ISC 2015
MATHEMATICS

- **SOLUTION OF 2015**
- **COMMENTS OF COUNCIL EXAMINERS**
- **SUGGESTIONS FOR TEACHERS**

Dedicated to all my lovely students. May God help you always.

This small booklet contains solution of ISC 2015 Mathematics.

The comments from the council examiners under solution of every question makes this a very handy guide for students to understand what the council expects as answer from the students.

I hope that the students will find this to be useful.

- *Md. Zeeshan Akhtar*

03rd March, 2016.

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MATHEMATICS

SECTION A

Question 1 **[10×3]**

(i) Find the value of k if M = $\begin{matrix} 1 & 2 \\ 2 & 3 \end{matrix}$ and $M^2 - kM - I_2 = 0$

(ii) Find the equation of an ellipse whose latus rectum is 8 and eccentricity is $\frac{1}{3}$.

(iii) Solve: $\cos^{-1}(\sin \cos^{-1}x) = \frac{\pi}{6}$

(iv) Using L'Hospital's rule, evaluate: $\lim\limits_{x \to 0} \frac{x - \sin x}{x^2 \sin x}$

(v) Evaluate: $\int \frac{2y^2}{y^2 + 4}\, dy$

(vi) Evaluate: ${}^{3}_{0} f\ x\ dx$, where $f(x) = \begin{cases} cos2x, 0 \le x \le \frac{\pi}{2} \\ 3, \quad \frac{\pi}{2} \le x \le 3 \end{cases}$

(vii) The two lines of regressions are $4x + 2y - 3 = 0$ and $3x + 6y + 5 = 0$. Find the correlation co-efficient between x and y.

(viii) A card is drawn from a well shuffled pack of playing cards. What is the probability that it is either a spade or an ace or both?

(ix) If 1, ω and ω^2 are the cube roots of unity, prove that $\frac{a + b\omega + c\omega^2}{c + a\omega + b\omega^2} = \omega^2$

(x) Solve the differential equation: $\sin^{-1} \frac{dy}{dx} = x + y$

Comments of Examiners

(i) A number of candidates wrote M^2 as $\begin{bmatrix} 1 & 4 \\ 4 & 9 \end{bmatrix}$ instead of $\begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix}$. Some candidates took 'k' as matrix instead of 'k' as scalar value while some candidates wrote I_2 as $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ which was not correct. I_2 was an identity matrix of the order 2×2.

(ii) Some candidates took the length of latus rectum as '4a' instead of $\frac{2b^2}{a}$. A few candidates wrote the equation

Suggestions for teachers

- Revise all matrix operations in the class. Pay heed to matrix multiplication.
- Identity matrix, null matrix and their order should be explained thoroughly.
- The topic of ellipse and hyperbola should be taught separately and then their properties should be compared.
- Horizontal and vertical ellipse should be explained thoroughly.
- Relation $b^2 = a^2(1-e^2)$ where e<1 should be explained.

of hyperbola in place of ellipse i.e. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ instead of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. A few candidates used eccentricity formula, which was incorrect.

(iii) Many candidates did this question well but a few used wrong value of cos $\frac{\pi}{6}$, while some wrote incorrect conversion of $\cos^{-1}x$ in terms of $\sin^{-1}$ x. A few candidates wrote $\cos^{-1}x$ in terms of $\tan^{-1}x$ which made the expression complicated.

(iv) Some candidates wrote incorrect differentiation of numerator and denominator. They wrote differentiation of '1-cosx' as '1-sinx' which was not correct. Some wrote the differentiation of sinx as (-cosx).

(v) The power of numerator and denominator was equal in the given integral so division was a must or addition and subtraction of constant could also work but many candidates forgot and tried to solve it as it was.

(vi) Candidates were able to score marks in this question.

(vii) Many candidates answered this question correctly. However, a few candidates wrote incorrect regression coefficient.

(viii) Many candidates found this topic difficult. Some did not understand the meaning of 'either or term' in the question.

(ix) Many candidates wrote the formula but were not able to apply it correctly. They put the value of $w = -1-w^2$ and $w^2 = -1-w$, which made the equation very complicated.

(x) A number of candidates wrote sin (x + y) as, sin x + sin y which was incorrect. On the other hand, some candidates were not able to substitute x + y = t. A few candidates made calculation mistakes in this question

- Teach students derivations of inverse Trigonometric functions.
- Indeterminate forms i.e. $\frac{0}{0},\frac{\infty}{\infty}$ etc. should be explained properly and revision of differentiation chapter must be done for practice. L' Hospital's rule must be taught giving appropriate conditions to deal with different indeterminate forms.
- Teach students the properties of definite integrals properly and their use in area.
- Coefficient of regression of lines y on x and x on y should be explained by explaining $r = \pm\sqrt{b_{yx}b_{xy}}$ and that the value of 'r' should be less than 1; b_{yx} and b_{xy} both positive, 'r' will be positive otherwise negative.
- Theorem 'either or' and theorem 'AND' should be explained properly to students. Number of outcomes and number of favourable outcomes should be explained properly. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(A \cap B)= P(A).P(B)$ and $P(A/B)= \frac{P(A\cap B)}{P(B)}$.
- Complex numbers should be divided into different parts then explained step by step. Application of cube roots of unity needs to be explained thoroughly. Stress upon the techniques of solving such questions.
- Differential equations and various forms i.e. separation of variables, homogenous, linear differential equations and their reducible forms need to be revised by doing different types of questions based on them.

MARKING SCHEME

Question 1.

(i)

$$M^2 - kM - I_2 = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 5 & 8 \\ 8 & 13 \end{bmatrix} - \begin{bmatrix} k & 2k \\ 2k & 3k \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 5-k & 8-2k \\ 8-2k & 13-3k \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 5-k=1,\ 8-2k=0,\ 13-3k=1$$

$$\Rightarrow k = 4$$

(ii)

$\frac{2b^2}{a} = 8$; e = $\frac{1}{3}$

$b^2 = 4a$

$b^2 = a^2(1 - e^2)$

$\Rightarrow 4a = a^2\left(1 - \frac{1}{9}\right)$

$\Rightarrow a = \frac{9}{2}$

$b^2 = 18$.

