BHU MATHEMATICS 2018

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1. Let $\phi: G \rightarrow G'$ be a homomorphism of groups such that Ker $\phi = \{e\}$. Then
   (1) ϕ is onto
   (2) ϕ is one-one
   (3) ϕ is one-one and onto both
   (4) ϕ maps every element of G to identity of G'

2. The number of elements of order 12 in a cyclic group of order 12 is
   (1) 3
   (2) 2
   (3) 4
   (4) 1

3. Let H be a finite subgroup of a group G and let $g \in G$ If $gHg^{-1} = \{ghg^{-t}|h \in H$ then
   (1) $|gHg^{-1}| = |H|$
   (2) $|gHg^{-1}| < |H|$
   (3) $|gHg^{-1}| > |H|$
   (4) $|gHg^{-1}| = 1$

4. The remainder of $(37)^{49}$ when divided by 7 is
   (1) 3
   (2) 1
   (3) 2
   (4) 6

5. Which one of the following is an incorrect statement?
   (1) Every subset of a linearly independent set is linearly independent.
   (2) {O} is a linearly dependent set.
   (3) Every set which contains a linearly dependent subset is linearly dependent.
   (4) Every set containing O is linearly independent.

6. If u and v are vector in an inner-product space such that $||u+v|| = 10||u-v|| = 2$ and $||v|| = 4$, then $||u|| =$
   (1) 6
   (2) 4
   (3) 2
   (4) 8

7. If W is a subspace of a vector space V over the field $(\mathbb{Z}_3, +_3, \times_3)$ such that dim(V)=7 and dim (W)=4, then the number of element in V/W is
   (1) 9
   (2) 81
   (3) 49
   (4) 27

8. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^4 - 2A^3 - A^2 + 2I =$
(1) 2A
(2) $2(I - A)$
(3) $2(I + A)$
(4) $2(A - I)$

9. If W is the subspace of $M_{n\times n}(\mathbb{R})$ consisting of skew-symmetric matrices, then
(1) $\dim(W) = \frac{n(n+1)}{2}$
(2) $\dim(W) = n^2 - n$
(3) $\dim(W) = \frac{n(n+1)}{2}$
(4) $\dim(W) = (n-1)^2$

10. If a set A has n elements, then the number of all relations on A is
(1) 2^{n^2}
(2) n^2
(3)2^n
(4) $2n$

11. Total number of transpositions in the permutation
$f = \begin{pmatrix} 1 & 2 & 34 & 5 & 67 & 8 & 910 \\ 10 & 9 & 35 & 1 & 68 & 2 & 7\ 4 \end{pmatrix}$ are
(1) 8
(2) 6
(3) 9
(4) 6

12. The number of generators is an infinite cyclic group is
(1) 8
(2)7
(3)9
(4)6

13. If V is a real inner-product space and $\alpha, \beta \in V$ such that $||\alpha|| = ||\beta||$then $\langle \alpha + \beta, \alpha - \beta \rangle =$
(1) $2||\alpha||^2$
(2) $2||\alpha||$
(3) 0
(4) $||\alpha||^2$

14. If T is a linear transformation from the vector space $\mathbb{R}^2(\mathbb{R})$ into the vector space $\mathbb{R}^3(\mathbb{R})$ such that $T(x, y) = (x + y, x - y, 2y)$ then rank of T is
(1) 3
(2) 2
(3) 1
(4) 0

15. In a group of order 66, the number of Sylow-11 subgroups is
(1) 1
(2) 3
(3) 2
(4) 6

16. If R is a ring such that $a^2 = a$ for all $a \in R$, then characteristic of R is
(1) 0
(2) ∞
(3) 2
(4) 4

17. Total number of group homomorphism from the group $\mathbb{Z}_{12}$ *to* $\mathbb{Z}_{30}$ are
(1) 6
(2) 3
(3) 2
(4) 1

18. The order of the subgroup $\langle 5 \rangle \oplus \langle 3 \rangle$ of the group $\mathbb{Z}_{12}$ *to* $\mathbb{Z}_{30}$ are
(1) 6
(2) 3
(3) 2
(4) 1

19. Total number of roots of the polynomial $\bar{2}\,x^2 + \bar{4}\,x + \bar{4}$ over the $(\mathbb{Z}_{10} +_{10}, \times_{10})$ are
(1) 1
(2) 2
(3) 3
(4) 4

