Register Number :

Name of the Candidate :

6 1 7 9

M.Sc. DEGREE EXAMINATION, 2012

(MATHEMATICS)

(SECOND YEAR)

(PAPER - VII)

230. GRAPH THEORY

(Including Lateral Entry)

December] [Time : 3 Hours

Maximum : 100 Marks

SECTION - A $(8 \times 5 = 40)$

Answer any EIGHT questions.
ALL questions carry EQUAL marks.

1. Define k-cube. Show that the k-cube has 2^k vertices, $k2^{k-1}$ edges and its bipartite.

2. Show that if G is a tree, then $\in = \gamma - 1$.

Turn Over

3. Show that a non-empty connected graph is Eulerian if and only if, it has no vertices of odd degree.

4. Find the number of different perfect matchings in K_{2n} and $K_{n,n}$.

5. Show that if G is bipartite, then $\psi' = \Delta$.

6. Define Ramsey number. Prove that

$$r(k, l) \leq \binom{k + l - 2}{k - 1}$$

7. Show that for any graph G, $\Psi \leq \Delta + 1$.

8. State and prove Euler formula for plane graphs.

9. Describe planarity algorithm.

10. Show that if G is a loopless plane graph with a Hamiltonian cycle C, then

$$\sum_{i=1}^{\gamma} (i-2)(\phi_i' - \phi_i'') = 0,$$

where ϕ_i' and ϕ_i'' are the numbers of faces of degree i contained in Int C and Ext C respectively.

SECTION - B $(3 \times 20 = 60)$

Answer any THREE questions.
ALL questions carry EQUAL marks.

11. (a) Show that a graph is bipartite if and only if, it contains no odd cycle. (15)

(b) Show that if G is simple, then G contains a cycle of length atleast $k + 1$. (5)

12. (a) State and prove Hall's theorem for matching. (12)

(b) Show that if G has Hamilton path, then for every proper subset S of V,

$$w(G - S) \leq |S| + 1.$$ (8)

13. Show that if G is simple, then either $\Psi' = \Delta$ (or) $\Psi' = \Delta + 1$.

14. (a) State and prove Brook's theorem. (15)

(b) Show that if G is a simple planar graph with $\gamma \geq 3$, then $\in \geq 3\gamma - 6$. (5)

15. (a) Show that every planar graph is five vertex colourable. (10)

(b) Show that a digraph D contains a directed path of length $\psi - 1$. (5)