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(2063)

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B.Tech 2nd Semester Examination

Applied Mathematics-II

AS-1006

Time : 3 Hours **Max. Marks : 100**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/ continuation sheet will be issued.

Note : Candidates are required to attempt five questions in all, selecting one question from each of the sections A, B, C & D of the question paper and all the subparts of the questions in section E. Use of non-programmable calculator is allowed.

SECTION - A

1. (a) Find the curvature and torsion for the curve $\vec{R}(t) = a(3t - t^3)\hat{i} + 3at^2\hat{j} + a(3t + t^2)\hat{k}$

   (b) Find the directional derivative of $\phi = xy^2 + yz^3$ at a point (2, –1, 1), in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (–1, 2, 1). **(10,10)**

2. (a) If $F = 2y\,I - zJ + xK$, evaluate $\int F \times dR$ along the curve x = cost, y = sint, z = 2cost from $t = 0$ to $t = \frac{\pi}{2}$.

   (b) Use divergence theorem to evaluate $\int_S F.dS$, where $F = x^3 I + y^3 J + z^3 K$, and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(10,10)**

SECTION - B

3. (a) Expand the function f(x) = xsin x, as a Fourier series in the interval $-p \le x \le \pi$. Hence deduce that
$$\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\frac{1}{7.9}+.... =\frac{\pi-2}{4}.$$

(b) For $0 \le x \le \pi$, express $f(x) = x(\pi - x)$ into half range cosine series and hence deduce that $\sum_{n=1}^{\infty}\frac{1}{4}=\frac{\pi^4}{90}$ **(10,10)**

4. (a) Find the Laplace transform of the triangular wave function of period 2c given by
$$f(x)=\begin{cases} t, & 0<t<c \\ 2c-t, & c<t<2c \end{cases}$$

(b) Solve the integral equation
$$\int_0^{\infty} F(x)\cos px\, dx=\begin{cases} 1-p, & 0\le p\le 1 \\ 0, & p>1 \end{cases}$$
Hence deduce that $\int_0^{\infty}\frac{\sin^2 t}{t^2}dt=\frac{\pi}{2}$. **(8,12)**

SECTION - C

5. (a) Solve in series (by Frobenious method) the differential equation $4x\frac{d^2y}{dx^2}=2\frac{dy}{dx}+y=0$.

(b) Prove that
$$\int_{-1}^{1} P_m(x)P_n(x)dx=\begin{cases} 0, & \text{if } m\neq n \\ \frac{2}{2n+1}, & \text{if } m=n \end{cases}$$, where Pm(x) and Pn(x) are Legendre's polynomials. **(10,10)**

6. (a) Find the general solution of the Bessel's equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}(x^2-n^2)y=0$ when n is not an integer.

(b) Prove that $nP_n = xP_n^{'} - P_{n-1}^{'}$ **(12,18)**

SECTION - D

7. (a) A bar of 20 cm long, with insulated sides, has its ends A and B maintained at 30°C and 80°C, until steady state conditions prevail. The temperatures of ends are changed to 40°C and 60ºC respectively. Find the temperature distribution in the rod at time t.

(b) A string is stretched and fastened to two fixed points, l distant apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released from rest at t = 0. Find the displacement y(x, t) at any time t. **(10,10)**

8. (a) Solve the following partial differential equation $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0,$ by method of separation of variable.

(b) A rectangle plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along edge y = 0 is given by u(x, 0) = 100 sin $\frac{\pi x}{8}, 0 < x < 8.$ While the two long edges x = 0 and x = 8 as well as the other short edge are kept at 0 C. Find temperature u(x, y) at any point in the plate. **(8,12)**

SECTION - E

9. (1) Define scalar point and vector point functions and give physical interpretation of curl of vector point function.

(2) State Green's theorem and Stokes theorem.

(3) Define irrotational and solenoidal fields.

(4) Evaluate $\int_0^x \frac{\sin t}{t} dt$ using Laplace transform.

(5) Find the inverse Laplace transform of $\frac{s}{\left(s^2+a^2\right)^2}$.

(6) Define Fourier sine . Fourier cosine transforms and also inverse Fourier sine and inverse Fourier cosine transforms.

(7) Show that $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

(8) Show that $P_{2n}(0) = (-1)^n \frac{2n!}{2^{2n}(n!)^2}$.

(9) Solve the equation $((\partial f/\partial x) = 2(\partial f/\partial t) = f$, given f(x, 0) = 6exp (–3x).

(10) Show that wave equation $(\partial^2 u/\partial t^2) = c^2(\partial^2 u/\partial x^2)$ is of hyperbolic nature. **(2×10=20)**