MATHEMATICS

1. A variable line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is such that $a + b = 4$. The locus of the midpoint of the portion of the line intercepted between the axes is

   1) $x + y = 4$

   2) $x + y = 8$

   3) $x + y = 1$

   4) $x + y = 2$

2. The point $(5, -7)$ lies outside the circle

   1) $x^2 + y^2 - 8x = 0$

   2) $x^2 + y^2 - 5x + 7y = 0$

   3) $x^2 + y^2 - 5x + 7y - 1 = 0$

   4) $x^2 + y^2 - 8x + 7y - 2 = 0$

3. If the circles $x^2 + y^2 = 9$ and $x^2 + y^2 + 2\alpha x + 2y + 1 = 0$ touch each other internally, then $\alpha =$

   1) $\pm\dfrac{4}{3}$

   2) 1

   3) $\dfrac{4}{3}$

   4) $\dfrac{-4}{3}$

4. The locus of the midpoints of the line joining the focus and any point on the parabola $y^2 = 4ax$ is a parabola with the equation of directrix as

   1) $x + a = 0$

   2) $2x + a = 0$

   3) $x = 0$

   4) $x = \dfrac{a}{2}$

5. The tangents drawn at the extremeties of a focal chord of the parabola $y^2 = 16x$

   1) intersect on $x = 0$

   2) intersect on the line $x + 4 = 0$

   3) intersect at an angle of 60^0

   4) intersect at an angle of 45^0

(Space for Rough Work)

6. On the set Z of all integers $*$ is defined by $a*b = a+b-5$. If $2*(x*3)=5$ then $x =$

   1) 0

   2) 3

   3) 5

   4) 10

7. Which of the following is false ?

   1) Addition is commutative in N.

   2) Multiplication is associative in N.

   3) If $a*b = a^b$ for all $a, b \in N$ then $*$ is commutative in N.

   4) Addition is associative in N.

8. If $\vec{a} \cdot \hat{i} = \vec{a} \cdot \left(\hat{i} + \hat{j} \right) = \vec{a} \cdot \left(\hat{i} + \hat{j} + \hat{k} \right) = 1$ then $\vec{a} =$

   1) $\hat{i} + \hat{j}$

   2) $\hat{i} - \hat{k}$

   3) $\hat{i}$

   4) $\hat{i} + \hat{j} - \hat{k}$

9. If $\vec{a}$ and $\vec{b}$ are unit vectors and $\left| \vec{a} + \vec{b} \right| = 1$ then $\left| \vec{a} - \vec{b} \right|$ is equal to

   1) $\sqrt{2}$

   2) 1

   3) $\sqrt{5}$

   4) $\sqrt{3}$

10. The projection of $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ on $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ is

    1) $\dfrac{8}{\sqrt{35}}$

    2) $\dfrac{8}{\sqrt{39}}$

    3) $\dfrac{8}{\sqrt{14}}$

    4) $\sqrt{14}$

(Space for Rough Work)

11. $f: R \to R$ is defined by $f(x) = x^3$ then $f^{-1}(8) =$

   1) $\{2\}$

   2) $\{2, 2w, 2w^2\}$

   3) $\{2, -2\}$

   4) $\{2, 2\}$

12. R is a relation on N given by $R = \{(x, y) \mid 4x + 3y = 20\}$. Which of the following belongs to R ?

   1) $(-4, 12)$

   2) $(5, 0)$

   3) $(3, 4)$

   4) $(2, 4)$

13. If $Log_{10} 7 = 0.8451$ then the position of the first significant figure of 7^{-20} is

   1) 16

   2) 17

   3) 20

   4) 15

14. $\dfrac{1}{2.5} + \dfrac{1}{5.8} + \dfrac{1}{8.11} + \ldots\ldots$ upto n terms $=$