Equation of an ellipse:

$\frac{4x^2}{81} + \frac{y^2}{18} = 1$ or $8x^2 + 9y^2 = 162$

(iii)

$\cos^{-1}(\sin \cos^{-1} x) = \frac{\pi}{6}$

$\sin(\cos^{-1} x) = \cos = \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\sqrt{1 - x^2} = \frac{\sqrt{3}}{2}$

$1 - x^2 = \frac{3}{4} \Rightarrow x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

(iv)	$\underset{x \to 0}{Lt} \frac{x - sinx}{x^2 sinx}$ $= \underset{x \to 0}{Lt} \frac{x - sinx}{x^3} \left[\underset{x \to 0}{Lt} \frac{sinx}{x} = 1 \right]$ $= \underset{x \to 0}{Lt} \frac{1 - cosx}{3x^2}$ $= x \underline{Lt} \ 0 \ \frac{\sin x}{6x} = \frac{1}{6}$
(v)	$\int \frac{2y^2}{y^2+4} dy$ $2\int \frac{y^2+4-4}{y^2+4}$ dy $= 2y - 8. \frac{1}{2} \tan^{-1} \frac{y}{2} + c$ $= 2y - 4 \tan^{-1} \frac{y}{2} + c$
(vi)	$\int_0^3 f(x)\,dx$ $= \int_0^{\pi/2} \cos 2x\,dx + \int_{\pi/2}^{3} 3dx$ $= \left[\frac{sin2x}{2}\right]_0^{\pi/2} + [3x]_{\pi/2}^{3}$ $(0-0) + 3\left(3 - \frac{\pi}{2}\right)$ $= 9 - \frac{3\pi}{2}$
(vii)	Let the line of regression of x on y be $4x + 2y - 3 = 0$ $x = \frac{-1}{2}y + \frac{3}{4}$ $\Rightarrow b_{xy} = \frac{-1}{2}$ let the line of regression of y on x be $3x + 6y + 5 = 0$ $y = \frac{-1}{2}x - \frac{5}{6}$ $b_{yx} = \frac{-1}{2}$

	$\therefore\ r^2 = b_{yx} \times b_{xy} = \left(\frac{-1}{2}\right)\times\left(\frac{-1}{2}\right) = \frac{1}{4}$ $r = -\frac{1}{2}$, since b_{xy} *and* b_{yx} are negative.
(viii)	$P(E) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$
(ix)	$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}$ $= \frac{\omega^2(a+b\omega+c\omega^2)}{\omega^2(c+a\omega+b\omega^2)}$ Multiplying numerator and denominator by ω^2 $= \frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a\omega^3+b\omega^4}$ since $\omega^3 = 1$ and $\omega^4 = \omega^3.\omega = \omega$ $= \frac{\omega^2(a+b\omega+c\omega^2)}{c\omega^2+a+b\omega} = \omega^2$
(x)	$\sin^{-1}\frac{dy}{dx} = x+y$ $\frac{dy}{dx} = \sin(x+y)$ Let $x+y=v$ $1+\frac{dy}{dx} = \frac{dv}{dx}$ $\frac{dv}{dx} - 1 = \sin v$ $\frac{dv}{dx} = 1+\sin v$ $\int\frac{dv}{1+\sin v} = \int dx$ $\int\frac{(1-\sin v)\,dv}{1-\sin^2 v} = \int dx$ $\int\frac{(1-\sin v)\,dv}{\cos^2 v} = \int dx$ $\int \sec^2 v - \tan v \sec v\, dx = \int dx$ $\tan v - \sec v = x + c \Rightarrow \tan(x+y) - \sec(x+y) = x + c$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

(b) Given two matrices A and B **[5]**

$$A = \begin{vmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{vmatrix} \text{ and } B = \begin{vmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{vmatrix},$$

find AB and use this result to solve the following system of equations:

$$x - 2y + 3z = 6,\ x + 4y + z = 12,\ x - 3y + 2z = 1$$

Comments of Examiners

(a) Properties of determinants were not correctly implemented by several candidates. A few expanded the determinants directly without applying any property. They were not able to get zeroes in row or column. Some applied useless properties which did not lead to result. Rows and columns were not correctly identified by several candidates.

(b) A few candidates found the product of AB incorrectly. Many did not use the product of AB to solve the equation system. They found A^{-1} by using matrix inverse method. Several candidates found incorrect cofactors hence their values of x, y, z were incorrect. Some candidates could not obtain adjoint and inverse of a matrix correctly.

Suggestions for teachers

- Plenty of practice must be given in using determinant properties. The idea of obtaining two zeroes in a row or a column is to be taught for easiest simplification.
- Inverse of a square matrix needs to be taught step by step. Utilisation of the inverse to correctly find the unknown matrix needs to be grasped properly. Product of two matrices needs attention. Sufficient practice is a must.

MARKING SCHEME

Question 2.

(a) Replace $C_1 \to C_1 - bC_3$, $C_2 \to C_2 + aC_3$ and take $(1+a^2+b^2)$ *common from each* C_1 *and* C_2

$$D = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$$

Replace R_3 by $R_3 - bR_1$ to get

$$= (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$$ Expanding by C_1, we get

$$= (1+a^2+b^2)^2 (1-a^2+b^2+2a^2) = (1+a^2+b^2)^3$$

(b)	$AB = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix} = \begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix}$ $AB = -8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ AB= -8 I $A^{-1} = \frac{1}{-8}B$ $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{and } C = \begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix}$ $\mathbf{X = A^{-1}\,C}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-8}\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}\begin{bmatrix} 6 \\ 12 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = 1\ , y = 2, \quad z = 3$

Question 3

(a) Solve the equation for x: $sin^{-1}\frac{5}{x} + sin^{-1}\frac{12}{x} = \frac{\pi}{2}, \quad x \neq 0$ **[5]**

(b) A, B and C represent switches in 'on' position and A', B' and C' represent them in 'off' position. Construct a switching circuit representing the polynomial ABC + AB′C + A′B′C. Using Boolean Algebra, prove that the given polynomial can be simplified to C(A + B′). Construct an equivalent switching circuit. **[5]**

<u>Comments of Examiners</u>

(a) Some candidates made mistakes while converting $\sin^{-1}$ to $\cos^{-1}$ or vice versa. Many candidates got incorrect algebraic equation independent from inverse function. As a result they could not solve the equation further. Some candidates applied $\sin^{-1}$ formula but they could not solve further.