20. If $W_1 = \{(x, y, z, x, t) | x, y, z, t \in R\}$ and $W_2 = \{(0, x, y, z, t) | x, y, z, t \in R\}$ are two subspaces of $\mathbb{R}^5$ $(\mathbb{R})$, then $\dim(W_1 \cap W_2) =$
(1) 4
(2) 3
(3) 2
(4) 1

21. Let $T: V \to W$ be a linear transformation, where $\dim(V) = m$, $\dim(W) = n$ and $m < n$ Then
(1) T is surjective but not injective
(2) T can be injective but not surjective
(3) T=0
(4) T is both injective and surjective

22. The order of the group $\mathbb{Z}/30\mathbb{Z}$ is
(1) ∞
(2) 6
(3) 5
(4) 30

23. Let $T:\mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x,y) = (x-y, x-2y)$ is non-singular. Then $T^{-1}(x,y) =$
(1) $(y-x, 2x-y)$
(2) $(x-y, 2x-y)$
(3) $(x+y, y-2x)$
(4) $(2x-y, x-y)$

24. If the order of every element of a element of a group is 2, then this group
(1) is Abelian
(2) is cyclic
(3) is of infinite order
(4) is definitely non-abelian

25. Let R be a relation defined on the set of integers by a R b if $a = kb$ for some positive integer k, then
(1) R is reflexive and transitive but not symmetric
(2) R is reflexive and symmetric but not transitive
(3) R is symmetric
(4) R is an equivalence relation

26. If a is an elements of group G such that $o(\alpha) = n = 2m$ then which one of the following is also of order n?
(1) a^2
(2) a^m
(3) a^4
(4) a^3

27. If the characteristic values of an invertible $n \times n$ matrix A are $\lambda_1, \lambda_2, \ldots, \lambda_n$, then the characteristic values of Adj(A) are
(1) $\frac{1}{\lambda_r}, 1 \le r \le n$
(2) $\frac{1}{\lambda_r |A|}, 1 \le r \le n$
(3) $\frac{|A|}{\lambda_r}, 1 \le r \le n$
(4) $|A|\lambda_r, 1 \le r \le n$

28. Which one of the following rings is field ?
(1) $(\mathbb{Z}_4, +_4, \times_4)$
(2) $(\mathbb{Z}_6, +_6, \times_6)$
(3) $(\mathbb{Z}_7, +_7, \times_7)$
(4) $(\mathbb{Z}_9, +_9, \times_9)$

29. Let T be a linear operator on R^3 defined by $T(x,y,z) = (3x-3y, x-y, 2x+y+z)$ Then the rank and nullity of T are respectively
(1) 3,0
(2) 1,2
(3) 2,1
(4) 0,3

30. The number of invertible elements in the ring $(\mathbb{Z}_{24}, +_{24}, \times_{24})$ is
(1) 24
(2) 8
(3) 6
(4) 3

31. $\overline{\lim}$ and $\underline{\lim}$ of the sequence $\cos\frac{n\pi}{4} + \sin\frac{n\pi}{4}$ are respectively
(1) $\sqrt{2}, -\sqrt{2}$
(2) $\sqrt{2}, -1$
(3) $1, -\sqrt{2}$
(4) $1, -1$

32. If a sequence $\{a_n\}_{n+}^{\infty}$ of elements in the interval $(-1,1)$ is given, then which one of the following is true?
(1) Every limit point of $\{a_n\}$ is in (-1,1)
(2) Every limit point of $\{a_n\}$ is in (-1,1)
(3) The limit points of $\{a_n\}$ can only be in $\{-1,0,1\}$
(4) the limit point of $\{a_n\}$ cannot be in $\{-1,0,1\}$

33. Which of the following statements is true?
(1) The functions $\sin x$ and x^2 are uniformly continuous on $[0,\infty)$
(2) The function $\sin x$ and e^{-x} are uniformly continuous on $[0,\infty)$
(3) The Function e^{-x} and $\frac{1}{x}$ are uniformly continuous on $[0,\infty)$
(4) The function x^2 and $\frac{1}{x}$ are uniformly continuous on $[0,\infty)$

34. If C is the circle $|z| = 4$, then $\oint_C \frac{dz}{z^2+4}$ is equal to
(1) $4\pi i$
(2) $2\pi i$
(3) πi
(4) 0

35. Let A be a closed subset of $\mathbb{R}, A \neq \phi\ A \neq \mathbb{R}$ Then A is
(1) the closure of the interior of A
(2) a countable set.
(3) a compact set
(4) not open.

