This question paper contains **3** printed pages]

Your Roll No

6177

B.Sc. (Hons.) / I Sem. / Computer Science J

Paper 103 – Calculus – I

(Admissions of 2001 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt **all** questions

All questions carry equal marks

1 Evaluate the limits

(i) $\lim_{\theta \to 3^+} \frac{\lfloor \theta \rfloor}{\theta}$

(ii) $\lim_{x \to -2^+} \frac{1}{x^2 - 4}$

(iii) $\lim_{x \to 2^-} \frac{1}{x^2 - 4}$, $\lfloor \theta \rfloor$ being floor function.

2 For what values of a, m and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypothesis of the Mean Value Theorem on the interval [0, 2] ?

3 Find a function whose derivative is $\frac{1}{x^2} + 2x$ and whose graph passes through the point $(-1, 1)$

4 Is any real number exactly 1 less than its cube ?

5 Show that the sequence $\langle a_n \rangle$ defined by the recurrence relation $a_{n+1} = \sqrt{3a_n}$, $a_1 = 1$ converges to 3

6 Graph the equation $y = x^3(x + 2)$ Mark the co-ordinates of local extreme points and inflexion points, if they exist

7 Test the convergence or divergence of the series

(i) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$

(ii) $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

8 Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ is conditionally convergent Estimate the error when this series is approximated by its first nine terms Find the number of terms required to compute the value of the series with allowable error 0 05.

9 Find the quadratic approximation of the function $f(x, y) = e^x \cos y$ near the origin How accurate is the approximation if $|x| \leq 0\ 1$ and $|y| \leq 0\ 1$?

10 Find the linearization $L(x, y)$ of the function $f(x, y) = x^2 - xy + \frac{y^2}{2} + 3$ at $(3, 2)$ Find an upper bound for the magnitude of the error in the approximation $f(x, y) \approx L(x, y)$ over the rectangle R $|x - 3| \leq 0.1, |y - 2| \leq 0.1$

11 Find the parametric equations for the line tangent to the curve of intersection of the surfaces $x + y^2 + 2z = 4$ and $x = 1$ at the point $(1, 1, 1)$

12 Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

13 Find $\lim_{x \to \infty^+} (x)^{\frac{1}{\log x}}$

14 Solve the initial value problem for $\vec{r}$ as a vector function of t,

$$\frac{d\vec{r}}{dt} = -\hat{t}\hat{i} - \hat{t}\hat{j} - \hat{t}\hat{k}, \quad \vec{r}(0) = \hat{i} + 2\hat{j} + 3\hat{k}$$

15 Show that for $y > 0$,

$$\int_0^\infty e^{-xy} \frac{\sin x}{x} dx = \frac{\pi}{2} - \tan^{-1} y$$