(b) A few candidates made errors while constructing a switching circuit. They made mistakes while simplifying the given polynomial. They were not able to write distributive law at this step (AB + AB′ + A′B′)C while a few wrote B + B′ = 0 which was incorrect. Some candidates made simplification errors while expanding the Boolean function by applying incorrect properties of Boolean algebra.

<u>Suggestions for teachers</u>

– All algebraic and trigonometric laws need to be revised thoroughly before learning inverse trigonometric functions and their operations. Application of formula for inverse trigonometric functions needs attention. Domains and range needs to be explained properly.

– All properties of Boolean algebra need to be well understood before application. Sufficient practice is a must.

MARKING SCHEME

Question 3.

(a) $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$, $x \neq 0$

$$\sin^{-1}\frac{5}{x}=\frac{\pi}{2}-\sin^{-1}\frac{12}{x}$$

$$\frac{5}{x}=\sin\left(\frac{\pi}{2}-\sin^{-1}\frac{12}{x}\right)$$

$$\frac{5}{x}=\cos\left(\sin^{-1}\frac{12}{x}\right)$$

$$\frac{5}{x}=\cos\left(\cos^{-1}\sqrt{1-\left(\frac{12}{x}\right)^2}\right)$$

$$\frac{5}{x}=\sqrt{1-\left(\frac{12}{x}\right)^2}$$

squaring on both sides

$$\frac{25}{x^2}=1-\frac{144}{x^2}$$

$$x^2=169$$

$$x=\pm 13$$

$x = 13$ is the required answer

(b) ABC+AB′C +A′B′C

= ACB+ACB′ + A′B′C

=AC (B+B′) + A′B′C (B+B′ =1)

= AC + A′B′C

= (A+A′B′)C

=(A+A′)(A+B′) C

= (A+B′) C

Question 4

(a) Verify Lagrange's Mean Value Theorem for the following function: **[5]**

$f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

(b) Find the equation of the hyperbola whose foci are $(0, \pm\sqrt{10})$ and passing through the point (2, 3). **[5]**

Comments of Examiners

(a) Many candidates failed to state all the criteria for application of Lagrange's theorem correctly. The concept of closed or open was not clear to many candidates. A few candidates got confused with Rolle's theorem' condition f (a) = f (h).

(b) Some candidates did not have proper knowledge of hyperbola and conjugate hyperbola. They wrote incorrect equation of hyperbola, hence got incorrect answer; a few took 2ae = $\sqrt{10}$ which was incorrect (where 2ae is the distance between the two foci). A few candidates found the value of 'a' & 'b' correctly but substituted incorrectly.

Suggestions for teachers

- Help students enumerate the criteria for mean value theorem correctly. Firstly, the given function has to be continuous in the closed interval, secondly, the derivation of given function needs to exist in open interval and thirdly, f' (c) = $\frac{f(b)-f(a)}{b-a}$ where 'c' exists in open interval. Explanation of geometrical interpretation of mean value theorem with the help of figure is a must.
- Stress upon conics noting details with regard to their sketching and derivation of their equations for standard form as well as for other modified forms. Regular practice of conics is a must.

MARKING SCHEME **Question 4.**	
(a)	$f(x) = (2\sin x + \sin 2x)$ is continuous in $[0, \pi]$ $f'(x)$ exists in $(0, \pi)$ $f'(x) = 2\cos x + 2\cos 2x$ $f(0) = 0$, $f(\pi) = 0$ All the conditions of Lagrange's Mean Value theorem are satisfied there exist ' c ' in $(0, \pi)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$ $2\cos c + 2\cos 2c = 0$ $2\cos^2 c + \cos c - 1 = 0$ $\cos c = -1, \frac{1}{2} \Rightarrow \cos c = \cos\pi \Rightarrow c = \pi$ (not possible) or $\cos c = \cos\frac{\pi}{3}$ $\Rightarrow c = \frac{\pi}{3} \in (0, \pi)$

	$c = \pi/3$ which lies between 0 to π , hence, LMV theorem is verified.
(b)	Foci $(0, \pm\sqrt{10})$ be $= \sqrt{10}$ $a^2 = b^2(e^2 - 1) = b^2e^2 - b^2$ $a^2 = 10 - b^2$ let the equation be: $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ $\Rightarrow \frac{9}{10-a^2} - \frac{4}{a^2} = 1 \Rightarrow 9a^2 - 40 + 4a^2 = 10a^2 - a^4$ $\Rightarrow a^4 + 3a^2 - 40 = 0$ $\Rightarrow (a^2 + 8)(a^2 - 5) = 0 \Rightarrow a^2 = -8$ or $a^2 = 5$, (a^2 can't be negative) $a^2 = 5$, $b^2 = 5$ $\therefore$ the required equation is $\frac{y^2}{5} - \frac{x^2}{5} = 1 \Rightarrow y^2 - x^2 = 5$

Question 5

(a) If $y = e^{m\cos^{-1}x}$, prove that: **[5]**

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$

(b) Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius 10 cm is a square of side $10\sqrt{2}$ cm. **[5]**

Comments of Examiners

(a) Some candidates wrote differentiation of $\cos^{-1} x$ incorrectly. Many candidates did not place $e^{m\cos^{-1}x}$ as y after first differentiation. Second order derivation was incorrectly shown by several candidates. Some made calculation mistakes while simplifying the equation.

(b) Many candidates were not able to write the expression in mathematical form. They were not able to express the equation in one variable. A number of candidates made calculation mistakes while differentiating.

Suggestions for teachers

- Differentiation rules for different functions and terms need attention. A through revision is a must.
- Explain to students the importance of finding the second derivative. They must show the condition of maxima or minima as per the requirement.

MARKING SCHEME

Question 5.

