(English Version)

Math Mar. 2012

Instructions : i) The question paper has *five* **Parts – A, B, C, D** and **E.** Answer all the parts.

ii) **Part – A** carries 10 marks, **Part – B** carries 20 marks, **Part – C** carries 40 marks, **Part – D** carries 20 marks and **Part – E** carries 10 marks.

PART – A

Answer *all* the *ten* questions : $10 \times 1 = 10$

1. Write the prime power factorisation of the composite number 45.

2. Define a diagonal matrix.

3. Why the usual division is not a binary operation on the set of real numbers ?

4. Simplify $\left(\hat{i} \times \hat{j} \right) \times \hat{k}$.

5. Find the centre of the circle $3x^2 + 3y^2 - 6x + 3y - 4 = 0$.

6. For the parabola $2y^2 = 5x$, what is the length of the latus rectum ?

7. Evaluate $\sin\left[\frac{1}{2} \cos^{-1}\left(\frac{4}{5} \right) \right]$.

8. Express in the polar form $Z = 2 + i(2)$.

9. If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$.

10. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x + \sin x)\, dx$.

PART – B

Answer any *ten* questions : $10 \times 2 = 20$

11. If $(a, b) = 1$, $(a, c) = 1$ then prove that $(a, bc) = 1$.

12. Solve using Cramer's rule :

$$3x - y = 4$$

$$5x + 4y = 1.$$

13. Prove that the identity element of a group $(G, *)$ is unique.

14. Find the area of parallelogram whose two diagonals are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$.

15. $3x + 4y = 2$ and $x - y = 3$ are equations of two diameters of a circle whose radius is 5 units. Find the equation of the circle.

16. Find the centre of the ellipse $25x^2 + 16y^2 - 100x + 32y - 284 = 0$.

17. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$ then show that $x + y + xy = 1$.

18. If $x + \frac{1}{x} = 2 \cos \theta$ then show that one of the values of x is $e^{i\theta}$.

19. If $y = \sqrt{\sin 2x + \sqrt{\sin 2x + \sqrt{\sin 2x + \ldots \text{ upto } \infty}}}$, then show that $\frac{dy}{dx} = \frac{2 \cos 2x}{(2y - 1)}$.

20. Find the y-intercept of the tangent drawn to the curve $y = 3x^2 - 4x$ at $(1, -1)$ on it.

21. Evaluate : $\int \frac{\cos x \, dx}{\sqrt{8 + \cos^2 x}}$.

22. Find the order and degree of the differential equation

$$a^2 \frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{4}}.$$

PART – C

I. Answer any *three* questions : $3 \times 5 = 15$

23. Find the G.C.D. of 111 and 409 and express it in two ways in the form $111m + 409n$ where $m, n \in Z$.

24. Solve by matrix method :

$$x + 2y + 3z = 10$$

$$2x - 3y + z = 1$$

$$3x + y - 2z = 9.$$

25. Show that the set $G = \{ 2^n / n \in Z \}$ forms an Abelian group w.r.t. usual multiplication.

26. a) Prove that

$$\left[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a} \right] = 2 \left[\vec{a} \quad \vec{b} \quad \vec{c} \right].$$ 3

b) θ being angle between $\vec{a} = \hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, find the value of $\cos \theta$. 2

II. Answer any *two* questions : $2 \times 5 = 10$

27. a) Find the equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point $P(x_1, y_1)$ on it.

3

b) What is the value of k if the circles $x^2 + y^2 + kx + 4y + 2 = 0$ and $2x^2 + 2y^2 - 4x - 3y + k = 0$ cut each other orthogonally ?

2

28. a) Find the equation of ellipse in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if distance between foci is 8 and the distance between directrices is 32.

3

b) Find a point on parabola $y^2 = 2x$ whose focal distance is $\frac{5}{2}$.

2

29. a) Solve : $\sin^{-1} x + \sin^{-1} (2x) = \frac{\pi}{2}$. 3

b) Find the general solution of

$$\tan \theta + \tan 2\theta + \sqrt{3} \tan \theta \tan 2\theta = \sqrt{3} .$$ 2

III. Answer any *three* of the following questions : $3 \times 5 = 15$

30. a) Differentiate $\tan (ax)$ w.r.t. x from first principles. 3

b) Differentiate w.r.t. x : $(\cos x)^x$. 2

31. a) If $y = e^{m \tan^{-1} x}$ then show that

$$(1 + x^2)^2 y_2 + 2x (1 + x^2) y_1 - m^2 y = 0.$$ 3

b) Differentiate $\cos (3x + 2)$ w.r.t. $3 \cos x$. 2

32. a) If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$ and $x \neq y$ then show that

$$\frac{dy}{dx} = \frac{-1}{(1 + x)^2} .$$ 3

b) Evaluate $\int \frac{(x - 2)\, dx}{\sqrt{x + 5}}$. 2

33. a) Evaluate $\int \left[e^x (1 + x) \log_e (xe^x) \right] dx$. 3

b) Evaluate $\int (\tan x + \cot x)^2 dx$. 2

34. Find the area of circle $x^2 + y^2 = a^2$ by the method of integration.

PART – D

Answer any *two* of the following questions : $2 \times 10 = 20$

35. a) Define hyperbola as locus of a point and derive its equation in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. 6

b) Prove that

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}.$$ 4

36. a) State and prove the De Moivre's theorem for all rational indices. 6

b) Prove by vector method :

$$\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta.$$ 4

37. a) A man 6 ft tall walks at a rate of 2 ft/sec away from the source of light which is hung 15 ft above the horizontal ground.

i) How fast is the length of his shadow increasing ?

ii) How fast is the tip of his shadow moving ? 6

b) Find the general solution of

$$\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0.$$ 4

38. a) Prove that $\int_0^1 \frac{\log_e (1+x)}{(1+x^2)}\, dx = \frac{\pi}{8} \log_e 2$. 6

b) Find the general solution of

$$\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2 y^2}\,.$$ 4

PART – E

Answer any *one* of the following questions : $1 \times 10 = 10$

39. a) Find the cube roots of $1 + i\sqrt{3}$ and respresent them in Argand diagram. 4

b) Find the length of the common chord of the intersecting circles

$$x^2 + y^2 - 4x + 6y - 12 = 0 \quad \text{and}$$

$$x^2 + y^2 + 2x - 2y - 23 = 0.$$ 4

c) Find the number of incongruent solutions and the incongruent solutions of linear congruence $5x \equiv 3 \pmod 6$. 2

40. a) If $\vec{a}$, $\vec{b}$ are two unit vectors such that $\left(\vec{a} + \vec{b}\right)$ is also a unit vector then show that angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and also show that $\left|\vec{a} - \vec{b}\right| = \sqrt{3}$. 4

b) Evaluate $\int \tan^4 (3x)\, dx$. 4

c) Differentiate w.r.t. x : $\log_5 \left(\log_e x\right)$. 2