20. Verify the condition of integrability and solve

$$\left(y^2 + y^2\right)dx + \left(xz + z^2\right)dy + \left(y^2 - xy\right)dz = 0.$$

21. Solve :

(a) $p^2 + q^2 = 4$

(b) $\left(x^2 + y^2\right)p + 2xyq = (x + y)z$.

22. Using Lapalce transform, solve :

$$\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 24y = 24x;\ y = \frac{dy}{dx} = 0 \text{ when } x = 0.$$

Paper III — MODERN ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Using the principle of induction, prove that $3^{2n+1} + 2^{n+2}$ is divisible by 7.

2. Prove that any permutation can be expressed as a product of disjoint cycles.

3. State and prove Euler's theorem.

4. Define centre of a group. Prove that the centre H of a group G is a normal subgroup of G.

5. Let $f : G \rightarrow G^1$ be an isomorphism of groups. Prove the following.

(a) $f(e) = e'$ where e and e' are the identity elements of G and G' respectively.

(b) $f\left(a^{-1}\right) = \left(f(a)\right)^{-1}$.

6. Prove that one characteristic of an integral domain is either 0 or a prime number.

7. Solve : $x^2p^2 + 3xyp + 2y^2 = 0$.

8. Solve : $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = x^2$.

9. Solve : $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0$.

10. Solve : $pq = z$.

11. Find : $L(e^{-t}\cos 2t)$.

12. Find : $L^{-1}\left[\frac{1}{s(s+a)}\right]$.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. Show that $f : R \to R$ defined by $f(x) = 2x - 3$ is a bijection and find its inverse. Also compute. $f^{-1} \circ f$ and $f \circ f^{-1}$.

14. Show that the union of two subgroups of G is a subgroup of G if and only if one is contained in the other. Show also that the intersection of two subgroups of G is a subgroup of G.

15. (a) Prove that a group G has no proper subgroups if it is a cyclic group of prime order.

(b) Prove that the order of every subgroup of a finite group divides the order of the group.

16. Show that isomorphism between groups in an equivalence relation.

17. (a) Show that a finite integral domain is a field.

(b) If R is a ring such that $a^2 = a$ for all $a \in R$, prove that

(i) $a + a = 0$

(ii) $a + b = 0 \Rightarrow a = b$

(iii) $ab = ba$.

18. Solve :

(a) $(D^2 - 4D + 3)y = e^{-x}\sin x$

(b) $xyp^2 + (3x^2 - 2y^2)p - 6xy = 0$.

19. Solve : $\frac{dx}{dt} + 2y = -\sin 2t$

$\frac{dy}{dt} - 2x = \cos 2t$.

[7 pages)

7195/M22 OCTOBER 2008

Paper IV — STATISTICS

(For those who joined in July 2003 and after)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. Compute Quartile deviation and its coefficient from the following data :

X :	10-20	20-30	30-40	40-60	60-70	70-80
f :	12	19	5	10	9	6

2. The population of a country at ten year intervals is given below :

Year :	1881	1891	1901	1911	1921	1931	1941
Population in millions :	3.9	5.3	7.3	9.6	12.9	17.1	23.2

By Fitting the curve of the form $y = ab^x$ to this data estimate the population for 1951.

3. Find the correlation coefficient r for the following data :

X :	60	80	46	29	51	72	63	55	43	41
Y :	54	45	62	71	58	47	50	56	63	64

4. Find $\Delta^4 U_0$ and $\Delta^2 U_2$ given $U_0 = 6, U_1 = 7, U_2 = 16, U_3 = 39, U_4 = 82$.

5. From the following data, calculate the remaining class frequencies $N = 100$, $(A) = 50$, $(B) = 70$ and $(AB) = 30$.

6. For the following p.d.f compute mean and variance

$$f(x) = Ax(1-x) \quad \text{if} \quad 0 \le x \le 1$$
$$= 0 \quad \text{otherwise.}$$

7. Find the standard deviation of the Poisson distribution.

8. Find β and r coefficients for the binomial distribution and discuss the results with special reference to Skewness and Kurtosis.