(a)

y = $e^{m\cos^{-1}x}$

$\frac{dy}{dx} = e^{m\cos^{-1}x} \times \frac{-m}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx}$ - my differentiating again wrt x.

$$\Rightarrow \sqrt{1-x^2}\frac{d^2y}{dx^2} - \frac{2x}{2\sqrt{1-x^2}}\frac{dy}{dx} = -m\frac{dy}{dx}$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -m\frac{dy}{dx}\sqrt{1-x^2}$$

$$= -m \times (-my)$$

$$= m^2y$$

(b)

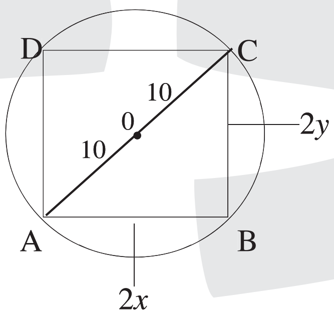
AB = $2x$; BC = $2y$

$x^2 + y^2 = 10^2 \quad 4x^2 + 4y^2 = 400$

$\therefore (2x)^2 + (2y)^2 = (20)^2$

P = $4x + 4y$

$= 4x + 4\cdot\sqrt{100 - x^2}$



$$\frac{dP}{dx} = 4 - \frac{4x}{\sqrt{100-x^2}} = 0 \Rightarrow x = 5\sqrt{2}$$

$$\frac{d^2P}{dx^2} = -4\left\{\frac{\sqrt{100-x^2} - \frac{x(-x)}{\sqrt{100-x^2}}}{100-x^2}\right\} = \frac{-4\times 100}{(100-x^2)^{\frac{3}{2}}} < 0$$

Hence, perimeter is maximum when x = $5\sqrt{2}$

$\therefore$ y = $5\sqrt{2} \Rightarrow x = y$

$\therefore$ ABCD is square of side $10\sqrt{2}$ cm

Question 6

(a) Evaluate: **[5]**

$$\int \frac{sec\,x}{1 + cosec\,x}\,dx$$

(b) Find the smaller area enclosed by the circle $x^2 + y^2$ and the line $x + y = 2$. **[5]**

Comments of Examiners

(a) Many candidates were not able to integrate the given expression. Some candidates could not decompose the problem into partial fraction. Errors were also made while integrating factors.

(b) Many candidates attempted this part correctly by taking arbitrary value of the radius of the circle.

Suggestions for teachers

- Partial fraction rule need to be understood and applied correctly. Methods of proper substitution need attention.
- A lot of practice of such problems must be given by the teachers.

MARKING SCHEME

Question 6.

(a)

$$I = \int \frac{\sec x}{1+\cos ec\, x}\,dx$$

$$= \int \frac{sinx}{\cos\ x(1+\sin\ x)}dx$$

$$= \int \frac{sinxcosx}{cos^2x(1+sinx)}$$

Put $t = sin\ x$

$dt = cos\ x\ dx$

$$= \int \frac{t\,dt}{(1-t^2)(1+t)}$$

$$= \int \frac{t\,dt}{(1-t)(1+t)^2}$$

$$\frac{t}{(1-t)(1+t)^2} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{(1+t)^2}$$

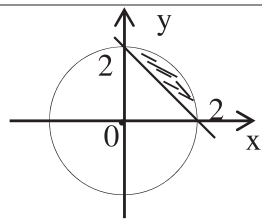
$t = A(1+t)^2 + B(1-t^2) + c(1-t) \Rightarrow t = A(1+2t+t^2) + B - Bt^2 + C - Ct$

$$\Rightarrow t = t^2(A-B) + t(2A-C) + (A+B+C)$$

$$\therefore A-B=0,\ 2A-C=1,\ A+B+C=0$$

Solving equations we get

$$\therefore A = \frac{1}{4}, B = \frac{1}{4}, C = \frac{-1}{2}$$

	$\therefore\ I = \frac{1}{4}\int\left(\frac{1}{1-t}+\frac{1}{1+t}-\frac{2}{(1+t)^2}\right)dt$ $= \frac{1}{4}\left\|-log\|1-t\|+log\|1+t\|+\frac{2}{1+t}\right\|+c$ $I = \frac{1}{4}\left\|log\left\|\frac{1+t}{1-t}\right\|+\frac{2}{1+t}\right\|+c$
(b)	The required area: $= \int_0^2 \sqrt{4-x^2}\ dx - \int_0^2 (2-x)dx$ $= \left\|\frac{x}{s}\sqrt{4-x^2}+\frac{4}{2}sin^1\frac{x}{2}\right\|_0^2 - \left\|2x-\frac{x^2}{2}\right\|_0^2$ $= 2\times\frac{\pi}{2} - 4+2 = \pi - 2$ sq. units 

Question 7

(a) Given that the observations are: **[5]**

(9, −4), (10, −3), (11, −1), (12, 0), (13, 1), (14, 3), (15, 5), (16, 8).

Find the two lines of regression and estimate the value of y when $x = 13{\cdot}5$.

(b) In a contest the competitors are awarded marks out of 20 by two judges. The scores of the 10 competitors are given below. Calculate Spearman's rank correlation. **[5]**

Competitors	A	B	C	D	E	F	G	H	I	J
Judge A	2	11	11	18	6	5	8	16	13	15
Judge B	6	11	16	9	14	20	4	3	13	17

Comments of Examiners

(a) Many candidates found b_{yx} and b_{xy} incorrectly, as a result, the two regression lines were incorrect. Some candidates found the value of y from given value of x by using regression equation of x on y instead of y on x. Several candidates were unable to calculate the correct values of $\sum xy$, $\sum x^2$, $\sum y^2$, b_{yx} and b_{xy} which led to wrong results.
(b) Some candidates calculated the ranks incorrectly. Correction factor for $\sum d^2$ was either incorrect or applied incorrectly in the formula for r. Some candidates wrote incorrect formula for spearman's rank correlation.

Suggestions for teachers
- Various methods of finding b_{yx} and b_{xy} should be taught giving examples. Students should be careful about the formulae for b_{yx} and b_{xy} as well as the regression equation of x on y and that of y on x.
- Students should be given adequate practice to understand which formula is to be applied when ranks are repeated and when ranks are not repeated.

MARKING SCHEME

Question 7.

