(b) Let a be an odd integer, then prove that 5

(i) $a \in Q_2$

(ii) $a \in Q_4$ iff $a \equiv 1 \pmod 4$

(iii) If $c \geq 3$, then $a \in Q_{2^c}$ iff $a \equiv 1 \pmod 8$

(c) Prove that Q_n is a subgroup of U_n. 5

UNIT - V

9. (a) Define Mersenne numbers. 2

(b) State Hurwitz Theorem 2

(c) Define g(k) and G(k) 2

(d) Define Disphantine equations and give two examples. 2

(e) Define Legendre's symbol 2

(f) Prove that Z_n is a group 2

(g) Find generators of U_{25} 2

(h) Find order of Q_{256} 2

Roll No. ....

76456

M.Sc. Mathematics 3rd Semester Examination–December, 2015

ANALYTICAL NUMBER THEORY-I

Paper : MM-515(CI)

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all selecting **one** from each Unit.

UNIT - I

1. (a) Prove that every natural number N > 1 has a prime divisor and, hence, primes are infinite. 8

(b) If $\frac{h}{k}, \frac{h'}{k'}$ are two consecutive members of F_n, then prove that $k \neq k'$ if $n > 1$. 8

2. (a) Let α be any rational number then prove that $\exists$ only a finite number of pairs of integers (x,y) such that 8

$x > 0$, $\gcd(x,y) = 1$ and $\left|\alpha - \frac{y}{x}\right| < \frac{1}{x^2}$

(b) Prove that F_5 is composite and $F_n \equiv 7 \pmod{10}$ for $n \geq 2$, where F_n represents nth Fermat number. 8

UNIT - II

3. (a) State and prove Euler's theorem and, hence, deduce Fermat's theorem. 8

(b) If all primes of type 4k + 3 occur to an even degree in the prime factorization of a natural number n, then n can be written as a sum of two squares. 8

4. (a) Prove that $G(2) = 4$. 8

(b) Let p be a prime such that p> 2, then

$G(p^0(p-1) \geq p^{0+1})$ 8

UNIT - III

5. (a) Let p be an odd prime and let g.c.d (a,p) = 1, then 8

$\lfloor p-1 \equiv -\left(\frac{a}{p}\right) a^{\frac{1}{2}(p-1)} \pmod{p}$

(b) State and prove Gauss lemma. 8

6. State and prove quadratic law of reciprocity and, hence, evaluate 16

$$\left(\frac{202}{257}\right)$$

UNIT - IV

7. (a) If p is an odd prime, then prove that $U_p c$ is cyclic for all $c \geq 1$. 10

(b) If $e \geq 3$, then $U_2 c = \{\pm 5^i : 0 \leq i \leq 2^{c-2}\}$ 6

8. (a) Let $n = n_1 n_2 \ldots n_k$, where the integers n_i are mutually co-prime. Then $a \in Qn$ iff $a \in Qn_i$ for each i. 6