9. Explain short term fluctuations in time series with two classifications.

0. In a sample of 1000 people in Tamil Nadu 540 are ce eaters and the rest are wheat eaters. Can we ssume that both rice and wheat are equally popular in is state at 1% level of significance?

1. Is a correlation coefficient of 0.5 significant if btained from a random sample of 11 pairs of values om a normal population? Use t-test.

2. The following table gives the number of train ccidents in a country that occurred during the various ays of the week. Find whether the accidents are niformly distributed over the week.

ays :	Sun	Mon	Tue	Wed	Thur	Fri	Sat
o.of ccidents :	20	18	13	23	26	11	15

SECTION B — ($6 \times 10 = 60$ marks)

Answer any SIX questions.

3. (a) The weighted geometric mean of the four mbers 8, 25, 17 and 30 is 15.3. If the weights of the st three numbers are 5, 3, and 4 respectively. Find e weight of the fourth number.

(b) After calculation of the mean and S.D of ten servations a copying mistake was detected. When pying the values, 236 was written for 326 through ersight. If the incorrect mean and S.D calculated are 0.1 and 5.6 respectively. Find the correct values of the ean and S.D.

14. (a) x is a variate. Following a uniform distribution in the interval [0, a] ie, $f(x) = k$ a consent for all x in [0, a] and zero elsewhere. Find

(i) Its S.D

(ii) The average deviation and

(iii) The r^{th} moment about the mean.

(b) Fit a parabola of second degree to the following data :

X :	0	1	2	3	4	5	6
Y :	14	18	23	29	36	40	46

15. Given the following sets of data determine the equations of the lines of regression. Find Y when X = 164 and X when Y = 170.

Father's ht(X) :	158	160	163	165	167	170	167	172	177	181
Son's ht(Y) :	163	158	167	170	160	180	170	175	172	175

16. Find the coefficient of total and partial associations between education and employment in a group of 100 persons divided according to sex (A) = 60, (B) = 80, (AB) = 48, (C) = 48, (ABC) = 40, (AC) = 42, (BC) = 44.

17. Explain the four method's for measurement of trends in time series.

18. (a) Fit a normal curve to the following data and calculate the expected frequencies by

(i) The area method

(ii) The ordinate method.

Class intervals :	60-62	63-65	66-68	69-71	72-74
f :	5	18	42	27	8

(b) If x is normally distributed with mean – 8 and S.D – 4. Find

(i) $P(5 \le x \le 10)$

(ii) $P(x \ge 15)$

(iii) $P(x \le 5)$.

19. (a) A sample of 900 members has a mean 3.4 cm and S.D 2.61 cms. Is the sample from a large population of mean 3.25 cms. and S.D 2.61 cms? If the population is normal and its mean is unknown. Find the 95% and 98% Fiducial limits of true mean.

(b) The means of simples of 1000 and 2000 are 67.5 and 68.8 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches?

20. (a) The manufacturer of a certain make of electric bulbs claims that his bulbs have a mean life of 25 months with a S.D of 5 months. A random sample of 6 such bulbs gave the following values :

Life in months :	24	26	30	20	20	20	18

Can you regard the producer's claim to be valid at 1% level of significant? For v = 5, $t_{0.01} = 4.032$.

(b) A sample of 900 men is found to have a mean height of 64 cm. If this sample have been drawn from a normal population with S.D 20 cm. Find the 99% confidence limits for the mean height of the men in the population.

21. For the 2 × 2 contingency table

a	b
c	d

Prove that $\chi^2 = \frac{N(ad - bc)}{(a+c)(b+d)(a+b)(c+d)}$.

22. (a) Five coins are tossed 320 times. The number of heads observed is given below. Examine whether the coin is unbiased.