(a)

x	y	xy	x^2	y^2
9	-4	-36	81	16
10	-3	-30	100	9
11	-1	-11	121	1
12	0	0	144	0
13	1	13	169	1
14	3	42	196	9
15	5	75	225	25
16	8	128	256	64
$x = 100$	$y = 9$	$\sum xy = 181$	$x^2 = 1292$	$\sum y^2 = 125$

$\bar{x} = \frac{100}{8} = 12\cdot5$

$\bar{y} = \frac{9}{8} = 1\cdot125$

$$b_{yx} = \frac{\sum xy - \frac{1}{n}\sum x \sum y}{\sum x^2 - \frac{1}{n}(\sum x)^2}$$

$$= \frac{181 - \frac{1}{8}\times 100 \times 9}{1292 - \frac{1}{8}(100)^2}$$

$$= \frac{68\cdot5}{42}$$

$$= 1\cdot63$$

$$b_{xy} = \frac{\sum xy - \frac{1}{n}\sum x \sum y}{\sum y^2 - \frac{1}{n}(\sum y)^2}$$

$$= \frac{181 - \frac{1}{8}\times 100 \times 9}{125 - \frac{1}{8}(9)^2} = \frac{68\cdot5}{114\cdot875} = 0\cdot596$$

Line of regression of *y* on *x*

$$y - \frac{9}{8} = 1\cdot63\ (x - 12\cdot5)$$

Line of regression of *x* on *y*

$$x - 12\cdot5 = 0\cdot596\ (y - \frac{9}{8})$$

	$x = 0{\cdot}596\ y + 11{\cdot}83$ y, when $x = 13{\cdot}5$ $y = 1{\cdot}63\ x - 19{\cdot}25$ $y = 1{\cdot}63 \times 13{\cdot}5 - 19{\cdot}25$ $= 2{\cdot}755 = 2{\cdot}76$
(b)	see table and working below

Judge A	Judge B	R_x	R_y	$d = R_x - R_y$	d^2
2	6	10	8	2	4
11	11	5.5	6	-0.5	0.25
11	16	5.5	3	2.5	6.25
18	9	1	7	-6	36
6	14	8	4	4	16
5	20	9	1	8	64
8	4	7	9	-2	4
16	3	2	10	-8	64
13	13	4	5	-1	1
15	17	3	2	1	1
					$\sum d^2 = 196{\cdot}5$

$$r = 1 - 6\,\frac{[\sum d^2 + \frac{1}{12}\sum(m^3 - m)]}{n(n^2-1)}$$

$$= 1 - \frac{6[196{\cdot}5 + \frac{1}{12}(2^3 - 2)]}{10 \times 99}$$

$$= 1 - \frac{6 \times 197}{990} = -0 \cdot 194$$

Question 8

(a) An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn. Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black. **[5]**

(b) Three persons A, B and C shoot to hit a target. If A hits the target four times in five trials, B hits it three times in four trials and C hits it two times in three trials, find the probability that: **[5]**

(i) Exactly two persons hit the target.

(ii) At least two persons hit the target.

(iii) None hit the target.

Comments of Examiners

(a) Many candidates handled this problem well but some could not understand the underlying principle.
(b) In some cases, the probability of hitting the target were not found correctly. In part (ii), several candidates could not understand the meaning of 'at least' while some did not apply 'AND theorem.

Suggestions for teachers
- Explain the correct interpretation of such problems.
- Laws of probability should be taught in detail with plenty of examples and illustrations.
- Terms such as 'at least', 'at most', 'exact', 'none' should be discussed and problems based on them practiced.

<table>
<tr><td colspan="3">MARKING SCHEME
Question 8.</td></tr>
<tr><td>(a)</td><td colspan="2">P(E) = P(WWB) + P(WBB) + P(BWB) + P(BBB)
$= \frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}+\frac{2}{4}\times\frac{2}{3}\times\frac{3}{4}+\frac{2}{4}\times\frac{2}{5}\times\frac{3}{4}+\frac{2}{4}\times\frac{3}{5}\times\frac{4}{6}$
$= \frac{1}{6}+\frac{1}{4}+\frac{3}{20}+\frac{1}{5}$
$= \frac{10+15+9+12}{60} = \frac{46}{60} = \frac{23}{30}$
Alternate solution:
P(E) = P(WWB) + P(WBB) + P(BWB) + P(BBB)
$= \left(\frac{2}{4}\times\frac{1}{3}\times\frac{2}{2}\right)+\left(\frac{2}{4}\times\frac{2}{3}\times\frac{2}{3}\right)+\left(\frac{2}{4}\times\frac{2}{4}\times\frac{2}{3}\right)+\left(\frac{2}{4}\times\frac{2}{4}\times\frac{2}{4}\right)$
$= \frac{1}{6}+\frac{2}{9}+\frac{1}{6}+\frac{1}{8}$
$= \frac{49}{72}$</td></tr>
<tr><td>(b)</td><td colspan="2">P(A) = $\frac{4}{5}$, P(B) = $\frac{3}{4}$, P(C) = $\frac{2}{3}$</td></tr>
<tr><td></td><td>(i)</td><td>$P\left(AB\overline{C}\right)+P\left(A\overline{B}C\right)+P\left(\overline{A}BC\right)$
$= \frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}$
$= \frac{26}{60} = \frac{13}{30}$</td></tr>
<tr><td></td><td>(ii)</td><td>$P\left(AB\overline{C}\right)+P\left(A\overline{B}C\right)+P\left(\overline{A}BC\right)+P\left(ABC\right)$</td></tr>
</table>

		$= \frac{13}{30} + \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{50}{60} = \frac{5}{6}$
	(iii)	$P\left(\overline{A}\overline{B}\overline{C}\right)$ $= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$

Question 9

(a) If $z = x + iy$, $w = \frac{2-iz}{2z-i}$ and $|w|=1$, find the locus of z and illustrate it in the Argand Plane. **[5]**

(b) Solve the differential equation: **[5]**

$$e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{x}{y}}\right)\frac{dx}{dy}=0 \text{ when } x = 0,\ y = 1$$

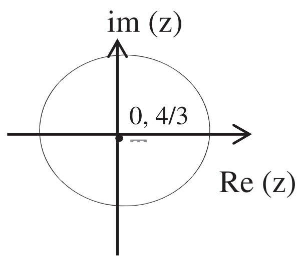
Comments of Examiners

(a) Most of the candidates made mistakes while finding the modulus as well as in the simplification to find the correct values of z. Illustration of z in the Argand plane was incorrectly shown by some candidates.

(b) Many candidates substituted y = vx and proceeded further to solve the given equation using the rules of homogeneous equation, which was an incorrect approach. The subsequent integrals were not correctly understood by some. A few candidates did not find the value of C (constant) under the given condition i.e. x=0, y=1.

