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I Semester M.Sc. Degree Examination, January 2017
(CBCS)
Mathematics
M101T : ALGEBRA – I

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any 5** questions.*
*2) **All** questions carry **equal** marks.*

1. a) Let $\phi : G \to G'$ be a homomorphism with Kernel K and let $\overline{N}$ be a normal subgroup of $\overline{G}$ and $N = \{ g \in G : \phi(g) \in \overline{N} \}$. Prove that $G|N \cong \overline{G}|\overline{N}$.

   b) Prove that $I(G) \cong G / Z(G)$, where I (G) is a group of inner automorphisms of G and Z (G) is the centre of G.

   c) Compute the group Aut (K_4), where K_4 is the Klein's 4-group. Hence illustrate that the automorphism group of an abelian group need not be abelian. **(5+4+5)**

2. a) State and prove the Cauchy-Frobenius Lemma.

   b) Derive the class equation for finite groups.

   c) Prove that every group of order p^2, for a prime p is abelian. **(5+5+4)**

3. a) Show that all p-sylow subgroups of a finite group are conjugate to each other.

   b) Show that the number of p-sylow subgroup of n_p of G is of the form $n_p \equiv 1$ (modp).

   c) Show that every group of order 15 is cyclic. **(6+6+2)**

4. a) Show that a normal subgroup N of G is maximal if and only if the quotient group $G|N$ is simple.

   b) If a group G has a composition series, then show that all its composition series are pairwise equivalent.

   c) Define a solvable group. Show that symmetric group S_4 is solvable, but not simple. **(5+6+3)**

P.T.O.

5. a) Define integral domain and a field P. Prove that every finite integral domain is a field.

   b) Let R be a commutative ring with unity whose ideals are {0} and R only. Prove that R is a field.

   c) Let U be the left ideal of a ring R and $\lambda(U)=\{x\in R : xu=0 \text{ for all } u\in U\}$. Prove that $\lambda(U)$ is an ideal of R. **(6+4+4)**

6. a) Define principal ideal of a ring R. Show that the ring Z of all integers is a principal ideal ring.

   b) Let R be an integral domain with ideal P. Show that P is a principal ideal of R if and only if $R|p$ is an integral domain.

   c) Show that any two isomorphic integral domains have isomorphic quotient fields. **(4+5+5)**

7. a) Show that every field is an Euclidean ring.

   b) Let R be an Euclidean ring and a, b $\in$ R be non-zero with 'b' non-unit. Then prove that d (a) < d (ab).

   c) If p is a prime number of the form 4n + 1, prove that $p = a^2 + b^2$ for some integers 'a' and 'b'. **(4+4+6)**

8. a) If F is a field, then show that F [x] is not a field.

   b) State and prove Eisenstein criterion for irreducibility of a polynomial.

   c) Let A $= (x^2 + x + 1)$ be an ideal generated by $x^2 + x + 1 \in z_2 [x]$. Verify that A is a maximal ideal in $z_2 [x]$. **(4+5+5)**

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I Semester M.Sc. Examination, January 2017
(CBCS)
MATHEMATICS
M102T : Real Analysis

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer* ***any five*** *questions.*
2) ***All*** *questions carry* ***equal*** *marks.*

1. a) Evaluate $\int_0^x x\, d\{[x]\}$ where [x] is the maximum integer function. 4

b) If $f \in R[\alpha]$ on [a, b], then prove that $\underline{\int_a^b} f\, d\alpha = \overline{\int_a^b} f\, d\alpha = \int_a^b f\, d\alpha = \lambda\,[\alpha(b) = \alpha(a)]$,

where $\lambda \in [m, M]$. 5

c) If P* is a refinement of partition P of [a, b], then show that

$L(P, f, \alpha) \leq L(P^*, f, \alpha) \leq U(P^*, f, \alpha) \leq U(P, f, \alpha)$. 5

2. a) Assuming f(x) is monotonic on [a, b] and $\alpha(x)$ is monotonically increasing and continuous functions on [a, b], prove that $f \in R[\alpha]$ on [a, b]. 5

b) If $f \in R[\alpha]$ on [a, b], $f \in [m, M]$ and ϕ is continuous function of f on [m, M] then prove that $\phi(f(x)) \in R[\alpha]$ on [a, b]. 7

c) Evaluate $\int_0^5 x^2 d\{[x] - x\}$. 2

3. a) Consider the functions $\beta_1(x)$ and $\beta_2(x)$ defined as follows :

$$\beta_1(x) = \begin{cases} 0 & \text{when } x \leq 0 \\ 1 & \text{when } x > 0 \end{cases}$$

$$\beta_2(x) = \begin{cases} 0 & \text{when } x < 0 \\ 1 & \text{when } x \geq 0 \end{cases}$$

verify whether $\beta_1(x) \in R[\beta_2(x)]$ on [–1, 1]. 7

P.T.O.