No. of heads :	0	1	2	3	4	5	Total
Frequency :	15	45	85	95	60	20	320

(b) The three samples below have been obtained from normal populations with equal variances. Test the hypothesis that the sample means are equal.

8	7	12
10	5	9
7	10	13
14	9	12
11	9	14

The table value of F at 5% level of significance for $v_1 = 2$ and $v_2 = 12$ is 3.88.

19. A particle projected with velocity u strikes at right angles a plane through the point of projection inclined at an angle β to the horizon. Show that the time of flight is $\dfrac{2u}{g\sqrt{1+3\sin^2\beta}}$.

20. Explain the geometrical representation of a simple harmonic motion.

21. A particle moving with S.H.M. has speeds v_1 and v_2 $(v_1 > v_2)$ and accelerations with magnitude f_1 and f_2 respectively at the points A and B which lie on the same side of the mean position 0. Show that $AB = \dfrac{v_1^2 - v_2^2}{f_1 + f_2}$.

22. A particle moves in a plane with an acceleration which is always directed to a fixed point O in the plane. Find the differential equation of its' path.

Paper V — MECHANICS

(For those who joined in July 2003 and afterwards)

Time : Three hours Maximum : 100 marks

SECTION A — (8 × 5 = 40 marks)

Answer any EIGHT questions.

1. State and prove triangle law of forces.

2. Three forces $\overline{X}, \overline{Y}, \overline{Z}$ acting at the vertices A, B, C respectively, each $\perp_r$ to the opposite side, keep the triangle in equilibrium. Prove that $\dfrac{X}{a} = \dfrac{Y}{b} = \dfrac{Z}{c}$.

3. AD is an altitude of a triangle ABC. Show that the force AD acting along AD has components $\dfrac{a^2+b^2+c^2}{2a^2}$. AB and $\dfrac{c^2+a^2-b^2}{2a^2}$ AC along AB and AC respectively.

4. The position of the resultant of two like parallel forces P and Q is unaltered, when the positions of P and Q are interchanged. Show that P and Q are of equal magnitude.

5. If three forces P, Q, R acting along the bisectors of the angles of a triangle ABC at the angular points A, B, C respectively keep it in equilibrium, show that $P:Q:R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$.

6. Write down any three laws of statical friction.

7. Define the following terms :

(a) Angle of projection

(b) Range of projection

(c) Time of flight.

8. If T is the time of flight, R the horizontal range and α the angle of projection, show that $gT^2 = 2R\tan\alpha$.

9. Define and explain the following :

(a) Impulse of a constant force

(b) Impulse of a variable force.

10. A simple harmonic motion has amplitude and its', maximum acceleration are 8 cm and 2 cm /sec². Find its period and maximum velocity.

11. If the velocities of a particle along and perpendicular to the radius vector are proportional to each other, find the accelerations along and perpendicular to the radius vector.

12. Derive the pedal equation of central orbit with pole at any point.

SECTION B — (6 × 10 = 60 marks)

Answer any SIX questions.

13. The resultant of two forces P and Q is of magnitude P. If P be doubled, show that the new resultant is at right angles to Q and its magnitude will be $\sqrt{4P^2 - Q^2}$.

14. Three forces P, Q, R act in the same sense along the sides BC, CA, AB of a triangle ABC. If their resultant passes through the orthocentre, then show that $P\sec A + Q\sec B + R\sec C = 0$.

15. Find the resultant of two unlike and unequal parallel forces acting on a rigid body.

16. State and prove Varignon's theorem of moments.

17. A weight can be supported on a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally. Show that the weight is $\frac{PQ}{\sqrt{Q^2\sec^2\lambda - P^2}}$, where λ is the angle of friction.

18. A stone is thrown with a velocity of 39.2 met/sec at an angle of 30° from the horizontal. Find at what times it will be a height of 14.7 metre (g = 9.8 m/sec²).