MSc Mathematics I SEM – 2019

Assignment

MM 211 LINEAR ALGEBRA

1. (a) . Prove that arbitrary intersection of subspaces of a vector space V is again a subspace of V
   (b). Prove that Union of two subspaces is a subspace if one of the space is contained in the other
2. Prove each of the following subsets of F^2 determine whether it is a subspace of (F^3)
   a. $S_1 = \{(x_1, x_2, x_3) \in F^3 : x_1 + 2x_2 + 3x_3 = 0\}$
   b. $S_2 = \{(x_1, x_2, x_3) \in F^3 : x_1 + x_2 + x_3 = 0\}$
3. Prove that the real vector space consisting of all continuous real functions as the interval $[0,1]$ is infinite dimensional .

(h) Prove that if $(v_1, v_2, ... v_n)$ is linearly independent in V so is

$(v_1, -v_2, v_2, -v_3 .... v_{n-1}, -v_n, v_n)$

4. Show that the mapping $T : R^2 \to R^3$ defined by $T(a,b) = (a+b, a-b, b)$ is linear Transformation Find range, rank, null space and nullity of T .
5. Find the matrix of livener transformation T on R^3 defined as $T(a,b,c) = (2b+c, a-4b, 3a)$with respect to the ordered basis B and also with respect to the orders basis, $\{(1,1,1), (1,1,0), (1,0,0)\}$
6. a). Suppose V is finite dimensional and $S, T \in L(V)$
   Prove that $ST = I$ if $TS = I$
   1) define $T \in L(C^2)$ by $T(w,z) = (z,0)$

   Find all generalized eigen vectors of T .
7. a) Find all Characteristic values and characteristic vectors of the following matrix

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

b). Give an example of an operator in c^4 whose characteristic polynomial is

$$(z-T)^2 (z-8)^2$$

8. a). Prove or give a counter example. If $S, T \in L(V)$ then det $(S+T) = \det S + \det T$
   b). If AB and BA are square matrices of the same order then prove that
   $AB = I \Rightarrow BA = I$

Assignment

MSc Mathematics – 2019

MM 212 REAL ANALYSIS (S_1)

1. a). Prove that a function of bounded variation is bounded

   b). Determine which of the following functions are of bounded variation on $[0,1]$

   i). $f(x) = x^2 \sin 1/x$ if $x \neq 0$, $f(0) = 0$

   ii). $f(x) = \sqrt{x} \sin x$ if $x \neq 0$, $f(0) = 0$

2. a). Let α be a continuous function of bounded variation as $[a,b]$. Assume $g \in R(\alpha)$ an $[a,b]$ and define $\beta(x) = \int_a^x g(t) d\alpha(t)$ if $x \in [a,b]$ show that $f \uparrow$ an $[a,b]$ and threre exists a point x_0 in $[a,b]$ such that

$$\int_a^b f d\beta = f(a) \int_a^{x_0} g d\alpha + f(x) \int_{x_0}^b g d_k$$

   b). If $\alpha \uparrow$ as $[a,b]$ and $f \in R(\alpha)$ as $[a,b]$, then prove that $f^2 \in R(\alpha)$on $[a,b]$.

3. a). If $f_n \to f$ uniformly and f_n is bounded an a set S prove that $[f_n]$ is uniformly founded.

   b). Let $f_n(x) = \dfrac{x}{1+4x^2}$ $yx \in R, n = 1,2,3....$

   Find the limit function f of the sequence $\{f_n\}$ and the limit function g of the sequence $\{f_n^1\}$. Also prove that $f'(0) \neq g(0)$.

4. a). Discuss the continuity of the function

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & for\ (x,y) \neq 0 \\ 0 & for\ (x,y) = 0 \end{cases}$$

   b). Let $f : R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} x+y & if\ x \neq 0 \\ 1 & if\ x = y \end{cases}$$

   Prove that $f(x,y)$ does not exists as $(x,y) \to (0,0)$

I.a). Prove that $f : R^2 \to R$ defined by

$f(x,y) = |xy|$ is differentiable at $(0,0)$ and $\nabla f(0,0) - (0,0)$

b). Find the directional derivative of at point $(0,0)$

$f : R^2 \to R$ defined for $f(x,y) = \sqrt{x^2 + y^2}$

Assignment

MSc Mathematics (S_1) – 2019

MM 213 DIFFERENTIAL EQUATIONS

1. a) Find a particular solution of

$$y'' - y' - by = e^{-x}$$

b). Find the exact solution of the initial value problem $y' = y^2$, $y(0) = 1$ starting with $y_0(x) = 1$. Apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$ and compare these results with exact solution.