   1) $\dfrac{n}{4n+6}$

   2) $\dfrac{1}{6n+4}$

   3) $\dfrac{n}{6n+4}$

   4) $\dfrac{n}{3n+7}$

15. The ten's digit in $1! + 4! + 7! + 10! + 12! + 13! + 15! + 16! + 17!$ is divisible by

   1) 4

   2) $3!$

   3) 5

   4) 7

(Space for Rough Work)

16. The equation $\dfrac{x^2}{2-\lambda} - \dfrac{y^2}{\lambda-5} = 0$ represents an ellipse if

   1) $\lambda > 5$ 2) $\lambda < 2$

   3) $2 < \lambda < 5$ 4) $2 > \lambda > 5$

17. The equation to the normal to the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ at $(-4, 0)$ is

   1) $2x - 3y = 1$ 2) $x = 0$

   3) $x = 1$ 4) $y = 0$

18. The converse of the contrapositive of the conditional $p \rightarrow \sim q$ is

   1) $p \rightarrow q$ 2) $\sim p \rightarrow \sim q$

   3) $\sim q \rightarrow p$ 4) $\sim p \rightarrow q$

19. The perimeter of a certain sector of a circle is equal to the length of the arc of the semicircle. Then the angle at the centre of the sector in radians is

   1) $\pi - 2$ 2) $\pi + 2$

   3) $\dfrac{\pi}{3}$ 4) $\dfrac{2\pi}{3}$

20. The value of $Tan\ 67\dfrac{1}{2}^0 + Cot\ 67\dfrac{1}{2}^0$ is

   1) $\sqrt{2}$ 2) $3\sqrt{2}$

   3) $2\sqrt{2}$ 4) $2 - \sqrt{2}$

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(Space for Rough Work)

21. If e_1 and e_2 are the eccentricities of a hyperbola $3x^2 - 3y^2 = 25$ and its conjugate, then

1) $e_1^2 + e_2^2 = 2$

2) $e_1^2 + e_2^2 = 4$

3) $e_1 + e_2 = 4$

4) $e_1 + e_2 = \sqrt{2}$

22. If p and q are prime numbers satisfying the condition $p^2 - 2q^2 = 1$, then the value of $p^2 + 2q^2$ is

1) 5

2) 15

3) 16

4) 17

23. If $A(adj\ A) = 5I$ where I is the identity matrix of order 3, then $|adj\ A|$ is equal to

1) 125

2) 25

3) 5

4) 10

24. The number of solutions for the equation $Sin\ 2x + Cos\ 4x = 2$ is

1) 0

2) 1

3) 2

4) Infinite

25. $\int e^x \cdot x^5\ dx$ is

1) $e^x \left[x^5 + 5x^4 + 20x^3 + 60x^2 + 120x + 120 \right] + C$

2) $e^x \left[x^5 - 5x^4 - 20x^3 - 60x^2 - 120x - 120 \right] + C$

3) $e^x \left[x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120 \right] + C$

4) $e^x \left[x^5 + 5x^4 + 20x^3 - 60x^2 - 120x + 120 \right] + C$

(Space for Rough Work)

26. If $f(x)$ is an even function and $f'(x)$ exists, then $f'(e) + f'(-e)$ is

   1) > 0 2) 0

   3) ≥ 0 4) < 0

27. If α is a complex number satisfying the equation $\alpha^2 + \alpha + 1 = 0$ then α^{31} is equal to

   1) α 2) α^2

   3) 1 4) i

28. The derivative of $Sin(x^3)$ w.r.t. $Cos(x^3)$ is

   1) $-Tan(x^3)$ 2) $Tan(x^3)$

   3) $-Cot(x^3)$ 4) $Cot(x^3)$

29. A unit vector perpendicular to both the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

   1) $\dfrac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$ 2) $\dfrac{\hat{i} + \hat{j} - \hat{k}}{3}$

   3) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ 4) $\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$

30. If $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $B = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ then

   1) $A = -B$ 2) $A = B$

   3) $B = 0$ 4) $B = A^2$

(Space for Rough Work)