b) State and prove fundamental theorem of integral calculus. **3**

c) Calculate the total variation function of $f(x) = x - [x]$ on [0, 2], where [x] is the minimum integral function. **4**

4. a) Define uniform convergence of sequences and series of functions. State Weirstrauss M-test for uniform convergence for infinite series

$\sum_{n=1}^{\infty} f_n(x)$ on [a, b]. **7**

b) Test for uniform convergence of the following : **7**

i) $\left\{\frac{nx}{1+n^2x^2}\right\}$ for $x \in [0, 1]$.

ii) $\left\{nxe^{-nx^2}\right\}$ for any real x.

iii) $\sum_{n=0}^{\infty} (1-x)\, x^n$ for $x \in [0, 1]$.

5. a) If $\{f_n(x)\}$ is uniformly convergent to f(x) on [a, b] and each $f_n(x)$ is continuous on [a, b] then prove that f(x) is continuous on [a, b]. **7**

b) Let $\{f_n(x)\}$ be a sequence of functions uniformly convergent to f(x) on [a, b] and each $f_n(x) \in R$ (a, b]. Prove the following :

i) $f(x) \in R\,[a, b]$,

ii) $\int_a^x \underset{n\to\infty}{\text{Lt}} f_n(t)\, dt = \underset{n\to\infty}{\text{Lt}} \int_a^x f_n(t)\, dt$. **7**

6. a) Define a k-cell in $\mathbb{R}^K$. Let $I_1 \supset I_2 \supset I_3 \supset ....$ be a sequence of k-cells in $\mathbb{R}^K$.

Show that $\bigcap_{n=1}^{\infty} I_n \neq \phi$. **7**

b) State and prove Heine-Borel theorem. **7**

7. a) Let $E \subset \mathbb{R}^n$ be an open set and $f : E \to \mathbb{R}^m$ is a map. Prove that if f is continuously differentiable if and only if the partial derivatives $D_j f_i$ exists and are continuous on E for $1 \leq i \leq m$, $1 \leq j \leq n$. **7**

   b) If $T_1, T_2 \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that

      i) $\| T_1 + T_2 \| \leq \| T_1 \| + \| T_2 \|$

      ii) $\| \alpha T_1 \| = | \alpha | \| T_1 \|$. **4**

   c) Let $f : [a, b] \to \mathbb{R}^k$, $f = (f_1, f_2 ...., f_k)$, f is differentiable iff each f_i is differentiable. **3**

8. State and prove the implicit function theorem. **14**

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I Semester M.Sc. Degree Examination, January 2017
(CBCS)
MATHEMATICS
M 103T : Topology – I

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) What do you mean by denumerable set ? Is Q, the set of rational numbers denumerable ? Justify. 4

 b) Show that :

 i) Superset of an infinite set is infinite.

 ii) Subset of a finite set is finite. 6

 c) Does every infinite set contains a denumerable ? If yes, explain. 4

2. a) State Schroder-Bernstein theorem.

 Use it to prove that $(0, 1) \sim [0, 1]$. 3

 b) Let C = card R. Show that C.C = C. 6

 c) If P(A) denote the power set of a set A then prove that card A < card P(A). 5

3. a) Let (X, d) be a metric space and $d(x,y) = \dfrac{d(x,y)}{1+d(x,y)}$.

 Show that (X, d_1) is a metric space by checking only triangle inequality for d_1. 3

 b) Show that if a convergent sequence in a metric space has infinitely many distinct points then its limit is a limit point of the set of elements of the sequence. 4

 c) Prove that a subspace Y of a complete metric space is complete if it is closed. Is the converse true ? Explain. 7

P.T.O.

4. a) Prove that if a metric space X is complete then for every nested sequence $\{F_n\}_1^{\infty}$ of a nonempty closed sets in X with $\delta(F_n) \to 0$, $\bigcap_{n=1}^{\infty} F_n$ is a singleton set. 6

b) Define a set of first category. State and prove Baire's category theorem. 8

5. a) Prove contraction mapping theorem. 6

b) Show that every metric space has a completion. 8

6. a) Show that intersection of two neighborhoods is a neighborhood. Is superset of a neighborhood is again a neighborhood ? Justify. 4

b) Show that :

i) an arbitrary intersection of closed sets is closed.

ii) a set containing all its limit points is closed. 6

c) Show that the interior of intersection of two sets is the intersection of their interiors, but this is not the case for the union of two sets. 4

