Maths July-2018

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This Question Paper contains 20 printed pages.
(Part - A & Part - B)
Sl.No. 0400006

050 (E)

(JULY, 2018)
SCIENCE STREAM
(CLASS - XII)

પ્રશ્ન પેપરનો સેટ નંબર જેની સામેનું વર્તુળ OMR શીટમાં ઘટ્ટ કરવાનું રહે છે.
Set No. of Question Paper, circle against which is to be darken in OMR sheet.

04

Part - A : Time : 1 Hour / Marks : 50
Part - B : Time : 2 Hours / Marks : 50

(Part - A)

1) The general solution of the differential equation $y.\frac{dy}{dx} = x.\left(\frac{dy}{dx}\right)^2 + 1$ is ______

(A) $y = cx - \frac{1}{c}$; (c is an arbitrary constant)

(B) $y = c + \frac{1}{c.x}$; (c is an arbitrary constant)

(C) $y = cx + \frac{1}{c}$; (c is an arbitrary constant)

(D) $y = c - \frac{1}{cx}$; (c is an arbitrary constant)

(1) Now $y = cx + \frac{1}{c}$ —(1)

$\therefore \frac{dy}{dx} = c$

$\therefore$ c ની કિંમત સમી. (1)માં મૂકતાં

$y = x\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}}$

$\therefore y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 1$ —(2)

જે આપેલ સમી. (2)નો ઉકેલ સમી (1) છે.

2) The particular solution of the differential equation $\sec^2 x.\tan y.\,dx + \sec^2 y.\tan x.\,dy = 0;\ y\left(\frac{\pi}{4}\right) = -\frac{\pi}{4}$ is ____

(A) $\tan x.\tan y = c$

(B) $\tan x.\tan y = 1$

(C) $\tan x.\tan y = -2$

(D) $\tan x.\tan y = \frac{\pi}{4}$

(2) (ii) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

$\therefore \sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy$

$\therefore \dfrac{\sec^2 x}{\tan x} dx = -\dfrac{\sec^2 y}{\tan y} dy$

$\therefore \int \sec^2 x \dfrac{1}{\tan x} dx = -\int \dfrac{\sec^2 y}{\tan y} dy$

$\therefore \log \tan x = -\log \tan y + \log c$ — (1)

$\therefore \log \tan x = \log \dfrac{c}{\tan y}$

$\therefore \tan x = \dfrac{c}{\tan y}$

$\therefore \tan x \cdot \tan y = c$ — (2)

સમી (2) માં $x = \frac{\pi}{4}$ અને $y = \frac{\pi}{4}$

મૂકતાં

$c = 1$

$\therefore$ સમી (1) પરથી

$\log \tan x = -\log \tan y + \log 1$

$\therefore \log \tan x = \log (\tan y)^{-1}$

$\therefore \tan x = \dfrac{1}{\tan y}$

$\therefore \tan x \cdot \tan y = 1$

3) Unit vectors orthogonal to both (1, 2, 3) and (2, –1, 4) are ______.

(A) $\pm\left(\frac{11}{5\sqrt{6}}, \frac{-2}{5\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(B) $\pm\left(\frac{11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(C) $\pm\left(\frac{-11}{5\sqrt{6}}, \frac{2}{5\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(D) $\pm\left(\frac{-11}{5\sqrt{6}}, \frac{-2}{5\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

(3) $\bar{x} = (1,2,3)$ અને $\bar{y} = (2,-1,4)$ બંનેને

લંબ એકમ સદિશ $= \pm \dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|}$ થાય

હવે $\bar{x}\times\bar{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & -1 & 4 \end{vmatrix} = \hat{i}\,(8+3) - \hat{j}\,(4-6) + \hat{k}\,(-1-4)$

$= 11\hat{i} + 2\hat{j} - 5\hat{k}$

$\therefore |\bar{x}\times\bar{y}| = \sqrt{121+4+25} = \sqrt{150} = 5\sqrt{6}$

$\therefore$ માંગેલ સદિશ $= \pm \dfrac{\bar{x}\times\bar{y}}{|\bar{x}\times\bar{y}|} = \pm \dfrac{1}{5\sqrt{6}}\,(11,2,-5)$

$= \pm\left(\dfrac{11}{5\sqrt{6}}, \dfrac{2}{5\sqrt{6}}, \dfrac{-5}{5\sqrt{6}}\right)$

$= \pm\left(\dfrac{11}{5\sqrt{6}}, \dfrac{2}{5\sqrt{6}}, -\dfrac{1}{\sqrt{6}}\right)$

4) If $\bar{a} = (-3,1,0)$ and $\bar{b} = (1,-1,-1)$ then $Comp_{\bar{a}}\,\bar{b} =$ __

(A) $\dfrac{\sqrt{3}}{4}$ (B) $\dfrac{4}{\sqrt{10}}$

(C) $\dfrac{-4}{\sqrt{10}}$ (D) $\dfrac{-\sqrt{3}}{4}$

(4) અહીં $\bar{a} = (-3,1,0)$, $\bar{b} = (1,-1,-1)$

હવે $Comp_{\bar{a}}\,\bar{b} = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|} = \dfrac{(-3,1,0)\cdot(1,-1,-1)}{\sqrt{9+1+0}} = \dfrac{-3-1+0}{\sqrt{10}} = \dfrac{-4}{\sqrt{10}}$

5) The area of the parallelogram whose adjacent sides are $2\hat{i} + 3\hat{j}$ and $3\hat{i} + 4\hat{k}$ is ______.

(A) 19 (B) 17

(C) 21 (D) 23

$\bar{a}$

$\bar{b}$

$\bar{a}\cdot\bar{b}$

(5) જેની સમાંતરબાજુ ચતુષ્કોણની પાસપાસેની બાજુ

$\bar{x} = 2\hat{i} + 3\hat{j}$ અને $\bar{y} = 3\hat{i} + 4\hat{k}$ છે

∴ ચતુષ્કોણનું ક્ષેત્રફળ $= |\bar{x} \times \bar{y}|$ થાય

હવે $\bar{x} \times \bar{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 3 & 0 & 4 \end{vmatrix} = \hat{i}(12-0) - \hat{j}(8-0) + \hat{k}(0-9)$

$= 12\hat{i} - 8\hat{j} - 9\hat{k}$

$\therefore |\bar{x} \times \bar{y}| = \sqrt{144+64+81}$

$= \sqrt{289}$

$= 17$

6) If A (3, 1), B (2, 3) and C (5, 1) then m $\angle$ B = ______

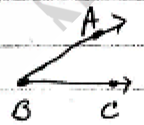
(A) $\cos^{-1}\left(\frac{7}{\sqrt{65}}\right)$ (B) $\cos^{-1}\left(\frac{3}{\sqrt{34}}\right)$

(C) $\sin^{-1}\left(\frac{5}{\sqrt{34}}\right)$ (D) $\frac{\pi}{2}$

(6) જો A(3,1), B(2,3) અને C(5,1) હોય તો ∠B

હવે $\overrightarrow{BA} = (1, -2)$

$\overrightarrow{BC} = (3, -2)$



હવે $\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}|\,|\overrightarrow{BC}| \cos B$

$\therefore 3 + 4 = \sqrt{5} \cdot \sqrt{13} \cdot \cos B$

$\therefore 7 = \sqrt{65} \cdot \cos B$

$\therefore \cos B = \frac{7}{\sqrt{65}}$

$\therefore \angle B = \cos^{-1}\left(\frac{7}{\sqrt{65}}\right)$

7) The measure of the angle between the line $\frac{x-2}{2} = \frac{2-y}{3} = \frac{z-1}{2}$ and plane $2x + y - 3z + 4 = 0$ is ______.

