MSc. Mathematics Degree (MGU-CSS-PG) Examination

MODEL QUESTION PAPER

1st SEMESTER

PC 2 - MT01C02 BASIC TOPOLOGY

PART A

(Answer any five Each question has weightage 1)

Time 3 hrs. Maximum Weight. 30

1.Define: i) Topographical space

ii) Closure of a set

iii) Interior point of a set

iv) Accumulation point of a set

2. If A,B are subsets of a space X show that $\overline{AUB} = \overline{A} \cup \overline{B}$

3. Let X,Y be topological spaces and $f: X \rightarrow Y$ a function. Show that f is continuous at $x_0 \in X$

iff forevery subset A$\subset$X, $x_0 \in \overline{A} \Rightarrow f(x_0) \in \overline{f(A)}$

4. Define separable space. Prove that every second countable space is separable

5. Let C be a connected subset of a space Xand A,B are mutually separated subsets of X. Then

$C \subseteq A \cup B$ implies either $C \subseteq A$ or $C \subseteq B$

6. Prove that components of open subsets of a locally connected space are open

7. Prove that a topological space X is T_1 if and only if every singleton set $\{x\}$ is closed in X

8. Prove that compact subsets in Hausdorff space are closed

PART B

(Answer any five Each question has weightage 2)

9. Prove that metrisability is a hereditary property

10. Define derived set of a subset A of space X. Prove that $\overline{A} = A \cup A'$

11. Let X,Y be topological spaces and $f : X \to Y$ a function. If f is continuous then the graph G=$\{(x, f(x)) : x \in X\}$ is homeomorphic to X

12. Every continuous real valued function on a compact space is bounded and attains its extrema

13. If X_1 and X_2 are connected spaces then $\mathbf{X_1}$ X $\mathbf{X_2}$ is connected

14. Every quotient space of a locally connected space is locally connected

15. Prove that all metric spaces are T_4

16. If F is a compact subset and C a closed subset of a completely regular space X and $F \cap C = \phi$, then there exist a continuous function f: X→[0,1] such that f(x)=0 $\forall x \in F$ and f(y)=1 $\forall$ y$\in C$

PART C

(Answer any three, Each question has weightage 5)

17. a)Every open cover of a second countable space has a countable subcover

b)In a metric space X, a point y is in the closure of a subset A iff there exist a sequence $\{x_n\}$ such that $x_n \in A$ $\forall$ n and $\{x_n\}$ converges to y in X

18. State and prove Lebesgue covering lemma

19. Let X be a space which is first countable at $x \in X$ and $f : X \to Y$ a function.Then f is

Continuous at x iff for every sequence $\{x_n\}$ which converges to x in X, the sequence $\{f(x_n)\}$ converges to f(x) in Y

20. A subset of R is connected iff it is an interval

21. Every regular, lindelöff space is normal

22. a)In a Hausdorff space limits of sequences are unique

b)Every completely regular space is regular

c)Let Y be a Hausdorffspace. Prove that for any space X and any two maps $f, g : X \to Y$ the set $\{x \in X : f(x) = g(x)\}$ is closed in X