18. Find the centre and radius of the circle $x^2 + y^2 + z^2 = 225$, $2x - 2y + z = 27$.

19. Find the equation of the right circular cylinder of radius 2 whose axis passes through $(1, 2, 3)$ and has direction cosines proportional to $(2, -3, 6)$.

20. (a) If the second order partial derivatives of $\bar{f}$ be continuous, then prove that, $div(curl\ \vec{f}) = 0$.

(b) Prove that

$div(\phi\bar{u}) = (grad\ \phi).\ \bar{u} + \phi\ div\ \bar{u}$.

21. Evaluate the surface integral $\iint_S [yz\bar{i} + zx\bar{j} + xy\ \bar{k}]\, ds$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

22. Verify Green's theorem in the plane for $\vec{F} = (xy + y^2)\,\vec{i} + x^2\ \vec{j}$ over the closed curve bounded by $y = x$ and $y = x^2$.

9197/M12 OCTOBER 2009

Paper II — TRIGONOMETRY, ANALYTICAL GEOMETRY OF 3 DIMENSIONS AND VECTOR CALCULUS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — $(8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Calculate $(1 + i)^{10}$.

2. Prove that $\dfrac{\sin 6\theta}{\sin\theta} = 32\cos^5\theta - 32\cos^3\theta + 6\cos\theta$.

3. If $\tan(x + iy) = u + iv$, prove that $\dfrac{u}{v} = \dfrac{\sin 2x}{\sinh 2y}$.

4. Find the equation of the plane through the point $(1, -2, 3)$ and the intersection of the planes $2x - y + 4z = 7$ and $x + 2y - 3z + 8 = 0$.

5. Find the equation of the plane which contains the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane $8x+2y+9z-1=0$.

6. Show that the lines $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane in which they lie.

7. Find the equation of the sphere which has its centre at $(6,-1,2)$ and touches the plane $2x-y+2z-2=0$.

8. Find the volume of tetrahedron whose vertices are $(3,2,3)$, $(0,3,4)$, $(6,1,4)$, $(6,3,2)$.

9. Show that the equation of the right circular cone which its vertex at $(0,0,0)$ with z-axis as its axis and semi vertical angle equal to α is $x^2+y^2=z^2\tan^2\alpha$.

10. Find the directional derivative of $f(x,y,z)=z^2x+y^3$ at $(1,1,2)$, in the direction $\frac{1}{\sqrt{5}}\vec{i}+\frac{2}{\sqrt{5}}\vec{j}$.

11. Calculate the line integral of $(y^2+z^2)\vec{i}+(z^2+x^2)\vec{j}+(x^2+y^2)\vec{k}$ over the straight line joining the points $(0,0,0)$, $(1,1,1)$.

12. Verify divergence theorem for $\vec{F}=\vec{r}$ and S is the surface of the unit sphere $x^2+y^2+z^2=1$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Separate with real and imaginary parts of $\tan^{-1}(\alpha+i\beta)$.

14. Sum the series upto n terms $\cos\alpha\ \cos 3\alpha+\cos 3\alpha\ \cos 5\alpha+\cos 5\alpha\ \cos 5\alpha+\ldots$.

15. (a) Solve $x^9+x^5-x^4-1=0$

(b) Find the value of i^i.

16. Find the equation of the plane which bisects the acute angle between the planes $3x-4y+12z=26$ and $x+2y-2z=9$.

17. Find the length and equations of the shortest distance between the lines

$$\frac{x-10}{1}=\frac{y-9}{3}=\frac{z+2}{-2};\ \frac{x+1}{2}=\frac{y-12}{4}=\frac{z-5}{1}.$$

21. (a) Solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec}x$ by using method of variation of parameters.

(b) Eliminate h and K from $z=(x^2+h)(y^2+K)$.

