46-06-520

Code No.: 6113

FACULTY OF SCIENCE

M.Sc. I Semester Examination, May 2006

COMPUTER SCIENCE

Paper 1.1

(Discrete Mathematical Structures)

Time : 3 *Hours*] [*Max. Marks* : 100

Answer **all** *questions.*

Section A – (Marks : 8 × 5 = 40)

1. Prove that $P \leftrightarrow Q$ and $(P \to Q) \wedge (Q \to P)$ are equivalent.
2. Prove that there are 2^{2^n} boolean functions on n-variables.
3. Prove that a tree always has one fewer edge than vertices.
4. Define Euler and Hamiltonian paths.
5. How many 9 letter words can be formed that contain 3, 4 or 5 vowels allowing repetition of letters?
6. From a group of 10 professors how many ways a committee of 5 members be formed so that at least one of professors A and B be included.
7. Find the coefficient of X^{14} in $(1 + X + X^2 + X^3)^{10}$.
8. Solve the recurrence relation $a_n = a_{n-1} + n$, $a_0 = 2$ by substitution method.

Section B – (Marks : 4 × 15 = 60)

9. (a) (i) Analyze the following argument and then determine whether it is a valid argument.

"If I buy a new car then I will not be able to go to Delhi in December. Since I am going to Delhi in December, I will not buy a new car".

(ii) Show that $p \vee (q \wedge r)$ is equivalent to $(p \vee {\sim}q) \vee {\sim} r$.

Or

(b) (i) In a boolean algebra with < + ordering, prove that $a + b$ is the least upper bound of a and b, and ab is the greatest lower bound.

(ii) Construct a minimal switching circuit for the boolean expression.

$xyzw + xyz'w + xyzw' + xyz'w + x'yz'w' + x'yzw' + xy'z'w' + x'y'z'w'$

[P.T.O.

10. (a) (i) Let $G = (V, E)$ be a graph, where $V = \{a, b, c, d, e\}$,

$E = \{(a, b), (b, a), (a, c), (a, d), (b, c), (d, e)\}$. Draw representation of G. Find the adjacency matrix for G and determine the in-degree and out-degree of each vertex.

(ii) Explain why Dijkstra's algorithm is of no use in solving the travelling salesman problem.

Or

(b) (i) Prove that every connected graph has at least one spanning tree.

(ii) Define a planar graph and show that $K_{3,3}$ is non-planar.

11. (a) (i) How many 4-digit telephone numbers will be formed with one or more repeated digits?

(ii) Find the number of integral solutions to $x_1 + x_2 + x_3 + x_4 = 50$ where $x_1 \geq -4$, $x_2 \geq 7$, $x_3 \geq -14$ and $x_4 \geq 10$.

Or

(b) (i) If A_i are finite subsets of a universal set U, then prove that

$$|A_1 \cup A_2 \cup \ldots\ldots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{i,j} |A_i \cap A_j| + \sum_{i,j,k} |A_i \cap A_j \cap A_k| + \ldots\ldots +$$

$$(-1)^{n-1} |A_1 \cap A_2 \cap \ldots\ldots \cap A_n|.$$

(ii) In usual notation prove that $D_5 = 44$.

12. (a) Explain Fibonacci sequence of numbers. If F_n $(n \geq 2)$ satisfies the Fibonacci relation then prove that there are constants C_1 and C_2 such that

$$F_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n.$$

Or

(b) (i) Find a particular solution to $a_n - 7a_{n-1} + 10a_{n-2} = 7.3^n + 4^n$.

(ii) Let $F = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, for $n \geq 0$, solve for the entries of F^n using recurrence relations.