(A) $\cos^{-1}\left(\sqrt{\frac{213}{238}}\right)$ (B) $\sin^{-1}\left(\frac{7}{\sqrt{238}}\right)$

(C) $\sin^{-1}\left(\sqrt{\frac{213}{238}}\right)$ (D) $\frac{\pi}{2}$

(7) રેખા $\frac{x-2}{2} = \frac{2-y}{3} = \frac{z-1}{2}$ પરથી

$$\frac{x-2}{2} = \frac{y-2}{-3} = \frac{z-1}{2}$$

$\therefore \bar{l} = (2, -3, 2)$

$\therefore |\bar{l}| = \sqrt{4+9+4} = \sqrt{17}$

તથા સમતલ $2x+y-3z+4=0$ પરથી

$\bar{n} = (2, 1, -3)$

$\therefore |\bar{n}| = \sqrt{4+1+9} = \sqrt{14}$

$\therefore \bar{l}\cdot\bar{n} = 4-3-6 = -5$

હવે જો રેખા અને સમતલ વચ્ચેનો ખૂણો θ હોય તો

$|\bar{l}\cdot\bar{n}| = |\bar{l}|\cdot|\bar{n}| \sin\theta$

$\therefore |-5| = \sqrt{17}\cdot\sqrt{14} \sin\theta$

$\therefore \sin\theta = \frac{5}{\sqrt{238}}$

$\therefore \theta = \sin^{-1}\left(\frac{5}{\sqrt{238}}\right)$

8) Direction of the line perpendicular to the plane $3x - 4y + 7z = 2$ and passing through $(-1,2,4)$ is ________.

(A) (4, –6, 3)
(B) (3, 4, 7)
(C) (–3, 4, –7)
(D) (–1, 2, 4)

(8) સમતલ $3x-4y+7z=2$

નો અભિલંબ $\bar{n} = (3, -4, 7)$ છે

$\therefore$ સમતલને લંબ રેખાની દિશા

$\bar{l} = (3, -4, 7)$ અથવા $(-3, 4, -7)$ છે

9) Line passing through (2, –3, 1) and (3, –4, –5) intersects YZ - plane at ______.

(A) (0, 1, –13)
(B) (0, –1, 13)
(C) (–1, 0, 19)
(D) (–1, 0, 13)

(9) હવે $P(2,-3,1)$ અને $Q(3,-4,-5)$ માંથી પસાર થતી રેખાની દિશા $\bar{l} = \overrightarrow{PQ} = (1,-1,-6)$ થાય

∴ રેખાનું સમી

~~$(x,y,z) = (1,-1,-6)$~~

$(x,y,z) = (2,-3,1) + k(1,-1,-6)$, $k \in R$ થાય

∴ $x = 2+k$, $y = -3-k$, $z = 1-6k$, $k \in R$

હવે YZ સમતલને રેખા જ્યાં છેદે ત્યાં x-યામ $= 0$ હોય

∴ $2+k=0$

∴ $k=-2$

∴ $y = -3-(-2) = -1$

$z = 1-6(-2) = 13$

∴ માંગેલ બિંદુ $(0,-1,13)$ છે

10) Lines L : $\frac{x+2}{3} = \frac{y-2}{-1}, z+1=0$ and

M : $\{(4+2k, 0, -1+3k) / k \in R\}$ then $L \cap M =$ ______

(A) $(4,-1,0)$ (B) $(0,4,-1)$

(C) $(-1,4,0)$ (D) $(4,0,-1)$

(10) હવે $L: \frac{x+2}{3} = \frac{y-2}{-1}$, $z+1=0$ પરથી

$L: \frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{0}$ થાય

∴ ~~[illegible]~~

તથા $M: \{(4+2k, 0, -1+3k), k \in R\}$

તે બંને રેખાનું છેદબિંદુ $P(x,y,z)$ ધારતાં

$(x,y,z) = (3k-2, -k+2, -1) = (4+2k', 0, -1+3k')$ હોય

∴ $-1+3k' = -1 \Rightarrow 3k'=0 \Rightarrow k'=0$

∴ છેદબિંદુ $P(x,y,z) = (4,0,-1)$ હોય

11) If set A = { x / x is a measure of an angle of scalen triangle}. then the number of equivalence relations containing (measure of minimum angle, measure of maximum angle) is _____.

(A) 2

(B) 1

(C) 3

(D) 8

(11) A = {x / x એ વિષમબાજુ ત્રિકોણના ખૂણા છે}

∴ A ની ત્રણ સભ્યો છે

જેમાં ક્રમયુક્ત જોડ (નાનામાં નાનો ખૂણો, મોટામાં મોટો ખૂણો)

ધરાવતા

આવા 2 સામ્ય સંબંધ મળે.

12) The number of binary operations on set $\{3k/1 \leq k \leq n;\ k, n \in N\}$ is _____.

$\frac{3k}{1} < k < n$

$3(1) < 1 < 1$

$(n)^3 (1)^3$

(A) ~~n^{n^2}~~

(B) 2^n

(C) n^3

(D) n^{2n}

(12) $A = \{3k / 1 \leq k \leq n\}$

∴ A ની n સભ્યો છે

∴ A પરની દ્વિક્રિયા $= n^{n^2}$

13) If $f : R \to R$, $f(x) = x^3$, $g : R \to R$, $g(x) = 3^x$ then $\{x / fog(x) = gof(x)\}$ = _______.

(A) { }

(B) $\{0, \sqrt{3}\}$

(C) $\{0, \sqrt{3}, -\sqrt{3}\}$

(D) R

(3) હવે $f(x)=x^3$ અને $g(x)=3^x$

$\therefore fog(x) = f(g(x)) = f(3^x) = (3^x)^3 = 3^{3x}$

અને $gof(x) = g(f(x)) = g(x^3) = 3^{x^3}$

જો $fog(x) = gof(x)$ તો

$3^{3x} = 3^{x^3}$

$\therefore 3x = x^3$

$\therefore \cancel{3=x}$

$\therefore x^3-3x=0$

$\therefore x(x^2-3)=0$

$\therefore x=0$ અથવા $x^2=3$

$\therefore x=0$ અથવા $x=\pm\sqrt{3}$

$\therefore$ ઉકેલગણ $= \{0, \sqrt{3}, -\sqrt{3}\}$

14) Range of $[\cos^{-1}x]$ is ____ (where [ ] = greatest integer part)

