Second PUC Mathematics July, 2014 Question Paper

(English Version)

Instructions : (i) The question paper has **five** parts namely A, B, C, D and E. Answer **all** the parts.

(ii) Use the graph sheet for the question on Linear Programming problem in Part E.

PART – A

I. Answer **all** the questions : **10 × 1 = 10**

1. Let $*$ be a binary operation defined on the set of rational numbers Q defined by $a * b = ab + 1$, Prove that $*$ is a commutative.

2. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$.

3. Define a diagonal matrix.

4. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of $|2A|$.

5. Find $\frac{dy}{dx}$, if $y = \cos(1 - x)$.

6. Evaluate $\int (2x - 3\cos x + e^x)\,dx$.

7. Define a unit vector.

8. If a line makes angle 90°, 60° and 30° with positive direction of x, y and z axis respectively, find its direction cosines.

9. In Linear programming problems, define linear objective function.

10. If $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$, find $P(A/B)$.

PART – B

II. Answer any **ten** questions : **10 × 2 = 20** III.

11. Show that the function $f : N \to N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one-one.

12. Prove that $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$, $x \in \mathbb{R}$.

13. Write the simplest form of

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ 0 < x < \pi$$

14. Find the equation of a line passing through (3, 1) and (9, 3) using determinants.

15. If $\sqrt{x} + \sqrt{y} = \sqrt{10}$, show that $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$.

16. Find $\frac{dy}{dx}$, if $y = (\log x)^{\cos x}$.

17. Approximate $\sqrt{36.6}$ by using differential.

18. Integrate $\sin x \cdot \sin(\cos x)$ with respect to x.

19. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$.

20. Find order and degree of differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right) + y = 0.$$

21. Find the area of parallelogram whose adjacent sides determine by the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

22. Obtain the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

23. Find the equation of the plane through the intersection of planes $3x - y + 2z - 4 = 0$ and $x + y - z - 2 = 0$ and the point (2, 2, 1).

24. A die is thrown. If E is the event 'the number appearing is a multiple of 3' and F be the event 'the number appearing is even', then prove that E and F are independent events.

PART – C

III. Answer any **ten** questions : **10 × 3 = 30**

25. Show that the relation R in the set Z of integers given by R = {(x, y) : 2 divides (x – y)} is an equivalence relation.

26. Prove that $\tan^{-1}(x) + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right),\ |x| < \frac{1}{\sqrt{3}}$.

27. For any square matrix A with real numbers, prove that

$A + A'$ is a symmetric and

$A - A'$ is a skew-symmetric

28. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$

Prove that $\frac{dy}{dx} = \tan\left(\frac{\theta}{2}\right)$.

29. Verify Mean Value theorem, if $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$, where $a = 1$ and $b = 4$.

30. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

31. Evaluate $\int \sin 3x \cdot \cos 4x \, dx$

32. Integrate $x^2 e^x$ with respect to x.

33. Determine the area of the region bounded by $y^2 = x$ and the lines $x = 1$ and $x = 4$ and the x – axis in the first quadrant.

34. Form the differential equation of the family of circles touching the x-axis at origin.

35. If two vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, find $|\vec{a} - \vec{b}|$.

36. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

37. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$.

38. Bag-I contains 3 red and 4 black balls while another Bag-II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag-II.

PART – D

IV. Answer any **six** questions : $6 \times 5 = 30$

39. Prove that the function f: N → N defined by $f(x) = x^2$ where $Y = \{y : y = x^2, x \in N\}$ is invertible. Also find the inverse of f.

40. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then

    Show that $A^3 - 23A - 40I = O$

41. Solve the following system of linear equations by matrix method :

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

42. Given $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

43. The length x of a rectangle is decreasing at the rate of 5 cm/minute and width y is increasing at the rate of 4 cm/minute. When $x = 8$ cm and $y = 6$ cm, find the rate of change of

    (i) the perimeter and

    (ii) the rate of the rectangle

44. Find the integral of $\frac{1}{\sqrt{x^2 + a^2}}$ with respect to x and hence evaluate $\int \frac{1}{\sqrt{x^2 + 7}}\,dx$.

45. Find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

46. Solve : $(x \log x)\frac{dy}{dx} + y = \frac{2}{x}(\log x)$.

47. Derive the equation of a line in a space through a given point and parallel to a vector both in the vector and cartesian form.

48. If a fair coin is tossed 10 times, find the probability of

    (i) exactly six heads and

    (ii) at least six heads

PART – E

V. Answer any **one** question : **1 × 10 = 10**

49. (a) Prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ and hence evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx.$$ **6**

(b) Determine the value of k, if

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$. **4**

50. (a) Minimise and Maximise : $Z = x + 2y$

Subjected to Constraints

$x + 2y \geq 100,$

$2x - y \leq 0,$

$2x + y \leq 200,$

$x \geq 0,\ y \geq 0,$

by the graphical method. **6**

(b) Prove that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3.$$ **4**
