15. Prove that every non-zero Hilbert space contains a complete orthonormal set with necessary results proved thereby.

Register Number :

Name of the Candidate :

6178

M.Sc. DEGREE EXAMINATION, 2012

(MATHEMATICS)

(SECOND YEAR)

(PAPER - VI)

220. SET TOPOLOGY AND FUNCTIONAL ANALYSIS

(Including Lateral Entry)

December] [Time : 3 Hours

Maximum : 100 Marks

SECTION - A $(8 \times 5 = 40)$

Answer any EIGHT questions.
ALL questions carry EQUAL marks.

1. Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.

2. State and prove Lindelof theorem.

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3. Prove that any closed subspace of a compact space is compact.

4. Let x be an infinite set with the topology $T = \{ \phi \} \cup \{ \cup \subseteq x / x - \cup \}$ is finite. Prove that (x, T) is a T_1 space but not a Hausdorff space.

5. Let x be a compact Hausdorff space. Prove that x has an open base whose sets are closed *iff* x is totally disconnected.

6. Prove that X_∞ is Hausdorff.

7. Let T be a linear transformation from a normed linear space N to another normed linear space N′. Then, prove the following conditions are equivalent :

   (a) T is continuous

   (b) T is continuous at the origin.

   (c) There exists a real number $k \geq 0$ such that $\| T(x) \| \leq k \| x \|$, for every $x \in N$.

8. State and prove the closed graph theorem.

9. State and prove the Schwarz inequality.

10. If T is an operator on H, then prove that T is normal *iff* its real and imaginary parts commute.

SECTION - B (3 × 20 = 60)

Answer any THREE questions.
ALL questions carry EQUAL marks.

11. (a) Let f be a mapping from a metric space X to another metric space Y. Then, prove that f is continuous *iff* $f^{-1}(G)$ is open in X where G is open in Y.

    (b) Prove that every separable metric space is second countable.

12. State and prove Tietze extension theorem.

13. State and prove the Weier strass approximation theorem.

14. Prove that the set $\mathcal{B}(N, N')$ of all continuous linear transformation T of a normed linear space N to another normed lienar space N′ with norm $\| T \| = \text{Sup} \{ \| T(x) \| / \| x \| \leq 1 \}$ forms a normed linear space. And hence prove that $\mathcal{B}(N)$ the set of all operators on a Banach space N, is an algebra.

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