36. Let $f: R \to R$ be twice continuously differentiable function with $f(0) = f(1) = f'(0) = 0$ Then
(1) f'' is the zero function
(2) $f''(0)$ is zero
(3) $f''(x) = 0$ for some $x \in (0,1)$
(4) f'' never vanished

37. Which one of the following statements is not correct for a real valued function f?
(1) If f is Riemann integrable on [a,b], then f^2 is also Riemann integrable on $[a,b]$
(2) If f^2 is Riemann integrable on $[a,b]$, then f is also Riemann integrable on $[a,b]$
(3) If f^3 is Riemann integrable on $[a,b]$ then f is also Riemann integrable on $[a,b]$
(4) If f is Riemann integrable on $[a,b]$, then $|f|$ is also Riemann integrable on $[a,b]$

38. If $g: \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = \begin{cases} \frac{\sin x}{x}, if\ x \neq 0 \\ 1, if\ x = 0 \end{cases}$ then
(1) g is not continuous
(2) g is continuous but not differentiable
(3) g is differentiable
(4) g is not bounded

39. Let for each $n \geq 1, S_n$ be the open disc in R^2, with center at a point $(n, 0)$ and radius equal to n. Then $S = \bigcup_{n\geq 1} S_n$ is
(1) $\{(x, y) \in R^2 : x > 0 \; and \; |y| < x\}$
(2) $\{(x, y) \in R^2 : x > 0\}$
(3) $\{(x, y) \in R^2 : x < 0 \; and \; |y| < 2x\}$
(4) $\{(x, y) \in R^2 : x > 0 \; and \; |y| < 3x\}$

40. Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous map. Choose the correct statements
(1) $f(A)$ is Bounded for all bounded subsets A to R
(2) f is bounded
(3) The images of f is an open subsets of $\mathbb{R}$
(4) $f^{-1}(A)$ is compact for all compact for all compact subsets A of B

41. If $V = (x^2 + y^2 + z^2)^{-1/2}$, then $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ is equal to
(1) 0
(2) V
(3) 2V
(4) 3V

42. If $u = x\phi(y/x) + \Psi(y/x)$ where $\phi(y/x)$ and $\Psi(y/x)$ are two function of $\frac{y}{x}$ th $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ is equal to
(1) 3
(2) 2
(3) 1
(4) 0

43. The envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, m bei parameter, is
(1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
(2) $x^2 + y^2 = a^2 + b^2$
(3) $x^2 + y^2 = a^2 - b^2$
(4) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

44. The value of the integral $\int_0^1 \left(log\frac{1}{y}\right)^5 dy$ is
(1) $\Gamma(10)$
(2) $\Gamma(6)$
(3) $\Gamma(3)$
(4) $\Gamma(18)$

45. The value of the integral $\iint \frac{xy}{\sqrt{1-y^2}} \; dx, dy$ over the first quadrant of the circle $x^2 + y^2 - 1$ is
(1) $\frac{1}{2}$
(2) $\frac{1}{3}$
(3) $\frac{1}{5}$
(4) $\frac{1}{6}$

46. By changing the order of integration in the integral $\int_0^4 \int_x^{2\sqrt{x}} f(x,y)\,dx,dy$ it can be expressed as

(1) $\int_0^4 \int_{y^2/2}^{y} f(x,y)\,dy,dx$

(2) $\int_0^2 \int_{y^2/2}^{y} f(x,y)\,dy,dx$

(3) $\int_0^4 \int_{y^2/4}^{y} f(x,y)\,dy,dx$

(4) $\int_0^4 \int_{y}^{y^2/4} f(x,y)\,dy,dx$

47. The value of the integral $\int_0^1 \int_{e^x}^{e} \frac{dx\,dy}{\log y}$ is

(1) e^2
(2) $e+1$
(3) e
(4) $e-1$

48. The real-valued function f, ϕ, Ψ are derivable in $[\alpha, b]$ then there exists at least one $c \in (a,b)$ such that

(1) $\begin{vmatrix} f'(a) & \phi'(a) & \Psi'(a) \\ f(b) & \phi(b) & \Psi(b) \\ f'(c) & \phi'(c) & \Psi'(c) \end{vmatrix} = 0$