(A) $[0,3]$ (B) $[0,\pi]$

(C) $\{1,2,3\}$ (D) $\{0,1,2,3\}$

(4) $[\cos^{-1}x]$ નો વિસ્તાર શું થાય

હવે $\cos^{-1}x$ નો વિસ્તાર $[0,\pi]$ માં હોય

અર્થાત્ $0 \le \cos^{-1}x \le \pi$ અથવા

$0 \le \cos^{-1}x \le 3.14$ હોય

$\therefore [\cos^{-1}x] = 0$ અથવા 1 અથવા 2 અથવા 3 હોય

$\therefore [\cos^{-1}x]$ નો વિસ્તાર $= \{0,1,2,3\}$

15) $\sec^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2}; x \neq 0$ then $x =$ ____

(A) 3

(B) 1

(C) 5

(D) 4

15) અહીં $\sec^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{4}{5}\right)=\frac{\pi}{2}$

$\therefore \sec^{-1}\left(\frac{5}{x}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$

$\therefore \frac{5}{x}=\frac{5}{4}$

$\therefore x=4$

16) If $2.\cos(2\tan^{-1}x)=1$ then $x=$ ____

(A) $1-\sqrt{3}$

(B) $1/\sqrt{3}$

(C) $1-1/\sqrt{3}$

(D) $\sqrt{3}$

16) અહીં $2\cos(2\tan^{-1}x)=1$

તો $\cos(2\tan^{-1}x)=\frac{1}{2}$

$\therefore 2\tan^{-1}x=\pm\frac{\pi}{3}$

$\tan^{-1}x=\pm\frac{\pi}{6}$

$\therefore x=\tan\left(\pm\frac{\pi}{6}\right) \Rightarrow x=\pm\frac{1}{\sqrt{3}}$

(આપેલ વિકલ્પો પૈકી) આપેલ જવાબ $=\frac{1}{\sqrt{3}}$

17) If $0<x<1$ and if $\tan^{-1}(1-x)$, $\tan^{-1}x$ and $\tan^{-1}(1+x)$ are in arithmetic progression then $x^3=$ ______.

(A) x^2-1

(B) $1+x^2$

(C) $1-x^2$

(D) x^2

(17) $\tan^{-1}(1-x)$, $\tan^{-1}x$ અને $\tan^{-1}(1+x)$ સમાંતર શ્રેણીમાં છે

$\therefore \tan^{-1}x - \tan^{-1}(1-x) = \tan^{-1}(1+x) - \tan^{-1}x$

$\therefore 2\tan^{-1}x = \tan^{-1}(1+x) + \tan^{-1}(1-x)$

$\therefore \tan^{-1}\dfrac{2x}{1-x^2} = \tan^{-1}\left(\dfrac{1+x+1-x}{1-(1-x^2)}\right)$

$\therefore \dfrac{2x}{1-x^2} = \dfrac{2}{1-1+x^2}$

$\therefore x^3 = 1-x^2$

18) If area of a triangle whose vertices are (8,2), (k,4) and (6,7) is 13 units then the possible integer value of k is _____.

(A) 1 (B) 0

(C) 2 (D) 3

(18) ધારો કે A(8,2), B(k,4), C(6,7) છે.

હવે ΔABC નું ક્ષેત્રફળ $= 13$ છે

હવે ΔABC નું ક્ષેત્રફળ $= \frac{1}{2}\left|\begin{vmatrix} 8 & 2 & 1 \\ k & 4 & 1 \\ 6 & 7 & 1 \end{vmatrix}\right|$

$= \frac{1}{2}\left|[8(4-7) - 2(k-6) + 1(7k-24)]\right|$

$= \frac{1}{2}\left|[-24 - 2k + 12 + 7k - 24]\right|$

$= \frac{1}{2}\left|-36 + 5k\right|$

$\therefore \frac{1}{2}\left|-36+5k\right| = 13$

$\therefore |5k-36| = 26$

$\therefore 5k-36=26$ અથવા $5k-36=-26$

$\therefore 5k = 62$ અથવા $5k = 10$

$\therefore k = \dfrac{62}{5}$ અથવા $k = 2$

આમ k ની શક્ય પૂર્ણાંક કિંમત 2 મળે

19) If $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix} = m(abc)^k$, then $m + k =$ _____

(A) 3 (B) 2

(C) 0 (D) 5

જવ : ધારો $\begin{vmatrix} 0 & ab^2 & ac^2 \\ a^2b & 0 & bc^2 \\ a^2c & b^2c & 0 \end{vmatrix} = m(abc)^k$

$\therefore a^2b^2c^2 \begin{vmatrix} 0 & a & a \\ b & 0 & b \\ c & c & 0 \end{vmatrix} = m(abc)^k$

$\therefore a^2b^2c^2 [0 - a(0-bc) + a(bc-0)] = m(abc)^k$

$\therefore a^2b^2c^2 (abc+abc) = m(abc)^k$

$\therefore a^2b^2c^2 (2abc) = m(abc)^k$

$\therefore 2a^3b^3c^3 = m a^k b^k c^k$

$\therefore m=2$ અને $k=3$

$\therefore m+k = 2+3 = 5$

20) $\begin{vmatrix} \sin\left(\frac{2\pi}{9}\right) & \cos\left(\frac{2\pi}{9}\right) \\ \sin\left(\frac{5\pi}{18}\right) & \cos\left(\frac{5\pi}{18}\right) \end{vmatrix} =$ ______

(A) $\tan\left(\pi/4\right)$ (B) $-\sin\left(\pi/18\right)$

(C) $\cot\left(3\pi/4\right)$ (D) $\sin\left(\pi/18\right)$

ઉકે. $\begin{vmatrix} \sin\frac{2\pi}{9} & \cos\frac{2\pi}{9} \\ \sin\frac{5\pi}{18} & \cos\frac{5\pi}{18} \end{vmatrix}$

$= \sin\frac{2\pi}{9}\cdot\cos\frac{5\pi}{18} - \cos\frac{2\pi}{9}\cdot\sin\frac{5\pi}{18}$

$= \sin\left(\frac{2\pi}{9} - \frac{5\pi}{18}\right)$

$= \sin\left(\frac{4\pi - 5\pi}{18}\right)$

$= \sin\left(-\frac{\pi}{18}\right)$

$= -\sin\frac{\pi}{18}$

21) If A= [x y z]; B = $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$; $C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and (AB) C is m×n matrix then ______.

(A) m < n (B) m > n

(C) m = n (D) m + n = 5

(21) $A = [x \quad y \quad z]$

$B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

$C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

હવે A નો 1×3 [redacted], B નો 3×3 અને C નો 3×1 છે

$\therefore (AB)$ નો 1×3 શ્રેણિક

$\therefore (AB)C$ નો 1×1 થાય

$\therefore m\times n = 1\times1$

આમ $m = n = 1$

22) If $\begin{bmatrix} a_1+a_2 & 4 \\ 3 & a_3+a_4 \end{bmatrix} - \begin{bmatrix} 3a_2 & 3a_1 \\ 3a_4 & 3a_3 \end{bmatrix} = \begin{bmatrix} -6 & -a_1 \\ -2a_4 & 1 \end{bmatrix}$ then

$\sum_{i=1}^{4} a_i =$ $a_1+a_2+a_3+a_4 = 2+4+$ ______.