31. The locus of a point which moves such that the sum of its distances from two fixed points is a constant is

    1) a circle   2) a parabola

    3) an ellipse   4) a hyperbola

32. The centroid of the triangle ABC where $A \equiv (2, 3)$, $B \equiv (8, 10)$ and $C \equiv (5, 5)$ is

    1) $(5, 6)$ 2) $(6, 5)$

    3) $(6, 6)$ 4) $(15, 18)$

33. If $3x^2 + xy - y^2 - 3x + 6y + K = 0$ represents a pair of lines, then $K =$

    1) 0 2) 9

    3) 1 4) -9

34. The equation of the smallest circle passing through the points $(2, 2)$ and $(3, 3)$ is

    1) $x^2 + y^2 + 5x + 5y + 12 = 0$ 2) $x^2 + y^2 - 5x - 5y + 12 = 0$

    3) $x^2 + y^2 + 5x - 5y + 12 = 0$ 4) $x^2 + y^2 - 5x + 5y - 12 = 0$

35. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are

    1) $1, 3, 6$ 2) $1, 2, 4$

    3) $4, 5, 6$ 4) $2, 4, 6$

(Space for Rough Work)

36. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^{-1} =$

    1) $\dfrac{-1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$
        2) $\dfrac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

    3) $\begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix}$
        4) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

37. The set $\{-1, 0, 1\}$ is not a multiplicative group because of the failure of

    1) Closure law
        2) Associative law

    3) Identity law
        4) Inverse law

38. The angle of elevation of the top of a TV tower from three points A, B and C in a straight line through the foot of the tower are α, 2α and 3α respectively. If $AB = a$, the height of the tower is

    1) $a\,Tan\,\alpha$
        2) $a\,Sin\,\alpha$

    3) $a\,Sin\,2\alpha$
        4) $a\,Sin\,3\alpha$

39. The angles A, B and C of a triangle ABC are in A.P. If $b : c = \sqrt{3} : \sqrt{2}$, then the angle A is

    1) 30^0
        2) 15^0

    3) 75^0
        4) 45^0

40. $Sin\left(2Sin^{-1}\sqrt{\dfrac{63}{65}}\right) =$

    1) $\dfrac{2\sqrt{126}}{65}$
        2) $\dfrac{4\sqrt{65}}{65}$

    3) $\dfrac{8\sqrt{63}}{65}$
        4) $\dfrac{\sqrt{63}}{65}$

(Space for Rough Work)

41. The general solution of $Sin\,x = Cos\,x$ (when $n \in Z$) given by

   1) $\quad n\,\pi + \dfrac{\pi}{4}$ 2) $\quad 2n\,\pi \pm \dfrac{\pi}{4}$

   3) $\quad n\,\pi \pm \dfrac{\pi}{4}$ 4) $\quad n\,\pi - \dfrac{\pi}{4}$

42. The real root of the equation $x^3 - 6x + 9 = 0$ is

   1) $\quad -6$ 2) $\quad -9$

   3) $\quad 6$ 4) $\quad -3$

43. The digit in the unit's place of 5^{834} is

   1) $\quad 0$ 2) $\quad 1$

   3) $\quad 3$ 4) $\quad 5$

44. The remainder when $3^{100} \times 2^{50}$ is divided by 5 is

   1) $\quad 1$ 2) $\quad 2$

   3) $\quad 3$ 4) $\quad 4$

45. $\displaystyle\int \dfrac{Sin\,x\,Cos\,x}{\sqrt{1 - Sin^4 x}}\,dx =$

   1) $\quad \dfrac{1}{2}\,Sin^{-1}\left(Sin^2 x\right) + C$ 2) $\quad \dfrac{1}{2}\,Cos^{-1}\left(Sin^2 x\right) + C$

   3) $\quad Tan^{-1}\left(Sin^2 x\right) + C$ 4) $\quad Tan^{-1}\left(2\,Sin\,x\right) + C$