22. Solve $y''-3y'+2y=e^{-t}$ given $y(0)=1$, $y'(0)=0$ using Laplace transforms.

9198/M21 OCTOBER 2009

Paper III — MODERN ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Define equivalence relation. Give an example.

2. If $f:A\to B$ and $g:B\to C$ are bijections then prove that $(g\circ f)^{-1}=f^{-1}\circ g^{-1}$.

3. Show that the union of two subgroups is a subgroup if and only if one is contained in the other.

4. Prove that every group of Prime order is cyclic.

5. If f is a homomorphism of a group G into a group G' with Kernel K then prove that K is a normal subgroup of G.

6. Prove that every finite integral domain is a field.

7. Solve : $(D^2-3D+2)y=\sin 3x$.

8. Solve : $(xp-y)^2 = a(1+p^2)\phi(x^2+y^2)$.

9. Solve : $(D^2+D+1)y = x$.

10. Show that the equation

$(y+z)dx+(z+x)dy+(x+y)dz=0$ is exact and solve the equation.

11. Find $L[8e^{8t}+\cosh 3t+\sin 5t]$.

12. Find $L^{-1}\left[\frac{1}{(S^2+a^2)(S^2+b^2)}\right]$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Prove that a cyclic group G with a generator of finite order n is isomorphic to the additive group of residue classes modulo n.

14. If G is an infinite cyclic group then prove that G has exactly two generators and G is isomorphic to additive group of integers.

15. State and prove Cayley's theorem.

16. (a) Let H and K be two subgroups of G of finite index in G. Prove that $H \cap K$ is a subgroup of finite index in G.

(b) State and prove Euler's theorem.

17. Show that every integral domain can be embedded in a field.

18. Solve the following equations :

(a) $(D^2-4D+3)y = e^x \cos 2x$.

(b) $(D^2+3D+2)y = x^2$.

19. Solve the following equations :

(a) $(2x+1)^2 y'' - 2(2x+1)y' - 12y = 6x$.

(b) $(D^2+3D-4)y = x^2-2x$.

20. Solve the simultaneous equations :

(a) $\frac{dx}{dt}+2x+3y = 2e^{2t}$

(b) $\frac{dy}{dt}+3x+2y = 0$.

9199/M22 **OCTOBER 2009**

Paper IV — STATISTICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Compute Quartile deviation and its coefficient from the following data :

Marks :	10	20	30	40	50	80
No. of students :	4	7	15	8	7	2

2. Calculate mode from the following data :

x :	0–10	10–30	30–50	50–60	60–70
f :	4	25	60	15	2

3. The first four moments of a distribution about the value 5 are 2, 20, 40 and 250 respectively. Find the mean, variance, μ_3 and μ_4.

4. From the following data, determine the two regression lines and estimate the value of y when $x = 22$.

	x	y
A.M.	25	40
S.D.	3	6

$r = 0.8$

5. Find out the divided difference of y_x, given that

x :	1	2	4	7	12
y_x :	22	30	82	106	206

6. If $N = 100$, $(A) = 50$, $(B) = 70$, $(AB) = 30$, calculate the remaining class frequencies.

7. For the probability density function $f(x) = C\,x^2(1-x);\ 0 < x < 1$, find

(a) the constant C.

(b) $P\left(X < \frac{1}{2}\right)$.

8. Let X be a discrete random variable with the following probability distribution :

x :	–3	6	9
$P(X = x)$:	1/6	1/2	1/3

Find $E(X)$, $E(X^2)$ and $E(2X+1)^2$.

9. The following mistakes per page were observed in a book :

No. of mistakes/page :	0	1	2	3	4
No. of times the mistake occurred :	211	90	19	5	0

Fit a Poisson distribution to fit data.

