8. (a) If $f(z)$ is an analytic function in a domain D containing z_0 and if $f'(z_0) \neq 0$ show that $w = f(z)$ is a conformal mapping at z_0

(b) State and prove Montel theorem.

UNIT - V

9. (a) Define Rectifiable curve.

(b) Show that the function $u(x,y) = c^x(x \cos y - y \sin y)$ is harmonic.

(c) State Taylor's theorem.

(d) Find the residue of $\frac{z^3}{z^2-1}$ at $z = \infty$

(e) Define Meromorphic function with an example.

(f) State Weierstrass theorem.

(g) What do you mean by an essential singular point ?

(h) Discuss the transformation $2z = w + \frac{1}{w}$

Roll No. ....

76453

M.Sc. Mathematics 3rd Semester Examination–December, 2015

COMPLEX ANALYSIS-I

Paper : MM-513

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **one** question from I-IV unit each. Unit-V is **compulsory**. All questions carry equal marks.

UNIT - I

1. (a) State and prove the sufficient conditions for a function to be analytic.

(b) Prove that the function $u = e^x \cos y$ is harmonic. Find the harmonic conjugate v and the analytic function $f(z) = u + iv$

2. (a) Define circle and radius of convergence of the power series. Discuss the behaviour of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^2}$ on the circle of convergence.

(b) Discuss the multi-valued function $f(z) = z^{1}/_{2}$ by obtaining its branches, branch points and branch cuts.

UNIT - II

3. (a) State and prove extension of cauchy's integral formula to multiply connected regions.

(b) If f(z) is continuous in a region D and if the integral $\int f(z)\,dz$ taken around any closed contour in D vanishes then f(z) is analytic in D.

4. (a) State and prove Poisson's Integral formula.

(b) Show that the only bounded entire functions are the constant functions.

UNIT - III

5. (a) State and prove Laurent's theorem.

(b) Prove that the function $f(z) = \cosh(z + z^{-1})$ can be expanded in a series of the type

$$\sum_{0}^{\infty} a_n z^n + \sum_{1}^{\infty} b_n z^{-n}$$

6. (a) Let f(z) be analytic inside and on a simple closed contour C except for a finite number of poles inside C and let $f(z) \neq 0$ on C then show that

$$\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz = N - P$$

(b) State and prove Maximum Modulus principle.

UNIT - IV

7. (a) Using calculus of residues show that

$$\int_0^{\pi} \tan(\theta + ia)\,d\theta = i\pi \text{ where } \mathrm{Re}(a) > 0$$

(b) Evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2},\ a > b > 0$