(A) 10 (B) 8

(C) 12 (D) 16

(22) જો $\begin{bmatrix} a_1+a_2 & 4 \\ 3 & a_3+a_4 \end{bmatrix} - \begin{bmatrix} 3a_2 & 3a_1 \\ 3a_4 & 3a_3 \end{bmatrix} = \begin{bmatrix} -6 & -a_1 \\ -2a_4 & 1 \end{bmatrix}$ ત્યારે

$\begin{bmatrix} a_1-2a_2 & 4-3a_1 \\ 3-3a_4 & a_4-2a_3 \end{bmatrix} = \begin{bmatrix} -6 & -a_1 \\ -2a_4 & 1 \end{bmatrix}$

$\therefore 4-3a_1 = -a_1$	$a_1-2a_2 = -6$	$3-3a_4 = -2a_4$	$a_4-2a_3 = 1$
$\therefore 2a_1 = 4$	$\therefore 2+6 = 2a_2$	$\therefore a_4 = 3$	$\therefore 3-1 = 2a_3$
$\therefore a_1 = 2$	$\therefore 2a_2 = 8$		$2a_3 = 2$
	$a_2 = 4$		$a_3 = 1$

$\therefore \sum_{i=1}^{4} a_i = a_1+a_2+a_3+a_4 = 2+4+1+3 = 10$

23) $A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$ then $A^{-1} =$ ______.

(A) $\begin{bmatrix} \frac{4}{23} & \frac{-3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{-4}{23} & \frac{-3}{23} \\ \frac{-5}{23} & \frac{-2}{23} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{5}{23} & \frac{2}{23} \end{bmatrix}$

(23)

અહીં $A = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = 8+15 = 23$

હવે $adj\,A = \begin{bmatrix} 4 & 3 \\ -5 & 2 \end{bmatrix}$

હવે $A^{-1} = \frac{1}{|A|} adj\,A = \frac{1}{23}\begin{bmatrix} 4 & 3 \\ -5 & 2 \end{bmatrix}$

$= \begin{bmatrix} \frac{4}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{2}{23} \end{bmatrix}$

24) $f(x) = \begin{cases} \frac{\sin 5x \cdot \tan kx}{x^2} & ; x \neq 0 \\ 1 & ; x = 0 \end{cases}$ is continuous at $x = 0$ then $k =$ ______ ; $(k \neq 0)$

(A) 1/5

(B) 1/3

(C) 1/15

(D) 5/3

(૨૪) $f(x) = \begin{cases} \dfrac{\sin 5x \cdot \tan kx}{x^2}, & x \neq 0 \\ 1 & x = 0 \end{cases}$

હવે $f(x)$ એ $x = 0$ આગળ સતત છે

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\therefore \lim_{x \to 0} \dfrac{\sin 5x \cdot \tan kx}{x^2} = 1$

$\therefore \lim_{x \to 0} \dfrac{\sin 5x}{5x} \cdot 5 \cdot \dfrac{\tan kx}{kx} \cdot k = 1$

$(1)\ 5\ (1)(k) = 1$

$\therefore k = \dfrac{1}{5}$

25) $\left\{\dfrac{d}{dx}(\sec x^\circ)\right\}_{(x=30^\circ)}$ = ______.

(A) $\pi/240$ (B) $\pi/180$

(C) $\pi/270$ (D) $\pi/90$

(25) $\dfrac{d}{dx}(\sec x^\circ) = \dfrac{d}{dx}\left(\sec \dfrac{x\pi}{180}\right)$

$= \sec \dfrac{x\pi}{180} \cdot \tan \dfrac{x\pi}{180} \cdot \dfrac{\pi}{180}$

$\therefore \dfrac{d}{dx}(\sec x^\circ) = \sec x^\circ \tan x^\circ \cdot \dfrac{\pi}{180}$

$\therefore \dfrac{d}{dx}\left[\sec x^\circ\right]_{x=30} = \sec 30^\circ \tan 30^\circ \dfrac{\pi}{180}$

$= \dfrac{2}{\sqrt{3}} \cdot \dfrac{1}{\sqrt{3}} \cdot \dfrac{\pi}{180}$

$= \dfrac{\pi}{270}$

26) If $x = at^2$, $y = 2at$ then y_2 = ______ $(t \neq 0)$

(A) $\dfrac{1}{2at^2}$ (B) $\dfrac{1}{2at}$

(C) $-\dfrac{1}{t^2}$ (D) $-\dfrac{1}{2at^3}$

26) અહીં $x = at^2,\ y = 2at$

$\therefore \dfrac{dx}{dt} = 2at$, અને $\dfrac{dy}{dt} = 2a$

હવે $y_1 = \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{2a}{2at} = \dfrac{1}{t}$

હવે $y_2 = \dfrac{d}{dx}(y_1) = \dfrac{d}{dx}\left(\dfrac{1}{t}\right) = -\dfrac{1}{t^2} \cdot \dfrac{dt}{dx} = -\dfrac{1}{t^2} \cdot \dfrac{1}{2at}$

$\therefore y_2 = \dfrac{-1}{2at^3}$

27) $\int \dfrac{x.(x\sin x + \cos x)^{-2}}{\sec x} dx =$ ______ $+ c$

(A) $\dfrac{-1}{\sin x + x.\cos x}$

(B) $\dfrac{-1}{x.\sin x + \cos x}$ ✓

(C) $\dfrac{x}{x.\sin x + \cos x}$

(D) $\dfrac{1}{\sin x + x.\cos x}$

27) $I = \int \dfrac{x.(x\sin x + \cos x)^{-2}}{\sec x} dx$

$= \int \dfrac{x\cos x}{(x\sin x + \cos x)^2} dx$

અહીં $x\sin x + \cos x = t$ લો

$(x\cos x + \sin x - \sin x)dx = dt$

$\therefore x\cos x\, dx = dt$

$\therefore I = \int \dfrac{1}{t^2} dt = -\dfrac{1}{t} + c = -\dfrac{1}{x\sin x + \cos x} + c$

28) $f'(x) = 3\sin x - 4\sin^3 x$ and $f(0) = \dfrac{1}{3}$ then c = ____

(where c is integrating constant)

$J = 3\sin c - 4\sin^3 c$

$= 3\sin(\frac{1}{3}) - 4(\frac{1}{3})^3$

(A) $\dfrac{2}{3}$

(B) $-\dfrac{2}{3}$

(C) 0 ✓

(D) $-\dfrac{3}{2}$

$= 3(\frac{1}{3}) - 4$ ...