10. Explain the different types of statistical hypothesis.

11. In a big city, 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

12. Test the equality of standard deviations for the data given below at 5% level of significance :

$n_1 = 10$, $n_2 = 14$, $s_1 = 1.5$, $s_2 = 1.2$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. From the following frequency distribution calculate the standard deviation :

X :	7.45	12.45	17.45	22.45	27.45	32.45
f :	2	9	29	54	11	5

14. Calculate Karl Pearson's coefficient of skewness.

x :	12.5	17.5	22.5	27.5	32.5	37.5	42.5	47.5
f :	28	42	54	108	129	61	45	33

15. Find the correlation coefficient for the following data :

x :	77	54	27	52	14	35	90	25	56	60
y :	35	38	60	40	50	40	35	56	34	42

16. If $N = 20$, $(A) = 9$, $(B) = 12$, $(C) = 8$, $(AB) = 6$, $(BC) = 4$, $(CA) = 4$ and $(ABC) = 3$, find the remaining positive class frequencies.

17. Fit a straight line trend equation by the method of least squares and estimate the trend values.

Year :	1961	1962	1963	1964
Value :	80	90	92	83
Year :	1965	1966	1967	1968
Value :	94	99	92	104

18. (a) State and prove Bayes' theorem on probability.

(b) The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases}\frac{x}{2}, & \text{if } \; 0<x<2\\ 0 & \text{otherwise.}\end{cases}$$

Find $E(X)$ and $E(3X^2-2X)$.

19. Obtain the moment generating function for a normal distribution and deduce its mean and variance.

20. Define a sample and explain the important types of sampling.

21. (a) Explain the test of significance for equality of population variances using F-distribution.

(b) For a sample of size 19, correlation coefficient = 0.36 was obtained. Can this be from a population with correlation coefficient = 0. ($t_{0.05}=2.11$)

22. A tea company appoints four salesmen A, B, C and D and observes their sales in three seasons – summer, winter and monsoon. The figures (in lakhs) are given below :

Seasons	Salesmen A	B	C	D	Season's total
Summer	36	36	21	35	128
Winter	28	29	31	32	120
Monsoon	26	28	29	29	112
Salesmen's total	90	93	81	96	

Carry out an analysis of variance.

21. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \frac{\pi}{2}$.

(b) Prove that $\int_0^{\infty} \frac{x^4 dx}{x^6 - 1} = \frac{\pi\sqrt{3}}{6}$.

22. State and prove Tailor's theorem.

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9201/M31 **OCTOBER 2009**

Paper VI — REAL AND COMPLEX ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Let M be the set of all bounded real valued functions defined on a non-empty set E. Define

$d(f, g) = \sup\{|f(x) - g(x)| / x \in E\}$ also prove that d is a metric on M.

2. Prove that any open subset of R can be expressed as the union of a countable number of mutually disjoint open intervals.

3. Prove that R^n with usual metric is complete.

4. Prove that any complete metric space is of second category.

5. Prove that a closed subspace of a compact metric space is compact.

6. Prove that metric spaces (0, 1) and (0, ∞) with usual metrices are homemorphic.

7. Prove that an analytic function whose real part is constant is itself a constant.

8. Prove that $u(x, y) = x^2 - y^2$ and $v(x, y) = -\frac{y}{x^2 + y^2}$ are both harmonic but $u + iv$ is not analytic.

9. Show that the transformation $w = \frac{iz + 2}{4z + i}$ maps the real axis in the z-plane to a circle in the w-plane. Find the centre and radius of the circle.

10. Evaluate $\int_C \frac{z+2}{z} dz$ where C is the semicircle $z = 2e^{i\theta}$ where $0 \le \theta \le \pi$.

11. Prove that $\frac{1}{2\pi i} \int_C \frac{e^{zt} dz}{(z^2+1)^2} = \frac{t \sin t}{2}$ if $t > 0$ and C is the circle $|z| = 3$.

12. Classify the various types of singularities and given one example for each type.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. State and prove Holder's inequality.

14. Prove that C with usual metric is complete.

15. A metric space M is connected iff there does not exists a continuous function f from M onto the discrete metric space $\{0, 1\}$.

