrow x=55 \text{ or } x=45.$ The numbers are 45 and 55. Required difference = (55 - 45) = 10.
 - 42. Let the numbers be x and (x + 2). Then, $x(x+2) = 24 \iff x^2 + 2x - 24 = 0 \iff (x-4)(x+6) = 0 \iff x = 4$. So, the numbers are 4 and 6.
 - 43. Let the numbers be a and b. Then, a+b=55 and $ab=5\times 120=600$.
 - :. Required sum = $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{55}{600} = \frac{11}{120}$
 - 44. Let H.C.F. be h and L.C.M. be l. Then, l = 45h and l + h = 1150.

... 45h + h = 1150 or h = 25. So, l = (1150 - 25) = 1125. (25×1125)

Hence, other number = $\left(\frac{25 \times 1125}{125}\right)$ = 225.

- **45.** First number = $(50 \times 2) = 100$. Second number = $\left(\frac{50 \times 250}{100}\right) = 125$.
 - 46. L.C.M. = $\frac{\text{Product of numbers}}{\text{H.C.F.}} = \frac{1320}{6} = 220.$
 - 47. H.C.F of co-prime numbers is 1. So, L.C.M. = $\frac{117}{1}$ = 117.

- Since H.C.F. is always a factor of L.C.M., we cannot have three numbers with H.C.F. 35 and L.C.M. 120.
- 49. H.C.F. of two numbers divides their L.C.M. exactly. Clearly, 8 is not a factor of 60.
- 50. Clearly, the numbers are (23×13) and (23×14) . :. Larger number = (23 × 14) = 322.
- 51. Since 16 is not a factor of 136, it follows that there does not exist any pair of numbers with H.C.F. 16 and L.C.M. 136.
- Product of numbers = 11 × 385 = 4235. Let the numbers be 11a and 11b. Then, $11a \times 11b = 4235 \implies ab = 35$. Now, co-primes with product 35 are (1, 35) and (5, 7). So, the numbers are $(11 \times 1, 11 \times 35)$ and $(11 \times 5, 11 \times 7)$. Since one number lies between 75 and 125, the suitable pair is (55, 77). Hence, required number = 77.
- Product of numbers = 29 × 4147.

Let the numbers be 29a and 29b. Then, $29a \times 29b = (24 \times 4147) \implies ab = 143$. Now, co-primes with product 143 are (1, 143) and (11, 13). So, the numbers are $(29 \times 1, 29 \times 143)$ and $(29 \times 11, 29 \times 13)$. Since both numbers are greater than 29, the suitable pair is $(29 \times 11, 29 \times 13)$ i.e. (319, 377).

:. Required sum = (319 + 377) = 696.

54. H.C.F. of two prime numbers is 1. Product of numbers = $(1 \times 161) = 161$. Let the numbers be a and b. Then, ab = 161. Now, co-primes with product 161 are (1, 161) and (7, 23). Since x and y are prime numbers and x > y, we have x = 23 and y = 7.

 \therefore 3y - x = (3 × 7) - 23 = -2.

- 55. H.C.F. of 2436 and 1001 is 7. Also, H.C.F. of 105 and 7 is 7. :. H.C.F. of 105, 1001 and 2436 is 7.
- 56. Required length = H.C.F. of 700 cm, 385 cm and 1295 cm = 35 cm.
- 57. Required measurement = (H.C.F. of 496, 403, 713) litres = 31 litres.
- 58. Required number of students = H.C.F. of 1001 and 910 = 91.
- 59. Largest size of the tile = H.C.F. of 378 cm and 525 cm = 21 cm.
- 60. Required number = H.C.F. of (91 43), (183 91) and (183 43) = H.C.F. of 48, 92 and 140 = 4.
- 61. N = H.C.F. of (4665 1305), (6905 4665) and (6905 1305) = H.C.F. of 3360, 2240 and 5600 = 1120. Sum of digits in N = (1 + 1 + 2 + 0) = 4.
 - 62. Required number = H.C.F. of (1356 12), (1868 12) and (2764 12) = H.C.F. of 1344, 1856 and 2752 = 64.
 - 63. Required number = H.C.F. of (1657 6) and (2037 5) = H.C.F. of 1651 and 2032 = 127.
 - 64. L.C.M. of 8, 16, 40 and 80 = 80.

$$\frac{7}{8} = \frac{70}{80}$$
; $\frac{13}{16} = \frac{65}{80}$; $\frac{31}{40} = \frac{62}{80}$.

$$\frac{7}{8} = \frac{70}{80}; \frac{13}{16} = \frac{65}{80}; \frac{31}{40} = \frac{62}{80}.$$
Since, $\frac{70}{80} > \frac{63}{80} > \frac{65}{80} > \frac{62}{80}$, so $\frac{7}{8} > \frac{63}{80} > \frac{13}{16} > \frac{31}{40}$.

So, $\frac{7}{8}$ is the largest.