(28) જો $f'(x) = 3\sin x - 4\sin^3 x$ અને $f(0) = \frac{1}{3}$

તો $f'(x) = 3\sin x - 4\sin^3 x = \sin 3x$

તો $f(x) = \int f'(x)\,dx = \int \sin 3x\,dx = \frac{-\cos 3x}{3} + c$

$\therefore f(0) = -\frac{\cos 3(0)}{3} + c$

$\therefore \frac{1}{3} = -\frac{1}{3} + c$

$\therefore c = \frac{2}{3}$

29) $\int \frac{x^4 + 5^{x-1}.\log_e 5}{x^5 + 5^x}.dx =$ ______ $+c$

(A) $-\frac{1}{5}\log|x^5 + 5^x|$

(B) $\frac{1}{5}\log|x^5 + 5^x|$

(C) $\frac{1}{\log 5}.\log|x^5 + 5^x|$

(D) $\frac{-1}{\log 5}.\log|x^5 + 5^x|$

(29) $\int \frac{x^4 + 5^{x-1}\log_e 5}{x^5 + 5^x}\,dx$

$= \frac{1}{5}\int \frac{5x^4 + 5^x \log_e 5}{x^5 + 5^x}\,dx$

$= \frac{1}{5}\log|x^5 + 5^x| + c \quad \left(\because \frac{d}{dx}(x^5 + 5^x) = 5x^4 + 5^x \log_e 5\right)$

30) $\int \frac{1}{x(x^{100}+1)}.dx = $ ______ $+c;(x>0)$

(A) $\frac{1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right|$

(B) $\frac{-1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right|$

(C) $\frac{1}{100}\log\left|\frac{x^{100}+1}{x^{100}}\right|$

(D) $\frac{-1}{100}\log\left|\frac{x^{100}+1}{x^{100}}\right|$

30) $I = \int \frac{1}{x(x^{100}+1)}\,dx = \text{----} + c$

$= \int \frac{x^{99}}{x^{100}(x^{100}+1)}\,dx$

લે $x^{100} = t$ ગણી $\quad 100x^{99}dx = dt$

$\therefore x^{99}dx = \frac{1}{100}dt$

$\therefore I = \int \frac{1}{100t(t+1)}\,dt$

$= \frac{1}{100}\int \frac{1}{t(t+1)}\,dt$

$= \frac{1}{100}\int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \frac{1}{100}\left[\log|t| - \log|t+1|\right] + c$

$= \frac{1}{100}\log\left|\frac{t}{t+1}\right| + c$

$= \frac{1}{100}\log\left|\frac{x^{100}}{x^{100}+1}\right| + c$

31) If P (A) = 0.25, P (B) = 0.55 and P (A ∪ B) = 0.65 then P (B'/A) = ______

$P(A\cap B) + P(A) + P(B)$

$P(A\cup B)$

$0.65 = P(A\cap B) + 0.25 + 0.55$

$P\left(\frac{B'}{A}\right)$

$P(A\cap B) = -0.15$

(A) 0.04

(B) 0.004

(C) 0.4

(D) 0.0004

31) અહીં $P(A) = 0.25$

$P(B) = 0.55$

$P(A \cup B) = 0.65$

હવે $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.65 = 0.25 + 0.55 - P(A \cap B)$

$\therefore P(A \cap B) = 0.80 - 0.65 = 0.15$

હવે $P(B'/A) = 1 - P(B/A) = 1 - \frac{P(A \cap B)}{P(A)}$

$= 1 - \frac{0.15}{0.25}$

$= 1 - \frac{15}{25}$

$= 1 - \frac{3}{5}$

$= \frac{5-3}{5}$

$= \frac{2}{5}$

$= 0.4$

32) If A_1 and A_2 are two independent events such that $P(A_1 \cup A_2) = 0.5$ and $P(A_1) = 0.2$ then $P(A_2) =$ ______

(A) $\frac{3}{5}$ (B) $\frac{3}{7}$

(C) $\frac{3}{4}$ (D) $\frac{3}{8}$

32) હવે A_1 અને A_2 નિરપેક્ષ ઘટનાઓ છે

$\therefore P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$

હવે $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$\therefore P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1) \cdot P(A_2)$

$\therefore 0.5 = 0.2 + P(A_2) - 0.2 P(A_2)$

$\therefore 0.3 = 0.8 P(A_2)$

$\therefore P(A_2) = \frac{0.3}{0.8} = \frac{3}{8}$

33) The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively then $P(X=3)=$ ______

(A) $\frac{13}{32}$ (B) $\frac{7}{32}$

(C) $\frac{17}{32}$ (D) $\frac{19}{32}$

(33) અહીં દ્વિપદી વિતરણના મધ્યક $= 4$ અને
વિચરણ $= 2$ છે

$\therefore np = 4$ અને $npq = 2$

$\therefore 4 \cdot q = 2$

$\therefore q = \frac{1}{2}$

હવે $p = 1 - q = 1 - \frac{1}{2} = \frac{1}{2}$

હવે $np = 4 \Rightarrow n\left(\frac{1}{2}\right) = 4 \Rightarrow n = 8$

હવે $P(X=x) = \binom{n}{x} p^x \cdot q^{n-x}$ પરંતુ

$\therefore P(X=3) = \binom{8}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^5 = \frac{8\times7\times6}{3\times2\times1} \times \frac{1}{2^8} = \frac{7}{32}$

34) If $P(x) = c.\binom{4}{x}$; where $x = 0,1,2,3,4$ then $c =$ ______

$) + c\binom{4}{1} + c\binom{4}{2} + c\binom{4}{3} \; c\cdot\binom{4}{4}$

(A) $\frac{1}{16}$ (B) $\frac{1}{22}$

$1 +$

(C) $\frac{1}{19}$ (D) $\frac{1}{21}$

(34) $P(x) = c\binom{4}{x}, \ x = 0,1,2,3,4$

હવે $\sum_{x=0}^{4} P(x) = 1$

$\therefore \sum_{x=0}^{4} c\binom{4}{x} = 1$

$\therefore c\sum_{x=0}^{4}\binom{4}{x} = 1$

$\therefore c\left[\binom{4}{0}+\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}\right] = 1$

$\therefore c \cdot 2^4 = 1$

$\therefore c = \frac{1}{2^4} = \frac{1}{16}$

35) Two unbiased coins are tossed. If one coin shows head, the probability that the other also shows head is ______.

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) 1

(35) અહીં $U = \{HH, HT, TH, TT\}$

A = પ્રથમ સીક્કા પર છાપ આવે તે ઘટના

$\therefore A = \{HH, HT\}$

અને B = બીજા સીક્કા પર છાપ આવે તે ઘટના

$\therefore B = \{HH, TH\}$

હવે $A \cap B = \{HH\}$

હવે $P(A) = \frac{2}{4}$

અને $P(A \cap B) = \frac{1}{4}$

હવે શરતી ઘટના $B|A$ છે

$\therefore$ શરતી સંભાવના $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$

36) The corner points of the feasible region determined by the system of linear constraints are (0,10), (10,15), (15,25), (0,30). Let $z = px + qy$, where $p,q > 0$. Condition on p and q so that the maximum of z occurs at both the points (15,25) and (0,30) is ______.