16. A metric space M is compact iff any family of closed sets with finite intersection property has non-empty intersection.

17. State and prove Heine Borel theorem.

18. A metric space (M, d) is totally bounded iff every sequence in M has a Cauchy subsequence.

19. State and prove Cauchy-Riemann equations.

20. (a) Show that $f(z) = \sqrt{r}\left[\cos\frac{\theta}{2} + i \sin\frac{\theta}{2}\right]$, where $r > 0$ and $0 < \theta < 2\pi$ is differentiable and find $f'(z)$.

(b) Prove that the real and imaginary parts of an analytic function are harmonic functions.

20. Why a linked list is called a dynamic data structure? What are the advantages of using linked list? Describe different types of linked lists.

21. What is a data file? How to open, write and close a data file in C. Give an example.

22. Write a C-program to multiply 2 matrices using pointers.

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9204/M34 **OCTOBER 2009**

Paper IX — PROGRAMMING IN C AND C++

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. What are the general characteristics of C-language? What are the major components of a C-program?

2. What is a string constant? How do string constants differ from character constant? Do string constants represent numerical values?

3. What are the commonly used input/output functions in C? How are they accessed?

4. What is meant by looping? Describe two different forms of looping.

5. What is a function? State several advantages to the use of functions.

6. What is meant by the storage class of variable? Name any two storage class specification in C.

7. In what way does an array differ from an ordinary variable? Summarise the rules for writing a one-dimensional array definition.

8. Write ac-program to find sum of n numbers.

9. How is a pointers variable declared? What is the purpose of the datatype included in the declaration?

10. What is a structure? How does a structure differ from an array?

11. What are static and register variables? Explain.

12. Write a function using pointer to exchange the values stored in two memory location.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Explain the various datatypes available in C with examples.

14. Explain :

(a) Increment and decrement operators.

(b) Conditional operators.

(c) Bitwise operators in C.

15. Explain :

(a) If statement

(b) Switch statement

(c) Go to statement in C.

16. Distinguish between :

(a) While and for

(b) Break and continue and

(c) Continue and goto in C.

17. Write ac-program to arrange the 'n' numbers into descending order.

18. Write ac-program to solve the quadratic equation $ax^2 + bx + c = 0$.

19. Write ac-program that will read the value of x and evaluate the following functions :

$$f(x) = \begin{cases} 1 & if \quad x > 0 \\ 0 & if \quad x = o \\ -1 & if \quad x < 0 \end{cases}$$

Using (a) Nested if statements (b) Else if statements.

19. From the following data, find the missing term

x :	2	3	4	5	6
$f(x)$:	45.0	49.2	54.1	–	67.4

20. Find the first and second derivative of the function tabulated below at $x = 0.6$.

x :	0.4	0.5	0.6	0.7	0.8
$f(x)$:	1.5836	1.7974	2.0442	2.3275	2.6511

21. Calculate $\int_0^1 \frac{dx}{1+x}$ using (a) Trapezoidal rule (b) Simpson's $\frac{3}{8}$ rule (c) Weddle's rule.

22. Find the values of a for which $ax^3 - 9x^2 + 12x - 5 = 0$ has equal roots and solve the equation in one case.

9205/M3A

OCTOBER 2009

Paper X — THEORY OF EQUATIONS AND NUMERICAL ANALYSIS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + \sqrt{-1}$ is a root.

2. If α, β, γ are the roots of $x^3 - 14x + 8 = 0$ find $\Sigma\alpha^2$ and $\Sigma\alpha^3$.

3. Remove the second term from

$$x^4 + 20x^3 + 143x^2 + 430x + 462 = 0.$$

4. Find the multiple roots of the equation $x^4 - 9x^2 + 4x + 12 = 0$.

5. Find the positive root of the equation $x = \cos x$, correct to 4 places of decimals using Newton-Raphson method.

9206/M3B **OCTOBER 2009**

Paper XI — GRAPH THEORY

(For those who joined in July 2003)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Explain the pictorial representation of a graph.