7. a) Let (X, τ) be a topological space. Show that a subfamily $\mathscr{B}$ of τ is a base for τ if and only if for every $U \in \tau$ and $x \in U$ there is a $B \in \mathscr{B}$ such that $x \in B \subseteq U$. 6

b) Show that a function $f : X \to Y$ is continuous if and only if inverse of every open set in Y is open in X. 4

c) Show that a bijective function $f : X \to Y$ is a homeomorphism if and only if $f(\overline{A}) = \overline{f(A)}$, for all $A \subseteq X$. 4

8. a) If C is a connected subset of (X, Y) which has a separation $X = A \cup B$ then prove that either $C \subseteq A$ or $C \subseteq B$. 4

b) Show that closure of a connected set is connected. 5

c) Prove that union of family of connected sets with non-empty intersection. 5

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First Semester M.Sc. Examination, January 2017
(CBCS)
MATHEMATICS
M 104T : Ordinary Differential Equations

Time : 3 Hours | Max. Marks : 70

Instructions : *1) **All** questions **have equal** marks.*
*2) Answer **any five** questions.*

1. a) Let $y_1, y_2, y_3 \ldots, y_n$ be a fundamental set of $L_n y = 0$. Then show that $z_1, z_2, z_3, \ldots, z_n$ also form a fundamental set of $L_n\, y = 0$ iff there exists a non singular matrix A such that

   $[z_1, z_2, z_3, \ldots, z_n]^T = A\,[y_1, y_2, y_3 \ldots, y_n]^T$.

   b) If the Wronskian of $y_1(x)$ and $y_2(x)$ is $3e^{4x}$ and if $y_1(x) = e^{2x}$, then find $y_2(x)$. **(9+5)**

2. a) State and prove Sturm's separation theorem.

   b) Let f(x) and g(x) be the two functions having n continuous derivatives in [a, b]. Then prove that

   $$\int_a^b g(x) L_n f(x)\,dx = \int_a^b f(x) L_n^* g(x)\,dx + \{[f, g](x)\}_a^b$$ **(7+7)**

3. a) If $y_1(x)$ is a solution of $y''(x) + a_1(x)y'(x) + a_2(x)y(x) = 0$. Then show that $y_2(x) = y_1(x)\,f(x)$ is also a solution of the same differential equation provided $f'(x)$ satisfies the equation $\left(y_1^2 y\right)' + a_1(x)\left(y_1^2\, y\right) = 0$.

   Also prove that $y_1(x)$ and $y_2(x)$ are linearly independent.

   b) Define a Lipschitz condition and test the validity of this condition with respect to y for $f(x, y) = \dfrac{\cos x}{x^2}(y + y^2)$; $|x - 1| < \dfrac{1}{2}$, $|y| \le 1$. **(7+7)**

4. a) Define the self-adjoint eigenvalue problem. Also prove that the eigenvalues of a self-adjoint eigenvalue problem are real.

   b) Show that the eigen functions corresponding to the distinct eigenvalues of a self-adjoint eigenvalue problem are orthogonal over the same interval. **(7+7)**

P.T.O.

5. a) Define ordinary, regular and irregular singular points of a differential equation and hence find the same for
$(1 - x^2)\ y'' - 2xy' + Ny = 0$, N is a constant.

b) Find the series solution of
$(x^2 - 1)\ y'' + 3xy' + xy = 0,\ y(0) = 4,\ y'(0) = 6.$ **(7+7)**

6. a) Obtain the general solution of the Laguerre differential equation.

b) Prove the following :

i) $xL'_n(x) = n\,L_n(x) - n\,L_{n-1}(x)$.

ii) $L'_n(x) = -\sum_{m=0}^{n-1} Lm(x)$. **(7+7)**

7. a) Express an n^{th} – order differential equation as a system of first order differential equation and hence obtain it for $y''' - 14y'' + 10y' - 16y = 16t$.

b) Find the fundamental matrix and the general solution of $\underset{\sim}{X}'(t) = A\underset{\sim}{X}(t)$

where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, $\underset{\sim}{X}(t) = \begin{bmatrix} x_1 & (t) \\ x_2 & (t) \end{bmatrix}$. **(7+7)**

8. a) Define the various types of critical points of the linear system

$$\frac{dx}{dt} = ax + by,$$

$$\frac{dy}{dt} = cx + dy,\ ad - bc \neq 0.$$

b) Locate the critical point and find the nature of the system

$$\frac{dx}{dt} = x + y,$$

$$\frac{dy}{dt} = 3x - y.$$

Also find the equation of phase path. **(7+7)**

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I Semester M.Sc. Degree Examination, January 2017
(CBCS)
MATHEMATICS
M105T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *i) Answer **any five full** questions.*
*ii) **All** questions carry **equal** marks.*