Suggestions for teachers

- Interpret the locus of a complex number clearly. Explain the concept of Argand plane. The procedure for finding modulus must be revised thoroughly.
- All forms of integration need rigorous practice. The constant of integration should not be ignored.

MARKING SCHEME **Question 9.**	
(a)	$\lvert W \rvert = 1$ $\Rightarrow \left\lvert\frac{2-iz}{2z-i}\right\rvert = 1 \quad \Rightarrow \lvert 2 - iz\rvert = \lvert 2z - i\rvert$ $\Rightarrow \lvert 2 - i(x+iy)\rvert = \lvert 2(x+iy) - i\rvert$ $\Rightarrow \lvert (2+y) - ix\rvert = \lvert 2x + i(2y-1)\rvert$ $\Rightarrow \sqrt{(2+y)^2 + x^2} = \sqrt{4x^2 + (2y-1)^2}$ Squaring $4 + 4y + y^2 + x^2 = 4x^2 + 4y^2 - 4y + 1$ $\Rightarrow 3x^2 + 3y^2 - 8y - 3 = 0$ $\Rightarrow x^2 + y^2 - \frac{8}{3}y - 1 = 0$ Circle, *Centre* $\left(0, \frac{4}{3}\right)$ and $r = \sqrt{\frac{16}{9} + 1}$ $= 5/3$ 

(b)	$e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)+\left(1+e^{\frac{x}{y}}\right)\frac{dx}{dy}=0$ Substitute $x=vy \Rightarrow \frac{dx}{dy}=v+y\frac{dv}{dy}$ $e^v - ve^v + v + ve^v + (1+e^v)y\frac{dv}{dy}=0$ $(v+e^v)+(1+e^v)y\frac{dv}{dy}=0$ $\left(\frac{1+e^v}{v+e^v}\right)dv+\frac{dy}{y}=0$ Integrating $\int\left(\frac{1+e^v}{v+e^v}\right)dv=-\int\frac{dy}{y}$ $log(v+e^v)=-logy+logc$ $log(v+e^v)+\ logy=logc$ $log\{(v+e^v)y\}=logc$ $log\left\{\left(\frac{x}{y}+e^{\frac{x}{y}}\right)y\right\}=logc$ $\left\{\left(\frac{x}{y}+e^{\frac{x}{y}}\right)y\right\}=c$ $x+e^{\frac{x}{y}}\ y=c$ when $x=0\ and\ y=1\ \Rightarrow c=1$ $x+e^{\frac{x}{y}}\ y=1$

SECTION B

Question 10

(a) Using vectors, prove that angle in a semicircle is a right angle. **[5]**

(b) Find the volume of a parallelopiped whose edges are represented by the vectors: **[5]**
$\vec{a}=2\hat{i}-3\hat{j}-4\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-\hat{k}$, and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$.

Comments of Examiners

(a) Most of the candidates were unable to proceed with the solution for a vector based geometrical question. Vector symbols were not used by many candidates. Some candidates did not show the arrow in the diagram drawn by them. The dot product of vectors was found incorrectly by some candidates.

(b) The concept of scalar triple product was clear to most of the candidates but some wrote dot product first and then cross product, which was incorrect. Some wrote [$\bar{a}\ \bar{b}\ \bar{c}$] in determinant form and made mistakes in calculation.

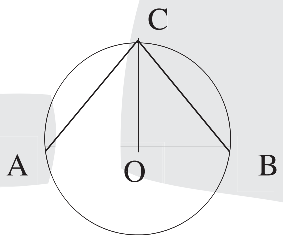
Suggestions for teachers

– Dot product and cross product should be explained well to students. Students must be told to give proper direction to the vectors.
– Vector algebra in totality needs to be explained well to students, especially the properties of scalar triple product. Combination of dot and cross product in scalar triple product needs thorough understanding as well as rigorous practice.

MARKING SCHEME

Question 10.

(a) Let O be the centre of the circle and AB be the diameter. C is a point on the circumference. Take O as the origin and let $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$



Therefore, $\overrightarrow{OB} = -\vec{a}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = \vec{a} - \vec{c}$

$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = -\vec{a} - \vec{c}$

$\overrightarrow{CA}.\overrightarrow{CB} = (\vec{a} - \vec{c}).(-\vec{a} - \vec{c})$

$= -\vec{a}.\vec{a} - \vec{a}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{c}$

$= -|\vec{a}|^2 + |\vec{c}|^2 = -r^2 + r^2 = 0$, Where r is radius

Therefore, angle ACB is a right angle.

(b) The volume of the parallolepiped is:

$$\begin{vmatrix} 2 & -3 & -4 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= 2\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} - 4\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

$= 2\cdot 5 + 3\cdot 5 - 4(-5)$

$= 2\cdot 5 + 3\cdot 5 - 4\ (-5)$

= 45 cubic units

Question 11

(a) Find the equation of the plane passing through the intersection of the planes: **[5]**

$x + y + z + 1 = 0$ and $2x - 3y + 5z - 2 = 0$ and the point $(-1, 2, 1)$.

(b) Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda\,(2\hat{i} + 3\hat{j} + 4\hat{k})$ **[5]**
and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \mu\,(4\hat{i} + 6\hat{j} + 8\hat{k})$

<u>Comments of Examiners</u>

(a) A number of candidates wrote incorrect equation of the plane passing through the intersection of planes. Some made mistakes in calculating the value of λ. A few candidates applied the condition of perpendicularity in this question which was incorrect.

(b) A number of candidates were unable to calculate the correct values of $\overline{a_1}, \overline{a_2} and \overline{b}$. Some made mistakes in calculating $(\overline{a_2} - \overline{a})$. The concepts of skew lines and parallel lines were not clear to many candidates.
Some candidates calculated $\overline{b_1} \times \overline{b} = 0$. They were unable to understand that the given lines are parallel. A few candidates applied wrong formula to calculate the shortest distance between the given lines.

> <u>*Suggestions for teachers*</u>
> – Teach the equation of plane thoroughly. Cartesian and vector of plane should be revised by practicing different types of questions.
> – The concept of parallel and non-parallel lines needs to be explained clearly to students.