2. Define Walk, path and trail in a graph. Illustrate them.

3. Define the distance between two vertices in a graph. What is the diameter of a graph G? What is the diameter of K_5.

4. Define euler graph. Give an example. Is K_4 an euler graph.

5. What is Chinese Postman problem?

6. Explain marriage problem. Give its graph theoretic interpretation.

7. If G is a tree, prove that $q = p - 1$.

8. Define cut-set space of a graph. Give an example.

9. Prove that the boundary of the face of a plane graph need not be a cycle.

10. What are the values of $\chi(K_{1,4})$, $\chi(K_{2,4})$, $\chi(K_4)$ and $\chi(K_{3,3})$?

11. Define edge colouring of a graph. Give an example.

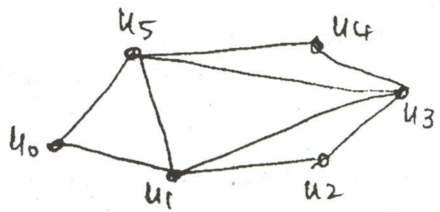
12. Explain the different types of connectedness in digraphs.

SECTION B — $(6 \times 10 = 60$ marks$)$

Answer any SIX questions.

13. If $q > \frac{p^2}{4}$, prove that every (p,q)- graph contains a triangle.

14.



Describe Fleury's algorithm. Using it, find a closed eulerian trail in the above graph.

15. If G is a (p,q)- graph with $p \geq 3$ such that deg (u) +d(v) $\geq p$ for every pair u,v of nonadjacent vertices in G, prove that G is a Hamiltonian graph.

16. Prove that a graph is bipartite if and only if it contains no odd cycles.

17. Describe Prim's algorithm. Prove that any tree constructed by this algorithm is an optimal tree.

18. If G is a connected graph, prove that the distance between v_i and v_j is the smallest integer $n(\geq 0)$ such that $[A^n]_{ij} \neq 0$.

19. Prove that there are exactly five regular polyhedra. What are they?

20. For any graph G, prove that $\chi(G) \leq \Delta(G) + 1$.

21. If G is a bipartite graph with $q \geq 1$, prove that $\chi_1(G) = \Delta(G)$.

22. Prove that a connected graph G is strongly orientable if and only if G has no cut-edges.

9207/M3C **OCTOBER 2009**

Paper XII — DATABASE MANAGEMENT SYSTEMS

(For those who joined in July 2003 and afterwards)

Time : Three hours Maximum : 100 marks

SECTION A — ($8 \times 5 = 40$ marks)

Answer any EIGHT questions.

1. Define (a) Database (b) Data record.

2. Explain the major components of a database system.

3. Define : (a) Data Dictionary (b) Redundancy.

4. List down the various functions of data description language.

5. Describe the concept of editing in database.

6. What are three types of data description needed? Give examples.

7. Explain the built in functions.

8. Explain the comparison operators in ORACLE.

9. Explain the SQL commands SELECT, WHERE clause.

10. Explain Triggers in SQL.

11. Write the types of reports in SQL.

12. Write the commands to edit only the NAME and CITY fields in the BIODATA database file.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Discuss the advantages obtained by having Database Management System.

14. Explain with examples the three general level of the architecture of a database system.

15. Write a program file for paybills preparation.

16. Write commands to the following :

(a) Loading a database

(b) Exiting from database

(c) Creating a database.

17. Explain the conversion functions TO-CHAR, TO-DATE and TO-NUMBER in ORACLE.

18. Explain the term :

(a) A simple plex structure

(b) A complex plex structure.

(c) A cycle.

19. Create an employee table and apply all DML queries.

20. Write a SQL for creating report which contains products sold by different salesman in a month.

21. Explain VIEWS in tables.

22. Discuss the purpose of clustering with suitable example.