(2) $\begin{vmatrix} f'(a) & \phi(a) & \Psi(a) \\ f(b) & \phi(b) & \Psi(b) \\ f'(c) & \phi'(c) & \Psi'(c) \end{vmatrix} = 0$

(3) $\begin{vmatrix} f(a) & \phi(a) & \Psi(a) \\ f'(b) & \phi'(b) & \Psi'(b) \\ f'(c) & \phi'(c) & \Psi'(c) \end{vmatrix} = 0$

(4) $\begin{vmatrix} f'(a) & \phi'(a) & \Psi'(a) \\ f'(b) & \phi'(b) & \Psi'(b) \\ f'(c) & \phi'(c) & \Psi'(c) \end{vmatrix} = 0$

49. With the help of mean value the theorem, for $x > 0, 0 < \theta < 1, log_{10}(x+1)$ can be expressed as

(1) $\frac{xlog_{10}e}{1+\theta x}$

(2) $\frac{x}{1+\theta x}$

(3) $\frac{\theta x}{1+x}$

(4) $\frac{\theta x}{(1+\theta x)}$

50. If $(x) = \sqrt{x}\,, \phi\,(x) = \frac{1}{\sqrt{x}}$ are defined on the interval $[1,2]$, then the value of C satisfying Cauchy's mean value theorem is

(1) $\sqrt{3}$
(2) $\sqrt{2}$
(3) $2+\sqrt{2}$
(4) $1+\sqrt{2}$

51. The value of $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\cdots+\frac{1}{\sqrt{n^2+n}}\right]$ is
(1) 1
(2) 0
(3) ∞
(4) $\frac{1}{2}$

52. The value of $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{1/n}$ is
(1) 1
(2) e
(3) 0
(4) $\sqrt[n]{e}$

53. The series $\sum_{n=2}^{\infty}\frac{1}{n(logn)^P}$ is
(1) convergent if $p > 1$ and divergent if $0 < p \le 1$
(2) convergent if $0 < p < 1$ and divergent if $p \ge 1$
(3) convergent $\forall\, p$
(4) divergent $\forall\, p$

54. For $a_1 > 0$ the sequence $\{a_n\}_{n=1}^{\infty}$ where $a_{n+1} = 1+\frac{1}{a_n}$ $\forall\, n \ge 1$ converges to
(1) $\frac{\sqrt{5}}{2}$
(2) $\frac{\sqrt{5}+1}{2}$
(3) $\frac{\sqrt{5}-1}{2}$
(4) $\frac{1}{\sqrt{5}}$

55. If $f(x) = \begin{cases} \frac{1}{a^{n-1}}, \frac{1}{a_n} < x \le \frac{1}{a^{n-1}}, n = 1,2,3,..,and\ a > I \\ 0, x = 0 \end{cases}$ then
(1) f is integrable on [0,1] and $\int_0^1 f\,dx = a$
(2) f is integrable on [0,1] and $\int_0^1 f\,dx = \frac{a}{a+1}$
(3) f is not integrable on [0,1]
(4) f is integrable on [0,1] and $\int_0^1 f\,dx = \frac{a+1}{a}$

56. If $f(x) = x[x]$ where $[x]$ denotes the greatest integer not greater than x then
(1) f is integrable on [0,2] and $\int_0^2 f\,dx = \frac{3}{2}$
(2) $f$is integrable on [0,2] and $\int_0^2 f\,dx = \frac{3}{2}$
(3) f is integrable on [0,2] and $\int_0^2 f\,dx = 0$
(4) f is not integrable on [0,2]

57. The integral $\int_a^b \frac{1}{(x-a)^n(b-x)^n}dx$ converges iff
(1) $n > 1$ and $m > 1$
(2) $n < 1$ and $m < 1$
(3) $n > 1$ and $m < 1$
(4) $n < 1$ and $m > 1$

58. If $f(z) = u + iv$ is an analytic function where $z = x + iy$ and $u - v = e^{-x}[(x-y)\sin y - (x+y)cosy]$ then $f(z)$ is
(1) $ze^{z} + c$
(2) $ize^{z} + c$
(3) $ize^{z} + c$
(4) $ze^{-z} + c$

59. The value of $\oint_{c} \frac{z^2+1}{z(2z-1)} dz$, where C is $|z| = 1$ and $z = x + iy$, is
(1) $\frac{\pi i}{2}$
(2) $\frac{5\pi i}{2}$
(3) $2\pi i$
(4) $\frac{7\pi i}{2}$

