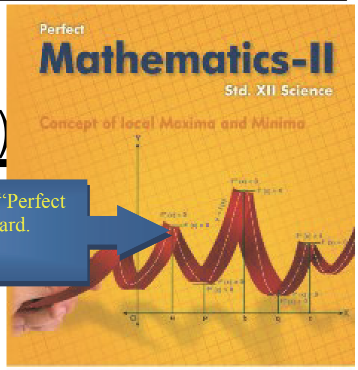
BOARD QUESTION PAPER: MARCH 2014
MATHEMATICS – II (12th Sci., HSC, Maharashtra)



Note:

i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Answers to both sections must be written in one answer book.
v. Answer to every new question must be written on a new page.

SECTION – II

Q.4. (A) Select and write the correct answer from the given alternatives in each of the following: (6)[12]

i. If $y = 1 - \cos\theta$, $x = 1 - \sin\theta$, then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(A) -1 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

ii. The integrating factor of linear differential equation

$\frac{dy}{dx} + y\sec x = \tan x$ is

(A) $\sec x - \tan x$ (B) $\sec x \cdot \tan x$

(C) $\sec x + \tan x$ (D) $\sec x \cdot \cot x$

iii. The equation of tangent to the curve $y = 3x^2 - x + 1$ at the point (1, 3) is

(A) $y = 5x + 2$ (B) $y = 5x - 2$

(C) $y = \frac{1}{5}x + 2$ (D) $y = \frac{1}{5}x - 2$

(B) Attempt any THREE of the following: (6)

i. Examine the continuity of the function

$f(x) = \sin x - \cos x$, for $x \neq 0$

$= -1$, for $x = 0$

at the point $x = 0$.

ii. Verify Rolle's theorem for the function

$f(x) = x^2 - 5x + 9$ on [1, 4]

iii. Evaluate: $\int \sec^n x \cdot \tan x \, dx$

iv. The probability mass function (p.m.f.) of X is given below:

X = x	1	2	3
P(X = x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

Find $E(X^2)$

v. Given that X ~ B (n = 10, p). If E (X) = 8, find the value of p.

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then prove that $y = f[g(x)]$ is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

ii. Obtain the differential equation by eliminating the arbitrary constants A, B from the equation: $y = A\cos(\log x) + B\sin(\log x)$

iii. Evaluate: $\int \frac{x^2}{(x^2+2)(2x^2+1)}\,dx$

(B) Attempt any TWO of the following: **(8)**

i. An open box is to be made out of a piece of a square card board of sides 18 cms by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

ii. Prove that: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

iii. If the function f(x) is continuous in the interval [–2, 2], find the values of a and b, where

$$f(x) = \frac{\sin ax}{x} - 2, \quad \text{for } -2 \le x < 0$$
$$= 2x + 1, \quad \text{for } 0 \le x \le 1$$
$$= 2b\sqrt{x^2+3} - 1, \quad \text{for } 1 < x \le 2$$

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. Solve the differential equation: $\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$

ii. A fair coin is tossed 8 times. Find the probability that it shows heads at least once.

iii. If $x^p y^q = (x+y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

(B) Attempt any TWO of the following: **(8)**

i. Find the area of the sector of a circle bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

ii. Prove that:

$$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$$

iii. A random variable X has the following probability distribution:

$X = x$	0	1	2	3	4	5	6
$P[X = x]$	k	3k	5k	7k	9k	11k	13k

(a) Find k

(b) Find $P(0 < X < 4)$

(c) Obtain cumulative distribution function (c.d.f.) of X.