(A) $p:q = 1:1$ (B) $p:q = 1:3$

(C) $p:q = 2:3$ (D) $p:q = 2:1$

36) ઉકેલ: ~~[illegible]~~ $Z = px + qy$ ની મહત્તમ કિંમત $(15, 25)$ અને $(0,30)$ છે

$\therefore Z_{(15,25)} = Z_{(0,30)}$

$\therefore p(15) + q(25) = p(0) + q(30)$

$\therefore 15p + 25q = 30q$

$\therefore 15p = 5q$

$\therefore \dfrac{p}{q} = \dfrac{1}{3}$

$\therefore p : q = 1 : 3$

37) For an LP problem the objective function $z = 3x + 2y$ the coordinates of the corner points of the bounded feasible region are A(3, 3), B(20,3), C(20,10), D(18,12) and E (12, 12) the minimum value of z is ______

(A) 15
(B) 49
(C) 10
(D) 05

(37)

બિંદુ	$z = 3x + 2y$	
(3,3)	15	← ન્યૂનતમ કિંમત
(20,3)	66	
(20,10)	80	← મહત્તમ કિંમત
(18,12)	78	
(12,12)	60	

38) There is 2 % error in measuring the period of a simple pendulum. The percentage error in length is ______ (where $T = 2\pi . \sqrt{\frac{l}{g}}$)

(A) 8 %
(B) 4 %
(C) 2 %
(D) 6 %

(38) હવે $T = 2\pi\sqrt{\frac{l}{g}}$

$\therefore T^2 = 4\pi^2 \frac{l}{g}$

$\therefore l = \frac{T^2 g}{4\pi^2}$

$\therefore$ l માં પ્રતિશત ત્રુટી = 2 (T માં પ્રતિશત ત્રુટી)

$= 2\ (2)$

$= 4\%$

39) The rate of change of length (R) of diagonal of square with respect to its area (A) is _____ (where A = 20 unit²)

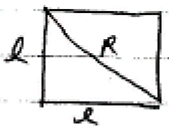
(A) $\frac{1}{\sqrt{10}}$ unit

(B) $\frac{1}{2\sqrt{10}}$ unit

(C) $\frac{1}{5\sqrt{2}}$ unit

(D) $\frac{1}{4\sqrt{5}}$ unit

(39) ધારોકે ચોરસની લંબાઈ l છે અને વિકર્ણની લંબાઈ R છે



$\therefore R = \sqrt{l^2 + l^2} = \sqrt{2l^2} = \sqrt{2}\,l$

$\therefore R^2 = 2l^2$

$\therefore R^2 = 2A$

$\therefore R = \sqrt{2} \cdot \sqrt{A}$

$\therefore \frac{dR}{dA} = \sqrt{2} \cdot \frac{1}{2\sqrt{A}} = \frac{1}{\sqrt{2A}} = \frac{1}{\sqrt{2 \times 20}} = \frac{1}{\sqrt{40}} = \frac{1}{2\sqrt{10}}$

40) $y = a.e^x$, $y = b.e^{-x}$ intersect at right angles if ____ $(a \neq 0, b \neq 0)$

(A) $a = b$

(B) $a = \frac{1}{b}$

(C) $a = \frac{-1}{b}$

(D) $a + b = 0$

ઉકેલ: $y = ae^x$ —① અને

$y = be^{-x}$ —②

છેદબિંદુ માટે સમી ① અને ② ને સરખાવતા

$ae^x = be^{-x}$

$\therefore ae^x = \frac{b}{e^x} \Rightarrow e^{2x} = \frac{b}{a} \Rightarrow e^x = \sqrt{\frac{b}{a}}$

ધારો કે છેદબિંદુનો x-યામ x લેતાં

$e^x = \sqrt{\frac{b}{a}}$

હવે સમી ① પરથી $\left(\frac{dy}{dx}\right) = ae^x$

$\therefore$ પ્રથમ વક્રના સ્પર્શકનો ઢાળ $m_1 = ae^x$

સમી ② પરથી $\frac{dy}{dx} = -be^{-x}$

$\therefore$ બીજા વક્રના સ્પર્શકનો ઢાળ $m_2 = -be^{-x}$

હવે બંને વક્રો કાટખૂણે છેદે છે

$\therefore m_1 \cdot m_2 = -1$

$\therefore (a \cdot e^x)(-be^{-x}) = 1$

$\therefore -ab = 1$

$\therefore a = -\frac{1}{b}$

41) $\int (\sqrt{e})^x \cdot \sin\left(\frac{x}{3}\right) dx =$ ________ $+ c$.

(A) $\frac{(\sqrt{e})^x}{\sqrt{13}} \cos\left(\frac{x}{2} - \tan^{-1}\frac{2}{3}\right)$

(B) $\frac{e^x}{\sqrt{13}} \sin\left(\frac{x}{2} - \tan^{-1}\frac{2}{3}\right)$

(C) $\frac{(\sqrt{e})^x}{\sqrt{13}} \sin\left(\frac{x}{2} - \cot^{-1}\frac{3}{2}\right)$

(D) $\frac{(\sqrt{e})^x}{\sqrt{13}} \sin\left(2x - \tan^{-1}\frac{2}{3}\right)$

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(41) $I = \int (\sqrt{e})^x \sin\frac{x}{3} dx = \int e^{\frac{x}{2}} \sin\frac{x}{3} dx$

લેતાં $a = \frac{1}{2}, b = \frac{1}{3}$

$\therefore \sqrt{a^2+b^2} = \sqrt{\frac{1}{4}+\frac{1}{9}} = \sqrt{\frac{9+4}{36}} = \frac{\sqrt{13}}{6}$

હવે $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{\sqrt{a^2+b^2}} \sin(bx+\alpha) + c$ જ્યાં $\alpha = \tan^{-1}\frac{b}{a}$

પરથી

$I = \frac{6e^{\frac{x}{2}}}{\sqrt{13}} \sin\left(\frac{x}{3} - \tan^{-1}\frac{\frac{1}{3}}{\frac{1}{2}}\right) + c$

$\therefore I = \frac{6(\sqrt{e})^x}{\sqrt{13}} \sin\left(\frac{x}{3} - \tan^{-1}\frac{2}{3}\right) + C$ આપેલ એક પણ વિકલ્પ સાચો નથી

42) $\int \frac{1}{(x-2020)\,(x-2018)}.dx =$ _______ $(x > 2020)$.

(A) $\frac{1}{2}\log\left|\frac{x-2018}{x-2020}\right| + c$

(B) $\frac{1}{2}\log\left|\frac{x-2020}{x-2018}\right| + c$

(C) $\log|(x-2020)(x-2018)| + c$

(D) $\log|2x-4038| + c$

42. $\int \frac{1}{(x-2020)(x-2018)}\,dx$

$= \frac{1}{2}\int \left(\frac{1}{x-2020} - \frac{1}{x-2018}\right)dx$

$= \frac{1}{2}\left[\log|x-2020| - \log|x-2018|\right] + c$

$= \frac{1}{2}\log\left|\frac{x-2020}{x-2018}\right| + c$

43) $\int\sqrt{\frac{x}{1-x^3}}\,.\,dx =$ ________ $+c;\ (0<x<1)$

(A) $\frac{3}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)$

(B) $\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)$

(C) $\frac{1}{3}\sin^{-1}\left(x^{\frac{2}{3}}\right)$

(D) $\frac{1}{2}\sin^{-1}\left(x^{\frac{3}{2}}\right)$

43) $\int\sqrt{\frac{x}{1-x^3}}\ dx$

$\therefore I = \int \frac{\sqrt{x}}{\sqrt{1-x^3}}\ dx$

$x^3 = t^2$ Gni

$x = t^{2/3} \Rightarrow \sqrt{x} = t^{1/3}$

$dx = \frac{2}{3}t^{-\frac{1}{3}}\ dt$

$\therefore I = \int \frac{t^{1/3}}{\sqrt{1-t^2}} \cdot \frac{2}{3} \cdot t^{-1/3} dt$