1. a) Explain the contradiction method of proof. Prove by method of contradiction the following statement, "$\sqrt{15}$ is not a rational number". 4

 b) Test the validity of the following arguments. "If I try hard and I have talent, then I will become a scientist. If I become scientist, then I will be happy. Therefore, I will not be happy then I did not try hard or I do not have talent". 5

 c) Show that if 11 numbers are chosen from the set {1, 2, 3, ..., 20}, one of them is a multiple of another. 5

2. a) A committee of k people is to be chosen from a set of 7 women and 4 men. How many ways are there to form the committee if

 i) The committee has 5 people, 3 women, 2 men ?

 ii) The committee can be of any positive size, but must have equal number of women and men ? 4

 b) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any amount can go in each of the other six boxes ? 5

 c) Find the number of ways to place 25 people into three rooms with atleast one person in each room. 5

3. a) Write a short note on modeling "The tower of Hanoi" problem and find an explicit formula for the solution of the problem. 4

 b) Solve the recurrence relation $a_n + a_{n-1} - 8a_{n-2} - 12\,a_{n-3} = 0$, for $n \geq 3$ with initial conditions $a_0 = 1$, $a_1 = 5$, $a_2 = 1$. 5

 c) Solve the recurrence relation using generating functions $a_{n+2} - 3a_{n+1} + 2a_n = 3^n$, with initial conditions $a_0 = 5$, $a_1 = 3$. 5

P.T.O.

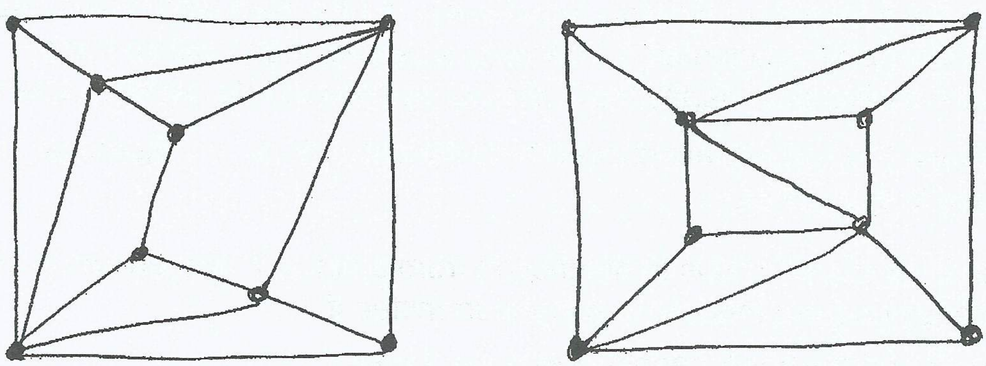
4. a) Define connectivity relation. If R is a relation on a set A and $|A| = n$, then prove that $R^{\infty} = R \cup R^2 \cup R^3 \cup \ldots \cup R^n$. 4

b) Give a step-by-step procedure of Warshall's algorithm. Using the same find the transitive closure of the relation R = {(x, x), (x, y), (y, z), (z, z), (x, u), (y, u), (u, u)} defined on a set A = {u, x, y, z}. 5

c) Define a Boolean algebra. Prove that the De' Morgan's Laws hold good in a Boolean algebra. 5

5. a) Define a graph. Prove that in a graph G, every u-v walk contains a u-v path. 4

b) What are isomorphic graphs ? Check for the isomorphism in the following graphs.



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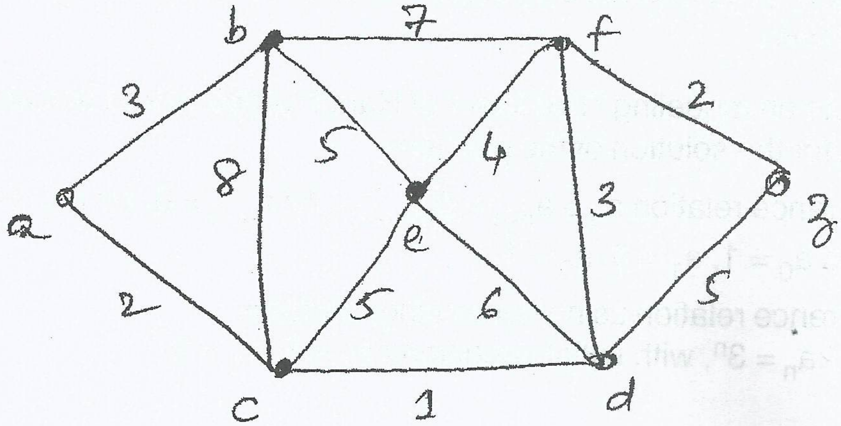
c) Define a bipartite graph. Prove that a graph is bipartite if and only if it contains no odd cycle. 5

6. a) Applying Dijkstra's algorithm find a shortest distance path between 'a' and 'z' for the following graph.