MARKING SCHEME **Question 11.**	
(a)	Equation of plane passing through the intersection of the given planes is: $(x + y + z + 1) + k\,(2x - 3y + 5z - 2) = 0$ If this plane passes through (-1,2,1) then $(-1+2+1+1) + k\,(-2 - 6 + 5 - 2) = 0$ $3 = 5k$ $K = 3/5$ $5(x+y+z+1) + 3(2x-3y+5z-2) = 0$ $11x - 4y + 20z - 1 = 0$ Or equivalent form
(b)	Here, $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{a_2} = \; = 2\hat{i} + 4\hat{j} + 5\hat{k}$ $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{a_2} - \vec{a_1} = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + 2\hat{j} + 2\hat{k}$ $\lvert\vec{b}\rvert = \sqrt{4 + 9 + 16} = \sqrt{29}$

	$(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = \widehat{2i} - 0\hat{j} - \hat{k}$ $\left|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\right| = \sqrt{4+1} = \sqrt{5}$ $\frac{\left|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\right|}{\left|\vec{b}\right|} = \sqrt{\frac{5}{29}}$ $\therefore$ shortest distance $= 0.415$

Question 12

(a) Box I contains two white and three black balls. Box II contains four white and one black balls and box III contains three white and four black balls. A dice having three red, two yellow and one green face, is thrown to select the box. If red face turns up, we pick up box I, if a yellow face turns up we pick up box II, otherwise, we pick up box III. Then, we draw a ball from the selected box. If the ball drawn is white, what is the probability that the dice had turned up with a red face? **[5]**

(b) Five dice are thrown simultaneously. If the occurrence of an odd number in a single dice is considered a success, find the probability of maximum three successes. **[5]**

Comments of Examiners

(a) The concept of Bayes' theorem was clear to most candidates but some candidates found incorrect probability. While some candidates found conditional probability for the happening of an event incorrectly, even probability of a specific known event was found wrongly by a few candidates.

(b) Many candidates were unable to understand the problem correctly. The concept of P ($x \leq 3$) was not clear to many candidates. Probability distribution theory was incorrectly applied by some candidates.

Suggestions for teachers

- Teach Bayes' theorem with proper explanation and illustration. Pay heed to the laws of total probability. Give adequate practice of Bayes' theorem.
- Revise Binomial theorem in the class thoroughly before teaching the probability distribution theory. Explain each term in the expansion. Train students about the situation of maximum three successes and minimum three successes.

MARKING SCHEME

Question 12.

(a)

P(A) = 3/6 , P(B) = 2/6,P(C) =1/6

Let D be the probability of drawing a white ball.

P (D/A) =2/5, P(D/B) = 4/ 5, P(D /C) = 3/7

$$P(A/D) = \frac{P(A) \times P(D/A)}{P(A)P(D/A) + P(B)P(D/B) + P(C)\,P(D/C)}$$

$$= \frac{3/6 \times 2/5}{(3/6 \times 2/5 + 2/6 \times 4/5 + 1/6 \times 3/7)}$$

= (6/30)x(210/113)

= 42/113 = 0.37

(b)

n = 5, p = ½, q = ½

$p(x \leq 3) = 1 - p(x=4,5)$

$$= 1 - {}^5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right) - {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= 1 - \frac{5}{32} - \frac{1}{32} = \frac{26}{32} = \frac{13}{16} \text{ or } 0.81$$

SECTION C

Question 13

(a) Mr. Nirav borrowed ₹ 50,000 from the bank for 5 years. The rate of interest is 9% per annum compounded monthly. Find the payment he makes monthly if he pays back at the beginning of each month. **[5]**

(b) A dietician wishes to mix two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below: **[5]**

Food	Vitamin A	Vitamin B	Vitamin C
X	1unit	2 units	3 units
Y	2 units	2 units	1 unit

One kg of food X costs ₹ 24 and one kg of food Y costs ₹ 36. Using Linear Programming, find the least cost of the total mixture which will contain the required vitamins.

<u>Comments of Examiners</u>

(a) Instead of present value of an annuity due, some candidates used the formula for present value of an ordinary Annuity. Number of instalments (n) was not calculated in terms of months, even rate of interest was not calculated per month by a few candidates. Many candidates used wrong formulae.

(b) Many candidates took incorrect inequality sign, hence they got incorrect feasible region and their corner points were also incorrect. Some candidates did not show any graphical representation of the inequalities. In some cases, the representation of the problem was not up to the mark, and the work was not systematic resulting in candidates missing the point of minimum cost.

<u>Suggestions for teachers</u>

- Explain the difference between annuities due and ordinary annuities by giving examples. Train students to read the question carefully, understand the meaning of the question and apply the formula accordingly. A thorough and regular practice is a must.
- Give practice to students in sketching of lines. They should be asked to express the line in intercept form i.e. x/a+ y/b =1, so that sketching is easy. Correct feasible region and its plotting is important.

MARKING SCHEME

Question 13.

(a) Here, $P = 50000$, $i = 0.0075$ and $n = 60$

Now, $P = \frac{A}{i}(1+i)[1-(1+i)^{-n}]$

$50000 = \frac{A}{0.0075}(1+0.0075)[1-(1+0.0075)^{-60}]$

$50000 = \frac{A}{0.0075}(1.0075)[1-(1.0075)^{-60}]$

$A = \frac{50000 \times 0.0075}{1.0075 \times [1-(1.0075)^{-60}]}$

$= 1030.2$

Thus, monthly installment should be Rs.1,030.2

(b) Let there be x units of food x and y units of food y.

Min $z = 24x + 36y$

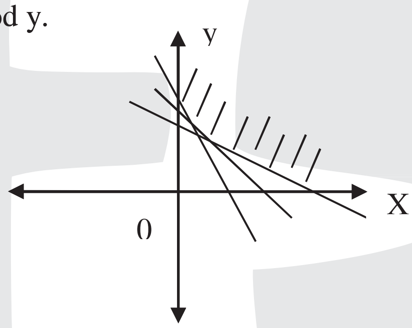
Subject to the constraints

$x + 2y \geq 10$

$2x + 2y \geq 12$

$3x + y \geq 8$

$x \geq 0, y \geq 0$



x	y	Z(cost)
10	0	240
0	8	288
1	5	204
2	4	192

→ (Min. cost)

Question 14

(a) A bill for ₹ 7,650 was drawn on 8th March, 2013, at 7 months. It was discounted on 18th May, 2013 and the holder of the bill received ₹ 7,497. What is the rate of interest charged by the bank? **[5]**

(b) The average cost function, AC for a commodity is given by AC = $x + 5 + \frac{36}{x}$, in terms of output x. Find: **[5]**

(i) The total cost, C and marginal cost, MC as a function of x.

