15. (a) Derive the hypergeometric series $F(a, b, c, x)$ as a solution of the differential equation.

$$x(1-x)\,y'' + [\,c - (a+b+1)\,x\,]\,y' - aby = 0.$$

(b) Prove that :

$$\int_0^1 x\,J_p(\lambda_m x)\;J_p(\lambda_n x)\,x$$

$$= \begin{cases} 0, & \text{if } m \neq n. \\ \dfrac{1}{2}\,J_{p+1}(\lambda_n{}^2), & \text{if } m \neq n. \end{cases}$$

Register Number :

Name of the Candidate :

6 1 7 5

M.Sc. DEGREE EXAMINATION, 2012

(MATHEMATICS)

(FIRST YEAR)

(PAPER - III)

130. DIFFERENTIAL GEOMETRY AND DIFFERENTIAL EQUATIONS

December] [Time : 3 Hours

Maximum : 100 Marks

SECTION - A $(8 \times 5 = 40)$

Answer any EIGHT questions.
ALL questions carry EQUAL marks.

1. Prove that $(\,r', r'', r'''\,) = k^2\,\tau$.

2. Find the centre and radius of oscillating sphere.

3. On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the section by the plane z = constant.

4. A particle is constrained to move on a smooth surface under no force except the normal reaction. Prove that its path is a geodesic.

5. Derive Rodrigue's formula.

6. Prove that the edge of regression of the polar developable is the locus of centres of spherical curvature of the given curve.

7. Find a particular solution of $y'' + y = \text{Cosec } x$.

8. The equation $x^2 y'' - 3xy' + (4x + 4) y = 0$ has only one Fnobenius series solution. Find it.

9. Prove that Legendre polynomial $P_n(x)$ is given by

$$P_n(x) = \frac{1}{2^n n!} \frac{n}{dx^n} (x^2 - 1)^n.$$

10. Prove that

$$P_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \text{ Cos } x.$$

SECTION - B (3 × 20 = 60)

Answer any THREE questions.

ALL questions carry EQUAL marks.

11. (a) State and prove Servet-Frenet formula.

(b) Prove that two curves with the same intrinsic equations are necessarily congruent.

12. State and prove Minding theorem.

13. (a) Prove that a necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is zero.

(b) Prove that if there is a surface of minimum area passing through a closed curve, it is necessarily a minimal surface.

14. (a) Find the general solution of

$$y'' - 2y' + 5y = 25x^2 + 12.$$

(b) State and prove Kepler's third law for planetary motion.