60. The number of roots of the equation $z^7 - 5z^3 + 12 = 0$, lying between the circles $|z| = 2$ is
(1) 3
(2)7
(3) 4
(4) 0

61. The locus of the complex number z, satisfying equation $|z-1| + |z+1| = 3$ is
(1) a line segment
(2) a circle
(3) an eillpse
(4) a straight line

62. Which one of the following iterative process cannot be used to determine the complex root of the equations? $f(x) = 0$?
(1) Bisection method
(2) Secant method
(3) Muller's method
(4) Lin-Bairstow method

63. If a root of the equation $f(x) = 0$ lies in the interval I then the condition under which the Newton-Raphson formula converges to the root in I is
(1) $|f(x)||f'(x)| = |f''(x)|^2, \forall x \in I$
(2) $|f(x)||f'(x)| < |f''(x)|^2, \forall x \in I$
(3) $|f(x)||f'(x)| > |f''(x)|^2, \forall x \in I$
(4) $|f(x)||f'(x)| \geq |f''(x)|^2, \forall x \in I$

64. If ω be the angular velocity at the nearest end of the major axis of the orbit of a planet with eccentricity e, then its period is
(1) $\frac{2\pi}{\omega} \sqrt{\frac{1-e}{1+e}}$
(2) $\frac{2\pi}{\omega} \sqrt{\frac{1+e}{1-e}}$
(3) $\frac{2\pi}{\omega} \sqrt{\frac{1-e}{(1+e)^3}}$
(4) $\frac{2\pi}{\omega} \sqrt{\frac{1+e}{(1-e)^3}}$

65. If the velocity at any point of a central orbit is $\frac{1}{n}th$ of what it would be for a circular orbit at the same distance r, then the central force varies inversely as
(1) r^n
(2) e^{2n^2+1}
(3) r^{π^2}
(4) $r^{\pi^2}-1$

66. If a particle describes the equiangular spiral $r = ae^{\theta cot a}$, under a force F to the pole where a and α are constant then the law of force is proportional to
(1) $\frac{1}{r}$
(2) $\frac{1}{r}$
(3) $\frac{1}{r^2}$
(4) $\frac{1}{r^4}$

67. The rate of convergence of the iterative method $x_{k+1} = Ax_k + \frac{\alpha B}{x_k^2}$ for computing $\alpha^{1/3}$ become as high as possible, if
(1) $A = \frac{1}{3}, B = \frac{1}{3}$
(2) $A = \frac{2}{3}, B = \frac{2}{3}$
(3) $A = \frac{1}{3}, B = \frac{2}{3}$
(4) $A = \frac{2}{3}, B = \frac{2}{3}$

68. In Givens method to calculated the eigenvalues for symmetric matrices, the maximum number of plane rotations required to brings a matrix of order n to its tri-diagonal form is
(1) $\frac{1}{2}(n-1)(n-2)$
(2) $(n-1)^2$
(3) $(n-2)^2$
(4) (n^2-1)

69. The radial and transverse velocities of a particle are non-zero constants, that the path of the particle is
(1) a spiral
(2) a circle
(3) a cardioid
(4) an ellipse

70. If the radial and transverse velocity of a particle at the point (r, θ) b respectively λr and $\mu\theta$, where $\lambda\,\mu$ are constants, then the radial and transvers accelerations are respectively
(1) $\lambda \; and \; \mu$
(2) $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\frac{\mu}{r} + \lambda\right)$
(3) $\lambda^2 r + \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\lambda - \frac{\mu}{r}\right)$
(4) $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\frac{\mu}{r} - \lambda\right)$

71. The Newton-Raphson algorithm to find square root of N is

(1) $x_{n+1} - \frac{1}{2}\left(2x_n \frac{N}{x_n}\right)$

(2) $x_{n+1} = \frac{1}{2}\left(2x_n + \frac{N}{x_n}\right)$

(3) $x_{n+1} = \frac{1}{2}\left(2x_n - \frac{N}{x_n}\right)$

(4) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$

72. For given two points $(a, f, (a)), (b, f(b))$, the linear Lagrange polynomial $P(x)$ that passes throught these two point is given by