(ii) The outputs for which AC increases.

Comments of Examiners

(a) A number of candidates calculated discounted days incorrectly. They were not able to calculate the rate of interest. Some tried to find 'r' by using B.G. while others used T.D. A few candidates took the difference of Rs. 7650 and Rs. 7497 as interest and applied the present worth formula which was not correct. Some candidates used formula T.D = $\frac{Ani}{1+ni}$ instead of B.D = Ani.

(b) Some candidates wrote incorrect formula of cost function, so their marginal cost was incorrect. Some wrote incorrect differentiation of the expression. Many candidates were not able to answer the second part of the question. They were confused with the maximum minimum condition. They found the derivative and put it equal to zero.

Suggestions for teachers

- Explain Bills of exchange in detail. Differentiate the B.D, T.D and B.G. The procedure for calculating the due date should be taught clearly.
- The concepts of Marginal Cost, Total Cost and Average Cost should be taught in depth for increasing and decreasing functions by giving sufficient examples.
- Familiarize students with the different terms used in this question by giving adequate practice.

MARKING SCHEME

Question 14.

(a)

Face value of the bill= ₹ 7650 = A

Discounted value of the bill =₹ 7497

Banker′s discount=(7650 -7497)

=₹ 153

Nominal due date is 8th October (∴8th October + 3 days of grace).

Legal due date of the bill is 11October

Number of unexpired days from 8 May to 11 October is 146 days   n =(2/5)year

Banker′s discount =Ani

153 = 7650 ×r ×(2/5)

r =(1/20) = 0.05   r = 5%

(b)

Cost function $C = AC \times x = (x + 5 + \frac{36}{x})x = x^2 + 5x + 36$

$$MC = \frac{dC}{dx} = \frac{d}{dx}(x^2 + 5x + 36) = 2x + 5$$

also, $\frac{d(AC)}{dx} = \frac{d}{dx}\left(x + 5 + \frac{36}{x}\right) = 1 - \frac{36}{x^2}$

For AC to be increasing $\frac{d(AC)}{dx} > 0 \Rightarrow 1 - \frac{36}{x^2} > 0$

$\Rightarrow x^2 - 36 > 0$

$\Rightarrow (x-6)(x+6) > 0$

$x < -6$ or $x > 6$

Hence, average cost increases if the output x is > 6.

Question 15

(a) Calculate the index number for the year 2014, with 2010 as the base year by the weighted aggregate method from the following data: **[5]**

Commodity	Price in ₹		Weight
	2010	2014	
A	2	4	8
B	5	6	10
C	4	5	14
D	2	2	19

(b) The quarterly profits of a small scale industry (in thousands of rupees) is as follows : **[5]**

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2012	39	47	20	56
2013	68	59	66	72
2014	88	60	60	67

Calculate four quarterly moving averages. Display these and the original figures graphically on the same graph sheet.

Comments of Examiners

(a) Some candidates used the weighted average of price relative method instead of weighted aggregate method to calculate the index number. A number of candidates wrote incorrect formula of weighted aggregate method.

(b) Some candidates did not calculate centered moving average. Several candidates made mistakes while finding the four yearly moving averages as well as centered moving averages. Plotting of the centered average was inaccurate in a few cases.

Suggestions for teachers

- A thorough and comprehensive practice for calculation of Index number by various method is a must.
- Students must be advised to read the question carefully so as to work out the question using the correct method.
- Students must be advised to practice various methods for finding moving averages rigorously. They must be taught to plot a neat graph for both actual and trend.

MARKING SCHEME

Question 15.

(a)

Commodity	**2010**		**2014**		**p_1w**	**p_ow**
	p_o	**w**	**p_1**	**w**		
A	2	8	4	8	32	16
B	5	10	6	10	60	50
C	4	14	5	14	70	56
D	2	19	2	19	38	38
					200	160

The index number for the year 2014 with 2010 as the base year is $\frac{200}{160} \times 100 = 125$

(b)

Year	Quarter	Quarterly profits	4 yearly moving total	4 yearly average	4 yearly centered moving average
2012	1	39			
	2	47			
			162	40.5	
	3	20			44.125
			191	47.75	
	4	56			49.25
			203	50.75	
2013	1	68			56.5
			249	62.25	
	2	59			64.25
			265	66.25	
	3	66			68.75
			285	71.25	
	4	72			71.375
			286	71.5	
2014	1	88			70.75
			280	70	
	2	60			69.375
			275	68.75	
	3	60			
	4	67			

Correct Graph

Note: For questions having more than one correct solution, alternate correct solutions, apart from those given in the marking scheme, have also been accepted.

GENERAL COMMENTS:

(a) Topics found difficult by candidates in the Question Paper:

- Determinant properties and their use.
- Conics (parabola, ellipse, hyperbola)
- Application of L Hospital's rule.
- Indefinite Integrals, Definite Integrals.
- Inverse trigonometric functions.
- Area of curves.
- Probability (Both sections) and probability distribution.
- Differential equations.
- Complex numbers.
- Vectors.
- 3D plane & straight-line.
- Annuities.
- Linear programming.
- Regression lines.

(b) Concepts between which candidates got confused:

- Conics (parabola, ellipse, hyperbola)
- Open & closed intervals for Mean value theorem.
- Conversion of inverse trigonometric functions.
- Regression coefficient b_{yx} & b_{xy} and r.
- Differential equations (Linear & Homogeneous form)
- Geometrical problem in vectors.
- Annuity due & ordinary annuity.
- Banker's discount & banker's gain.
- Price relative and aggregate method in Index No.
- Shortest distance between skew lines and parallel lines.
- Probability distribution (conceptual problem)

(c) Suggestions for candidates:

- Learn to use the easiest method with correct formula for solving a problem.
- Theorem, rules and laws to be well understood.
- In each chapter, go through the theory and concepts thoroughly followed by solving the illustrations, examples without looking at their solutions.
- Revise and practice from previous year's question paper and sample papers.
- Question paper needs to be read carefully and answered accordingly.
- Wise choices should be made from the options available.
- All steps of calculation need to be simplified before proceeding to the next step.
- Take sufficient rest before the examination.
- Utilize the reading time properly.