Quantitative Aptitude

65. L.C.M. of 12, 18, 21, 30 $= 2 \times 3 \times 2 \times 3 \times 7 \times 5 = 1260.$ $\therefore \text{ Required number} = (1260 \div 2) = 630.$ $2 \mid 12 - 18 - 21 - 30$ $3 \mid 6 - 9 - 21 - 15$ 2 - 3 - 7 - 5

- **66.** Required fraction = L.C.M. of $\frac{6}{7}$, $\frac{5}{14}$, $\frac{10}{21} = \frac{\text{L.C.M. of } 6, 5, 10}{\text{H.C.F. of } 7, 14, 21} = \frac{30}{7}$
- Least number of 5 digits is 10000. L.C.M. of 12, 15 and 18 is 180.
 On dividing 10000 by 180, the remainder is 100.

.. Required number = 10000 + (180 - 100) = 10080.

- Greatest number of 4 digits is 9999. L.C.M. of 15, 25, 40 and 75 is 600.
 On dividing 9999 by 600, the remainder is 399.
 - :. Required number = (9999 399) = 9600.

44

- 69. L.C.M. of 5, 6, 4 and 3 = 60. On dividing 2497 by 60, the remainder is 37.
 ∴ Number to be added = (60 37) = 23.
- 70. The least number divisible by 16, 20, 24
 = L.C.M. of 16, 20, 24 = 240 = 2 × 2 × 2 × 2 × 3 × 5.

To make it a perfect square, it must be multiplied by 3×5 .

- \therefore Required number = 240 × 3 × 5 = 3600.
- 71. Required number = (L.C.M. of 12, 16, 18, 21, 28) + 7 = 1008 + 7 = 1015.
- 72. Required number = (L.C.M. of 24, 32, 36, 54) -5 = 864 5 = 859.
- 73. Required number = (L.C.M. of 12, 15, 20, 54) + 8 = 540 + 8 = 548.
- 74. Greatest number of 4 digits is 9999. L.C.M. of 4, 7 and 13 = 364. On dividing 9999 by 364, remainder obtained is 171.
 - \therefore Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 171) = 9828. Hence, required number = (9828 + 3) = 9831.
- 75. Least number of 6 digits is 100000. L.C.M. of 4, 6, 10 and 15 = 60. On dividing 100000 by 60, the remainder obtained is 40.
 - : Least number of 6 digits divisible by 4, 6, 10 and 15 = 100000 + (60 40) = 100020.
 - \therefore N = (100020 + 2) = 100022. Sum of digits in N = (1 + 2 + 2) = 5.
- 76. L.C.M. of 6, 9, 15 and 18 is 90.

Let required number be 90k + 4, which is a multiple of 7.

Least value of k for which (90k + 4) is divisible by 7 is k = 4.

- ∴ Required number = 90 × 4 + 4 = 364.
- 77. Here (48 38) = 10, (60 50) = 10, (72 62) = 10, (108 98) = 10 & (140 130) = 10.
 - .. Required number = (L.C.M. of 48, 60, 72, 108, 140) 10 = 15120 10 = 15110.
- 78. Here (18-7) = 11, (21-10) = 11 and (24-13) = 11. L.C.M. of 18, 21 and 24 is 504. Let required number be 504k-11.

Least value of k for which (504k - 11) is divisible by 23 is k = 6.

- ∴ Required number = 504 × 6 − 11 = 3024 − 11 = 3013.
- 79. L.C.M. of 5, 6, 7, 8 = 840.
 - ∴ Required number is of the form 840k + 3.

Least value of k for which (840k + 3) is divisible by 9 is k = 2.

- :. Required number = $(840 \times 2 + 3) = 1683$.
- 80. L.C.M. of 16, 18, 20, 25 = 3600. Required number is of the form 3600k + 4. Least value of k for which (3600k + 4) is divisible by 7 is k = 5.
 - :. Required number = (3600 × 5 + 4) = 18004.

81. L.C.M. of 2, 4, 6, 8, 10, 12 is 120. So, the bells will toll together after every 120 seconds, i.e., 2 minutes.

|30| + 1| = 16 times. In 30 minutes, they will toll together

82. Interval after which the devices will beep together = (L.C.M. of 30, 60, 90, 105) min. = 1260 min. = 21 hrs.

So, the devices will again beep together 21 hrs. after 12 noon i.e., at 9 a.m.

83. L.C.M. of 252, 308 and 198 = 2772.

So, A, B and C will again meet at the starting point in 2772 sec. i.e., 46 min. 12 sec.

S. CHAND

NOW BRINGS TO YOU THE MOST UNIQUE

COMPETITION SERIES

- OBJECTIVE ARITHMETIC Dr. R.S. Aggarwal
- AN ADVANCED APPROACH Dr. R.S. Aggarwal TO DATA INTERPRETATION
- 3. A MODERN APPROACH TO VERBAL AND NON-VERBAL REASONING

Dr. R.S. Aggarwal

- OBJECTIVE GENERAL ENGLISH Dr. R.S. Aggarwal Vikas Aggarwal
- OBJECTIVE GENERAL KNOWLEDGE

Dr. R.S. Aggarwal

Dr. R.S. Aggarwal वस्तुनिष्ठ सामान्य हिन्दी Vikas Aggarwal (OBJECTIVE GENERAL HINDI)