(1) $P(x) = \frac{(x-b)}{(a-b)} f(a) + \frac{(x-a)}{(a-b)} f(b)$

(2) $P(x) = \frac{(x-b)}{(a-b)} f(a) + \frac{(x-a)}{(b-a)} f(b)$

(3) $P(x) = \frac{(x-b)}{(b-a)} f(a) + \frac{(x-a)}{(b-a)} f(b)$

(4) $P(x) = \frac{(x-a)}{(b-a)} f(a) + \frac{(x-b)}{(a-b)} f(b)$

73. If a particle moves along a circle $r = 2a\cos\theta$ such that its acceleration towards the origin is always zero, then the transverse acceleration of the particles varies as

(1) $cosec^2\theta$
(2) $\sin\theta$
(3) $cos\theta$
(4) $cosec^5\theta$

74. Let P be particle whose radial and transverse velocities as well as radial an transverse accelerations respectively proportional. If r is the radial distance then velocity is proportional to

(1) r
(2) log r
(3) some power of radius vector
(4) exp(r)

75. Let T_H be the time taken by a projectile up to the highest point and T be tl time of flight, then

(1) $T_H = \frac{2T}{3}$

(2) $T_H = \frac{1}{2}T$

(3) $T_H = \frac{3T}{2}$

(4) $T_H = 2T$

76. Let a particle be in SHM (Simple Harmonic Motion) between A and B and O the fixed middle point . Then as the particle moves beyond 0,

(1) velocity and acceleration both increase
(2) velocity and acceleration both decrease
(3) Velocity increases but acceleration decreases
(4) velocity decreases but acceleration increases

77. The rational approximation of the form $\frac{(a+bx)}{(1+cx)}$ to e^x is

(1) $\frac{\left(1+\frac{x}{2}\right)}{\left(1-\frac{x}{2}\right)}$

(2) $\frac{\left(1-\frac{x}{2}\right)}{\left(1+\frac{x}{2}\right)}$

(3) $\frac{(2+x)}{\left(1-\frac{x}{2}\right)}$

(4) $\frac{(2-x)}{\left(1-\frac{x}{2}\right)}$

78. A canal is 40 m wide. The The depth y(in meter) of the canal at a distance x from one bank is given by the following :

x	0	10	20	30	40
y	0.0	3.5	5.5	.5	0.5

The approximate area of cross-section in square metre of the canal using Simpson rule is
(1) 135.0
(2) 137.5
(3) 140.0
(4) 145.0

79. Approximating the first derivative of $f(t)$ as

$$f'(t) = \frac{4}{3h}\left\{f\left(t+\frac{h}{2}\right) - f\left(t-\frac{h}{2}\right)\right\} - \frac{1}{6h}\{f(t+h) - f(t)\}$$

The order of error in the approximation is:
(1) two
(2) three
(3) four
(4) five

80. Adams-Bashforth-Moulton predictor formula for $\frac{dy}{dx} = f(x), y(x_0) = y_0, y(x_1) = y_1, y(x_2) = y_2, y(x_3) = y_3$ is

(1) $y_4 = y_3 + \frac{h}{24}(59f_3 - 55f_2 + 37f_1 - 9f_o)$
(2) $y_4 = y_3 + \frac{h}{24}(55f_3 - 59f_2 + 37f_1 - 9f_o)$
(3) $y_4 = y_3 + \frac{h}{24}(55f_3 - 59f_2 - 37f_1 + 9f_o)$
(4) $y_4 = y_3 + \frac{h}{24}(59f_3 - 55f_2 - 37f_1 + 9f_o)$

81. A rocketis fired vertically upwards with a velocity u which exceeds $\sqrt{2gh}$, where g is gravity and h is the height of a target. If t_1 and t_2 are the instants at which the rocket reaches the target then

(1) $t_1 + t_2 = \frac{2u}{g}$

(2) $t_1 - t_2 = \frac{2u}{g}$

(3) $t_1 + t_2 < \frac{2u}{g}$

(4) $t_1 - t_2 > \frac{2u}{g}$

82. Which one of the following is not referred as explicit method to solve $\frac{dy}{dx} = f(x,y), y(x_0) = y_0$?
(1) Taylor's series method
(2) Picard's method
(3) Euler-Cauchy method
(4) Backward Euler method

83. If cube of side 4 meter is increased by 2% then the approximate increase in it volume is
(1) 2%
(2) 6%
(3) 8%
(4) 12%