$= \frac{2}{3}\int \frac{1}{\sqrt{1-t^2}}\ dt$

$= \frac{2}{3}\sin^{-1} t + c$

$= \frac{2}{3}\sin^{-1}(x^{3/2}) + c$

44) $\int_{-1}^{0} x.|x|.\,dx =$ ________

(A) -1

(B) $\frac{1}{3}$

(C) $-\frac{1}{3}$

(D) 0

(44) $\int_{-1}^{0} x|x|\,dx$

$= \int_{-1}^{0} x(-x)\,dx$

$= -\int_{-1}^{0} x^2\,dx$

$= -\left[\frac{x^3}{3}\right]_{-1}^{0}$

$= -\left[0-\left(-\frac{1}{3}\right)\right]$

$= -\frac{1}{3}$

45) $\int_{n}^{n+1} f(x).dx = 2n-1$, $(n \geq 1)$, then $\int_{1}^{11} f(x)\,dx =$ ______

(A) 111 (B) 100

(C) 123 (D) 113

(45) $\int_{n}^{n+1} f(x)\,dx = 2n-1$

Sol $\int_{1}^{11} f(x)dx = \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \ldots + \int_{10}^{11} f(x)dx$

$= \sum_{n=1}^{10} \int_{n}^{n+1} f(x)\,dx$

$= \sum_{n=1}^{10} (2n-1)$

$= 2\sum_{n=1}^{10} n - \sum_{n=1}^{10} 1$

$= 2\,\frac{10}{2}(10+1) - 10$

$= 10(11) - 10$

$= 10(11-1)$

$= 10(10)$

$= 100$

46) $\int_{0}^{4036} \frac{2^x}{2^x + 2^{4036-x}}\,dx =$ ______.

(A) 4036 (B) -2015

(C) 2017 (D) 2018

(46) $I = \int_0^{4036} \frac{2^x}{2^x + 2^{4036-x}}\, dx$ —— ①

$\therefore I = \int_0^{4036} \frac{2^{4036-x}}{2^{4036-x} + 2^{4036-4036+x}}\, dx$

$I = \int_0^{4036} \frac{2^{4036-x}}{2^{4036-x} + 2^x}\, dx$ —— ②

હવે ① અને ② નો સરવાળો કરતા

$2I = \int_0^{4036} \frac{2^x + 2^{4036-x}}{2^{4036} + 2^x}\, dx$

$\therefore I = \frac{1}{2}\int_0^{4036} dx = \frac{1}{2}\left[x\right]_0^{4036} = \frac{1}{2}(4036) = 2018$

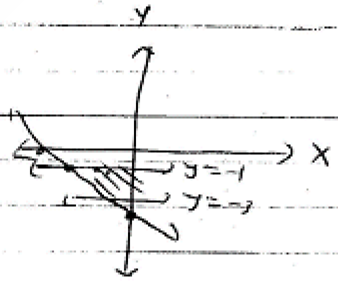
47) Area of the region bounded by the lines $x + 2y + 8 = 0$, $y = -3$, $y = -1$, and y-axis is ______ units.

(A) 4 (B) 6

(C) 8 (D) 16

(47) અહી $x + 2y + 8 = 0$

$\therefore x = -2y - 8$ —— ①



∴ આકૃતિની રચના આધારે ક્ષેત્રફળ $A = |I|$

જ્યાં $I = \int_{-1}^{-3} (-2y-8)\, dy = -2\left[\frac{y^2}{2}\right]_{-1}^{-3} - 8\left[y\right]_{-1}^{-3}$

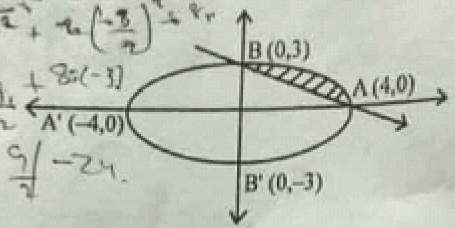
$= -(9-1) - 8(-3+1)$

$= -8 + 16$

$= 8$

$\therefore A = |I| = |8| = 8$

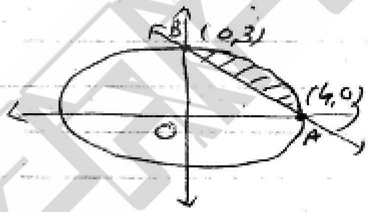
48) As shown in the figure of an ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$. The area of the shaded region is ______



(A) $3(\pi-2)$ (B) $12(\pi-2)$

(C) $4(\pi-2)$ (D) 12π

(48) હવે ઉપવલયનાં સમીકરણ



$\frac{x^2}{16}+\frac{y^2}{9}=1$ પરથી

$\frac{y^2}{9} = 1-\frac{x^2}{16} = \frac{16-x^2}{16}$

$\therefore y^2 = \frac{9}{16}(16-x^2)$

$\therefore y = \frac{3}{4}\sqrt{16-x^2}$ (પ્રથમ ચરણમાં)

∴ ઉપવલય નો પ્રથમ ચરણમાં આવેલા ભાગનું ક્ષેત્રફળ $A = |I|$

જ્યાં $I = \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx = \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$

$= \frac{3}{4}\left[8\sin^{-1}(1)+0-0-0\right] = \frac{3}{4}\times 8\times\frac{\pi}{2} = 3\pi$

$\therefore A = |3\pi| = 3\pi$

હવે ΔOAB નું ક્ષેત્રફળ $= \frac{1}{2}(OA)(OB)$

$= \frac{1}{2}(4)(3)$

$= 6$

∴ આકૃતિમાં દર્શાવેલ ક્ષેત્રફળ $A = 3\pi - 6$

$= 3(\pi-2)$

49) Area of the region bounded by the parabola $y^2 = 8x$ and line $x + y = 0$ is ______.

(A) $\frac{32}{3}$ (B) $\frac{37}{2}$

(C) $\frac{35}{2}$ (D) $\frac{39}{2}$

$y = \sqrt{8x}$

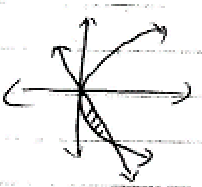
$2\sqrt{2}$

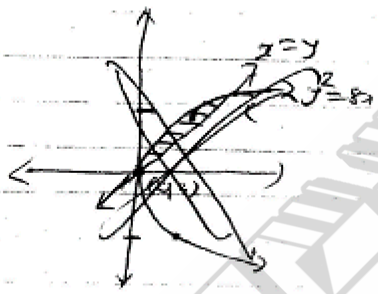
$2\sqrt{2}$

(49) $y^2 = 8x$ — ①

$x + y = 0$

$\therefore x = -y$ — ②





સમી① પરથી

$x = \frac{y^2}{8}$ — ③

સમી ② અને ③ પરથી

$\frac{y^2}{8} = -y$

$\therefore y^2 = -8y$

$\therefore y^2 + 8y = 0$

$\therefore y(y+8) = 0$

$\therefore y = 0,\ y = -8$

જરૂરી ક્ષેત્રફળ $A = |I|$

જ્યાં $I = \int_{-8}^{0} \left(\frac{y^2}{8} - (-y)\right) dy$

$= \left[\frac{y^3}{24}\right]_{-8}^{0} + \left[\frac{y^2}{2}\right]_{-8}^{0}$

$= \frac{1}{24}(0 - (-8)^3) + \frac{1}{2}(0 - (-8)^2)$

$= \frac{1}{24}(512) - \frac{64}{2}$

$= \frac{64}{3} - 32 = \frac{64-96}{3} = -\frac{32}{3}$

$\therefore I = \left|-\frac{32}{3}\right| = \frac{32}{3}$

50) The order and degree of the differential equation $\sqrt{y_2} = \sqrt[3]{y_1}$ are ______ respectively.