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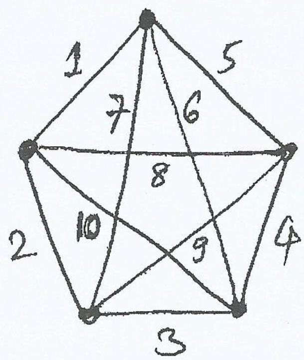
b) Define an Eulerian graph with an example. Does there exist an Eulerian graph with even number of vertices and odd number of edges ? Does there exist an Eulerian graph with odd number of vertices an even number of edges ? Justify your answer. 4

c) Define a Hamiltonian graph with example. State and prove Dirac's theorem for Hamiltonian graphs. 5

7. a) Using the nearest neighbour method, find the weight of a spanning cycle for the following graph.



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b) Define a planar graph. Show that K_5 and $K_{3,3}$ are non-planar graphs. 5

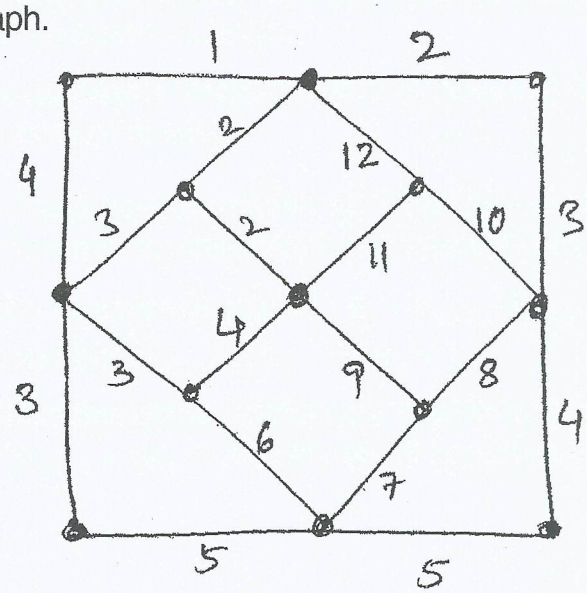
c) Define each term of the following and establish

$\alpha_0(G) + \beta_0(G) = p = \alpha_1(G) + \beta_1(G)$ 5

8. a) Define the terms : Tree, Bridge, Cut-vertex. Prove that a connected graph is a tree if and only if every edge is a bridge. 4

b) Prove that the centre of a tree is K_1 or K_2. 5

c) By applying Kruskal's algorithm find a minimal spanning tree of the following graph.



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I Semester M.Sc. Examination, January 2017
(CBCS)
MATHEMATICS
M108SC : Brief Biography of Eminent Mathematicians and History of Mathematics

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five** questions.*
*2) **All** questions carry **equal** marks.*

1. Present brief biographics of Ramanujan and Hilbert. **14**

2. Explain the various contributions of Euler to mathematics. Use illustrative examples. **14**

3. a) Explain the idea of Al-Khwarizmi for the development of Algebra.
   b) Write a short notes on the life of Omar Khayyam. **(8+6)**

4. Discuss briefly the contribution of ancient Greeks to mathematics through the works of Pythagoras, Euclid and Archimedes. **14**

5. a) Comment on the Rene Descartesian idea of real no. line, the Cartesian plane, Cartesian co-ordinates, Euclidean geometry and co-ordinates in three dimensional space.
   b) Elaborate through examples the need for the number systems of the natural numbers, the integers, the rational nos., the real numbers and the complex numbers. **(7+7)**

6. a) Write short notes on the life and works of Cauchy.
   b) If f(x) is continuous function with [a, b] as the domain, then using properties of real numbers prove that there are points m, M $\in$ [a, b] such that $f(m) \leq f(x) \leq f(M)$. **(8+6)**

7. a) Make a brief discussion on the life of child prodigy Henri poin care.
   b) Explain the concept of rubber sheet geometry and the idea of homotopy. **(8+6)**

8. Through suitable examples the proof of theorems by : **(3+3+4+4)**
   i) Direct proof
   ii) Contradiction
   iii) Contra positive
   iv) Mathematical induction.