2. 1) Express $\sin^{-1}(x)$ in the form of a power series $\sum a_n x^n$ by solving $y' = (1 - x^2)^{1/2}$ in two ways. Use this result to obtain the formula

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{32^3} \times \frac{1}{2}\frac{3}{4} \cdot \frac{1}{5 \times 2^5} \cdots\cdots\cdots$$

2). Find the general solution of $(1 + x^2)y^{11} + 2xy^1 - 2y = 0$ in terms of the power series x. Can you express this solution by mean of elementary function.

3. a). Show that $P_{2x+1}(0) = 0$ and

$$P_{2n}(0) = \frac{(-1)^n 1.3.5....(2n-1)}{2^n n!}$$

b). Draw the graph of $J_0(x)$ and $J_1(x)$

4. a). Prove the following

i). $\frac{d}{dx}\left(x^p J_p(x)\right) = x^p J_{p^{-1}}(x)$

ii). $J_p^1(x) - \frac{p}{x} J_p(x) = -J_{p+1}(x)$

b). Find the general integral of

i) $z(xp - yq) = y^2 - x^2$

ii) $yz\ dx + xz\ dy + xy\ dz = 0$

5. a) Show that the equations

$f = p^2 + q^{2-1} = 0$ and $g = (p^2 + q^2)x - pz = 0$ are compactable and find the corresponding one parameter family of common solution

b). Reduce the following into canonical form and solve whenever possible

$$4U_{xx} - 4U_{xy} + 5U_{xy} = 0$$

Assignment

MSc Mathematics – 2019

MM 214 - TOPOLOGY - I

1. suppose X is a metric space and A and B are subsets of X.
   (a). Prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$
   (b). Prove that $\overline{A \cup B} \subset \overline{A} \cap \overline{B}$
   (c). Find an example to show that we do not generally get equality in part (b)
2. Suppose X, Y and Z are metric spaces and $f : X \to Y$ and $g : X \to Z$ are continuous function. Prove that $g.f : X \to Z$ is continuous.
3. Prove that the definitions of closure that we gave for metric spaces is equivalent to the definition that we gave in for topological spaces.
4. Suppose that (X_n, d_n) is a metric spaces for each $n \in w$ and each d_n is bounded by

   Let $X = \pi_{n=0}^{\infty} X_n$ and define d on X by $d(x, y) = \sum_{n=0}^{\infty} \frac{d_{n(X_N U_N)}}{2^n}$. Prove that d is a metric an X.
5. Prove that any closed subspace of locally compact space is locally compact.
6. Show that any point in $[0,1]$ can be represented as $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ where for each u $n,\ a_n \in \{0,1,2\}$.
7. Give an example for topological space which is connected but not path wise connected Justify your answer.
8. Suppose u is a collection of open sets in $X \times Y$ where Y is compact and u covers $\{x\} \times Y$ where $x \in Y$. Show that there exists a finite sub collection $u_1 C u$ and an openset $V \subset \chi$ with $x \in \chi$ such that u_1 covers $V \times Y$.
9. Is it true that every separable first countable space is second countable space. Prove or find counter example.
10. Prove that if a and b are real numbers with $a < b$, then (a,b) is homcomorphic to R.

