**PD – 119**

# First Year M.Sc. Degree Examination, October 2012
## (R.S. Scheme)
## MATHEMATICS
## Paper – I : Algebra

Time : 3 Hours Max. Marks : 90

**Instructions :** 1) *Answer* ***any five*** *questions choosing at least* ***one*** *from* ***each*** *Part.*

2) ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) Define a normal subgroup of a group G. Prove that the subgroup N of G is a normal subgroup of G if and only if product of two right cosets of G is again a right coset of G. 6

b) If N and M are normal subgroups of a group G, then prove that

$$\frac{o(NM)}{o(M)} = \frac{o(N)}{o(N \cap M)}.$$ 6

c) State and prove Cayley's theorem for finite groups. 6

2. a) Verify the class equation for dihedral group $D_8$. 6

b) Define p-Sylow subgroups of a group. Show that any two p-Sylow subgroups are conjugate to each other. 6

c) Let G be a group of order pq, where p and q are distinct primes with $p < q$ and $q \neq 1 (\text{mod } p)$, then prove that G is abelian. 6

PART – B

3. a) Define an integral domain. Show that a finite integral domain is a field. 6

b) Define :

i) subring

ii) ideal of a ring. Show that any ideal S of a ring R is an ideal of a subring of R containing S. Deduce that an ideal S of a ring R is an ideal of S + T, where T is any subring of R. 6

c) If R is a commutative ring with unity, then prove that an ideal M of R is maximal if and only if R/M is a field. 6

P.T.O.

a) Prove that every integral domain can be imbedded into a field. 6

b) Define an Euclidean ring. Prove that z [i] is an Euclidean ring. 6

c) State and prove Einstein criteria for irreducibility of a polynomial. 6

PART – C

a) Define the degree of an extension K of a field F. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. More over [L : F] = [L : K] [K : F]. 6

b) Prove that a polynomial of degree n over a field F can have at most n-roots in any extension field K of F. 6

c) Define a splitting field of a polynomial $f(x) \in F[x]$. Determine the splitting field of $x^3 - 2$. 6

a) Prove that a polynomial $f(x) \in F[x]$ has a multiple root if and only if f(x) and $f'(x)$ have a non-trivial common factor. 6

b) If K is a finite Galois extension of a field F and if G(K, F) is the group of all F-automorphisms of K, prove that $o(G(K, F)) = [K : F]$. 6

c) Let K be a Galois extension of a field F and let L be an intermediate field of F and K. Prove that L is a normal extension of F if and only if G (K, L) is a normal subgroup of G (K, F). 6

PART – D

a) If V is an n-dimensional vectorspace over F, then show that for a given T in A (V), there exists a non trivial polynomial $q(x) \in F[x]$ of degree atmost $n^2$ such that q (T) = 0. 6

b) Define a characteristic root of $T \in A(V)$. Prove that the element $\lambda \in F$ is a characteristic root of $T \in A(V)$ if and only if for some $v \neq 0$ in V, $vT = \lambda v$. 6

c) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants. 6

ppose $V = V_1 \oplus V_2$, where $V_1$ and $V_2$ are subspaces invariant under T, $T_2$ are linear transformations induced by T on $V_1$ and $V_2$, respectiv $P_1(x)$, $P_2(x)$ are minimal polynomials of $T_1$ and $T_2$, prove that the min lynomial of T is l.c.m. of $P_1(x)$ and $P_2(x)$.

fine the unitary transformation $T \in A(V)$. Prove that the linear transforma on V is unitary if and only if it takes an orthogonal basis of V into hogonal basis of V.

ove that a Hermitian linear transformation $T \in A(V)$ is positive if and o its eigen values are positive.