MAY 2011 **P/ID 37455/PMAF**

Time : Three hours Maximum : 100 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

Each question carries 2 marks.

1. State Morera's theorem.

2. Define simply connected region.

3. Define a potential function.

4. State Weierstrass theorem.

5. Write Legendre's duplication formula.

6. State Arzela's theorem.

7. Define a free boundary arc.

8. Write Schwarz-Christoffel formula.

9. Write Legendre relation.

10. Define modular function.

SECTION B — (5 × 6 = 30 marks)

Answer ALL questions.

Each question carries 6 marks.

11. (a) State and prove the maximum principle.

Or

(b) State and prove the argument principle.

12. (a) Find the residues at the poles of $\frac{1}{z^2+5z+6}$.

Or

(b) State and prove Hurwitz theorem.

13. (a) For $\sigma = \text{Re } s > 1$, prove that

$$\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty}\left(1 - p_n^{-s}\right).$$

Or

(b) Prove that a locally bounded family of analytic functions has locally bounded derivatives.

14. (a) Prove that the zeros $a_1, a_2, \ldots\ldots, a_n$ and poles $b_1, b_2, \ldots\ldots b_n$ of an elliptic function satisfy $a_1 + a_2 + \ldots. + a_n \equiv b_1 + b_2 + \ldots. + b_n \pmod{M}$.

Or

(b) Prove that a continuous function $u(z)$ which satisfies $u(z_0) = \frac{1}{2\pi}\int_0^{2\pi} u(z_0 + re^{i\theta})d\theta$ is necessarily harmonic.

15. (a) Prove that

$$\frac{\wp'(z)}{\wp(z) - \wp(u)} = \zeta(z-u) + \zeta(z+u) - 2\zeta(z).$$

Or

(b) Prove that the set of all function elements (f, Ω) with $e^{f(\zeta)} = \zeta$ in Ω is a global analytic function.

SECTION C — (5 × 10 = 50 marks)

Answer ALL questions.

Each question carries 10 marks.

16. (a) State and prove Taylor's theorem.

Or

(b) If $pdx + qdy$ is locally exact in Ω prove that $\int_\gamma pdx + qdy = 0$ for every cycle $\gamma \sim 0$ in Ω.

17. (a) If u_1 and u_2 are harmonic in a region Ω, prove that $\int_\gamma u_1^* du_2 - u_2^* du_1 = 0$.

Or

(b) Evaluate $\int_0^\infty \frac{\cos x}{x^2 + a^2} dx$, a real.

18. (a) State and prove Mittag-Leffler theorem.

Or

(b) Prove that $\zeta(s) = 2^s \pi^{s-1} \sin\frac{\pi s}{2} \Gamma(1-s)\zeta(1-s)$

19. (a) State and prove Riemann Mapping theorem.

Or

(b) State and prove Harnack's principle.

20. (a) Prove that every point γ in the upper half plane is equivalent under the congruence subgroup mod 2 to exactly one point in $\overline{\Omega} \cup \Omega'$.

Or

(b) State and prove Monodromy theorem.