Assignment

MSc Mathematics (Semester) – 2019

MM 221 Algebra

1. Construct a Cayley table for U_{12}.
2. Show that the set of all positive rational numbers form an abelian group under the composition defined by $a+b=\frac{ab}{2}$
3. In z_{12}, find $<6>$, $<9>$, $<11>$. Is t_{12} cyclic ?
4. Prove that z_n has an even number of generators if $n>2$.
5. If $A=\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}$ $B=\begin{pmatrix}1&2&3\\3&1&2\end{pmatrix}$

   Find AB and BA
6. Show that the external direct product of groups is itself a group.
7. Prove that A_n is a normal subgroup p of s_n.
8. Prove that every group of order 65 is cyclic.
9. Find all possible direct products for G with order p^4
10. How many sylow 5 groups of s_5 are there ?
11. Find all abelian groups of order 27 upto isomorphism.
12. Give an example of a subset of a ring that is a subgroup under addition but not a subring.
13. Explain why a finite ring must have a non zero Characteristic.
14. Show that $R(x)/<x^2+1>$ is a field.
15. Determine all ring Homomorphism from Q and Q
16. Prove that the ideal $<x>$ in $Q[x]$ is maximal.
17. In $Z[i]$, show that 3 is irreducible but 2 and 5 are not.

Assignment

MSc Mathematics (Semester-2) – 2019

MM 223 Topology – II

1. Prove that the one point compactification of the real line R is homeomorphic to a circle.
2. Prove that the projection $\pi_1 : R^2 \to R$ is continues us open and onto an A neet not be closed.
3. If X is a topological space and $x \in X$. True prove that u_x at x is a filter base .
4. Prove that $\pi_1\left(s^1 \times s^1, (1, D)\right) \cong f \times f$
5. Prove that a simple with vertices $\{v_0, v_1, v_2 ..... v_n\}$ is the smallest convex set containing $\{v_0, v_1, v_2 ..... v_n\}$.
6. Prove that the bary centre of simplex is unique.
7. Prove that the quotient topology an X is the strongest topology an Y which will make f a continous map.
8. Prove that a filter f_x on the space X converges to x if and only if the net generated by the file f_x converge to x.
9. Prove that $\pi, \left(R^2, 0\right) = (e)$.
10. Prove that every tree is contractible.

Assignment

B.Sc Mathematics (Semester-1) – 2019

MM 1141

1. In Z define $a \sim b$ if $(1)\ ab > 0 (2) a+b$ is divisible by 3. Check whether '~' is an equivalence relation in both cases.
2. For all $n \geq 1$ find the sum of $1^4 + 2^4 + 3^4 .... + n^4$ by using induction.
3. Find the g.c.d of 17017 and 18900.
4. Find the least non negative residue of $m^{10} (\text{mod} 1)$ for each number $m, 1 \leq m \leq 10.$.
5. Solve $12x \equiv 5$ mod (47)
6. Find the natural domain of the function

   (a) $f(x) = \sqrt{\dfrac{x^2 - 4}{x - 4}}$ (b) $f(x) = \dfrac{3}{2 - \cos x}$
7. Sketch the graph of $y = |x-3| + 2$.
8. Use the graph of $y = x^{1/3}$ to sketch the graph of $y = |x|^{1/3}$.
9. Find the parametric equations for the portion of the parabola $x = y^2$ joining $(1,-1)$ and $(1,1)$ oriented down to up.
10. Let $f(x) = \begin{vmatrix} x-1 & if\ x \leq 3 \\ 3x-7 & if\ x > 3 \end{vmatrix}$

    Find (a) $\lim_{x \to \bar{3}} f(x)$ (b) $\lim_{x \to 3^x} f(x)$ (c) $\lim_{x \to 3} f(x)$
11. Show that the function defined by $f(x) = \sin(x^2)$ is a continuous function
12. Let $f(x) = \begin{vmatrix} x^2 \sin 1/x & if\ x \neq 0 \\ o\ if\ x = 0 \end{vmatrix}$

    a. Show that f is continuous at $x = 0$

    b. Find $f'(0)$

    c. Show that f' is not continuous at $x = 0$
13. Find all rational values of r such that $y = x^r$ satisfies the equation $3x^2 y'' + 4xy' = zy = 0$
14. Sketch the ellipse $a(x-1)^2 + 16(y-3)^2 = 144$
15. Rotate the co-ordinate axes to remove the $xy-$ term and name the conic $6x^2 + 24xy - y^2 - 12x + 24y + 11 = 0$
16.