(A) 3 and 2 (B) 2 and 3

(C) 2 and 2 (D) 3 and 3

(50) $\sqrt{y_2} = \sqrt[3]{y_1}$

$\therefore y_2^{1/2} = y_1^{1/3}$

$\therefore y_2^{3} = y_1^{2}$

$\therefore$ ક્રમ 2 અને પરિમાણ 3 છે

050 (E)

(JULY, 2018)
SCIENCE STREAM
(CLASS - XII)

(Part - B)

Time : 2 Hours] *[Maximum Marks : 50*

Instructions :

1) Write in a clear legible handwriting.
2) There are three sections in Part - B of the question paper and total 1 to 18 questions are there.
3) All the questions are compulsory. Internal options are given.
4) The numbers at right side represent the marks of the question.
5) Start new section on new page.
6) Maintain sequence.
7) Use of simple calculator and log table is allowed, if required.

SECTION - A

■ **Answer the following 1 to 8 questions as directed in the question (Each question carries 2 marks):** **[16]**

1) For the function $f : R - \left\{-\frac{2}{3}\right\} \to R,\ f(x) = \frac{4x+3}{6x+4}$ If $fof(a)$=1then find a.

2) Define binary operation $*$ on R by $m * n = m + n - (mn)^2$. Then find possible inverses of 1 $(m,n \in R)$

a b

$m * n = m + n - (mn)^2$

$e * a = a$

$a * e = a$
$m * e - (me)^2$
$= m$

3) Evaluate : $\int \frac{dx}{\cos x - \cos 3x}$

OR

Evaluate : $\int \frac{x^2 - 1}{x^4 + 6x^2 + 1} dx$

4) Find the number of times a fair coin must be tossed so that the probability of getting at least one tail is at least 0.95.

5) Maximize $z = x + y$ subject to $x + y \le 1, -3x + y \ge 3$ and $x \ge 0, y \ge 0$, if possible.

6) Where does the normal to $x^2 - xy + y^2 = 3$ at $(-1,1)$ intersect the curve again?

$(-1)^2 - (1)(1) + (1)^2 = 3$

$1 - 1 + 1 = 3 \quad 1 \ne 3$

OR

The position of a particle is given by $S = f(t) = t^3 - 6t^2 + 9t$, S is in meters, t is in seconds. Then find the distance travelled in first 5 seconds.

$-3x + y = 3$

x	0	-1	
y	3	0	6

7) Solve the differential equation:

$$x.\frac{dy}{dx} - y + x.\sin\left(\frac{y}{x}\right) = 0.$$

$-3x + 6 = 3$

$-3x = 3 - 6$

$-3x = -3$

$\boxed{x = 1}$

8) If the length of the subnormal of a curve is constant and such curve passes through the origin, then find its equation.

$-3x + y = 3$

$-3x = 3$

$x = \frac{3}{-3} \quad \boxed{x = -1}$

$-3x + 3 = 3$

$-3x = 3 - 3$

$-3x = 0$

$\boxed{x = 0}$

SECTION - B

- Answer the following 9 to 14 questions as directed in the question. (Each question carries 3 marks) **[18]**

$-3x + 6 = 3 \quad -3x = 3 - 6 \quad -3x = -3 \quad \boxed{x = 1}$

9) If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then prove that

$$x.\sqrt{1-x^2} + y.\sqrt{1-y^2} + z.\sqrt{1-z^2} = 2xyz.$$

$\sin\beta = y$

$\beta = \sin^{-1} y$

$\frac{x}{\sqrt{1-y^2}} + \frac{\sqrt{y}}{\sqrt{1-x^2}} = 2$

$y \quad 1 \quad \sqrt{1-y^2}$

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2$

10) Prove that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (z+x)^2 & yz \\ zx & yz & (x+y)^2 \end{vmatrix} = 2xyz.(x+y+z)^3$$

11) If the following system of equations has unique solution then find the solution set using matrix. $A^{-1}X = B$ ~~B~~ $BX = B^{-1}A$

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10$$

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10$$

$$\frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13 \qquad (xyz \neq 0)$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} \frac{1}{x} & -\frac{1}{y} & \frac{1}{z} \\ \frac{1}{x} & +\frac{1}{y} & \frac{1}{z} \\ \frac{1}{x} & -\frac{1}{y} & \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

12) A cylinder is inscribed in a sphere of radius R. Prove that its volume is maximum if its height is $\frac{2R}{\sqrt{3}}$

OR

Determine maximum and minimum values of $f(x) = x - 2\cos x, x \in [-\pi, \pi]$.

13) Find the area of the region enclosed by two parabolas $y^2 = 4ax$ and $x^2 = 4by$. $(a > 0, b > 0)$

OR

Find the area of the region : $\{(x,y) / 0 \leq y \leq x^2, 0 \leq y \leq x+2, 0 \leq x \leq 4\}$

14) $\overline{a}$ is a unit vector and $\overline{b} = (3,0,-4)$. The measure of the angle between them is $\frac{3\pi}{8}$. If the diagonals of the parallelogram are $(3\overline{a} + \overline{b})$ and $(\overline{a} + 3\overline{b})$, then obtain the area of the parallelogram.

SECTION - C

■ **Answer the questions nos. 15 to 18 as directed in the question (Each question carries 4 marks)** **[16]**

15) If $x = (a + bx).e^{y/x}$, prove $xy_2 = \left(y_1 - \frac{y}{x}\right)^2$, $(a > 0, b > 0, x \in R^+)$

OR

Find value of C when the mean value theorem is applied to $f(x) = 2\sin x + \sin 2x;\ x \in [0, \pi]$.

16) Evaluate : $\int (x+2).\sqrt{\frac{x+3}{x-3}}.dx; (x > 3)$

17) Prove that:

$$\int_0^{\pi/2} \log \sin x.\, dx = -\frac{\pi}{2}\log 2.$$

OR

Evaluate : $\int_{-1/2}^{1/2} \left\{ \left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2 \right\}^{1/2} .dx.$

18) Find the equation of the plane passing through the intersection of the planes $2x + 3y + z - 1 = 0$ and $x + y - z - 7 = 0$ and also passing through the point (1,2,3). Also obtain the equation of the line of these planes.

⊖⊖⊖