(Space for Rough Work)

46. The value of $\int_{-2}^{2} \left(ax^3 + bx + c \right) dx$ depends on the

    1)   value of b 2)   value of c

    3)   value of a 4)   values of a and b

47. The area of the region bounded by $y = 2x - x^2$ and the x-axis is

    1) $\dfrac{8}{3}$ sq. units   2) $\dfrac{4}{3}$ sq. units

    3) $\dfrac{7}{3}$ sq. units   4) $\dfrac{2}{3}$ sq. units

48. The differential equation $y\,\dfrac{dy}{dx} + x = c$ represents

    1)   a family of hyperbolas

    2)   a family of circles whose centres are on the y-axis

    3)   a family of parabolas

    4)   a family of circles whose centres are on the x-axis

49. If $f\left(x^5 \right) = 5x^3$, then $f'(x) =$

    1) $\dfrac{3}{\sqrt[5]{x^2}}$ 2) $\dfrac{3}{\sqrt[5]{x}}$

    3) $\dfrac{3}{x}$ 4) $\sqrt[5]{x}$

50. $f(x) = 2a - x \quad$ in $\quad -a < x < a$

          $= 3x - 2a \quad$ in $\quad a \le x$.

  Then which of the following is true?

    1) $f(x)$ is discontinuous at $x = a$ 2) $f(x)$ is not differentiable at x

    3) $f(x)$ is differentiable at all $x \ge a$ 4) $f(x)$ is continuous at all $x < a$

(Space for Rough Work)

51. The maximum area of a rectangle that can be inscribed in a circle of radius 2 units is (in square units)

    1) 4   2) 8π

    3) 8   4) 5

52. If Z is a complex number such that $Z = -\overline{Z}$, then

    1) Z is purely real

    2) Z is purely imaginary

    3) Z is any complex number

    4) Real part of Z is the same as its imaginary part

53. The value of $\sum_{K=1}^{6}\left[Sin\dfrac{2K\Pi}{7} - i\,Cos\dfrac{2K\Pi}{7}\right]$ is

    1) i 2) 0

    3) $-i$ 4) -1

54. $\displaystyle \mathop{Lt}_{x \to \infty} x\,Sin\left(\dfrac{2}{x}\right)$ is equal to

    1) ∞ 2) 0

    3) 2   4) $\dfrac{1}{2}$

55. A stone is thrown vertically upwards and the height x ft. reached by the stone in t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in

    1) 2 seconds   2) 2.5 seconds

    3) 3 seconds   4) 1.5 seconds

(Space for Rough Work)

56. The maximum value of $\dfrac{\log x}{x}$ in $(2, \infty)$ is

   1)   1   2) $\dfrac{2}{e}$

   3) e 4) $\dfrac{1}{e}$

57. If $f(x) = be^{ax} + ae^{bx}$, then $f''(0) =$

   1)   0   2) $2ab$

   3) $ab(a+b)$ 4) ab

58. If $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \dfrac{x}{y}$, then the value of $\tan A =$

   1) $\dfrac{x^2+y^2}{x^2-y^2}$ 2) $\dfrac{2xy}{x^2+y^2}$

   3) $\dfrac{2xy}{x^2-y^2}$ 4) $\dfrac{2xy}{y^2-x^2}$

59. $\displaystyle\int \dfrac{\sec x}{\sec x + \tan x}\, dx =$

   1) $\tan x - \sec x + C$ 2) $\log(1+\sin x) + C$

   3) $\sec x + \tan x + C$ 4) $\log \sin x + \log \cos x + C$

60. If $\displaystyle\int f(x)\, dx = g(x)$, then $\displaystyle\int f(x) g(x)\, dx =$

   1) $\dfrac{1}{2} f^2(x)$ 2) $\dfrac{1}{2} g^2(x)$

   3) $\dfrac{1}{2}\left[g'(x)\right]^2$ 4) $f'(x) g(x)$

(Space for Rough Work)