84. If a force F is resolved into components P and Q making angles α and β respectively with it, then
(1) $P = \sin(\alpha+\beta).Fcos\alpha, Q = \sin(\alpha+\beta).Fcos\beta$
(2) $P = \frac{F\sin\alpha}{\sin(\alpha+\beta)}\ Q = \frac{Fsin\beta}{\sin(\alpha+\beta)}$
(3) $P = \frac{F\sin\alpha}{\sin(\alpha+\beta)}\ Q = \frac{Fsin\beta}{\sin(\alpha+\beta)}$
(4) $P = Fcos\alpha, Q = F\ cos\beta$

85. Two forces F_1 and F_2 are acting at a point and are inclined to each other at an angle of 120^0 . if their resultant makes an angle 90^0 with the direction of F_1 then
(1) $F_1 = \sqrt{3}F_2$
(2) $F_1 = F_2$
(3) $3F_1 = F_2$
(4) $2F_2 = F_1$

86. The general solution of differential equation (2x-10y^3)$\frac{dy}{dx} + y = 0$ is (c being a constant)
(1) $x = 2y^3 + cy^{-2}$
(2) $y = 2x^3 + cx^{-2}$
(3) $y = 2x^{-3} + cx^3$
(4) $x = 2y^{-2} + cy^3$

87. The general solution of differential equation $(2x^2y + y)dx + xdy = 0$ is (c being a constant)
(1) $x + \log(xy) = c$
(2) $x^2 + \log(xy) = c$
(3) $logx + xy = c$
(4) $logy + xy = c$

88. The orthogonal trajectories of the curves $xy = c$ is (a being a constant)
(1) $x^2 + y^2 = a^2$
(2) $x^2 - y^2 = a^2$
(3) $x^2 + 2y^2 = a^2$
(4) $2\,x^2 + y^2 = a^2$

89. The general solution of differential equation $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$ is (A, B and C being constant)
(1) $y = Ae^x + Be^{2x} + Ce^{3x}$
(2) $y = Ae^x + Bxe^x + Ce^{2x}$
(3) $y = Ae^{-x} + Be^{2x} + Ce^{3x}$
(4) $y = Ae^{-x} + Be^x + Cxe^x$

90. Find the value of m for which $y = c_1e^{-3x} + c_2xe^{-3x}$ is a solution of differential equation $my" + 2y' + 3y = 0$ ⟨c_1 and c_2 being constants⟩
(1) $\frac{1}{2}$
(2) $-\frac{1}{2}$
(3) $\frac{1}{3}$
(4) $-\frac{1}{3}$

91. Let $y(x)$ be the solution of initial values problem $y" + 2y' - 3y = 0, y(0) = a, y'(0) = 3$. For which value of a $lim_{x\to\infty} y(x) = 0$
(1) 0
(2)1
(3) -1
(4) 2

92. The root of the indicial equation of differential equation $2x^2y" + (x^2 - x)y' + y = 0$ are
(1) 1 and $-\frac{1}{2}$
(2)$1\ and\ \frac{1}{2}$
(3) $-1\ and\ \frac{1}{2}$
(4) $-1\ and\ -\frac{1}{2}$

93. The value of $P'_n(1)$ where $P'_n(x)$ is first differential of Legendary polynomial of degree n is
(1) $\frac{1}{2}n(n+1)$
(2) $\frac{1}{2}n(n-1)$
(3) $\frac{1}{2}n^2(n+1)$
(4) $\frac{1}{2}\ n^2(n-1)$

94. The value of the Bessel's function $J_{1/2}(x)$ of order $\frac{1}{2}$ is
(1) $\sqrt{\frac{1}{2\pi x}} sinx$
(2) $\sqrt{\frac{2}{\pi x}} sinx$
(3) $\sqrt{\frac{1}{2\pi x}} \cos x$
(4) $\sqrt{\frac{2}{\pi x}} \cos x$

95. A solution of partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$ is
(1) $\cos(x - 3y)$
(2) $x^2 + y^2$
(3) $\sin(3x - y)$
(4) $e^{-3\pi x}\sin(\pi y)$

96. The partial differential equation for the surface $z = ax + by$ is
(1) $z = py + qx$
(2) $z = py - qx$
(3) $z = px + qy$
(4) $z = px - qy$

97. The solution of partial differential equation $(D^2 + 2DD' + D^2)z = e^{2x+3y}$ is
(1) $z = \phi_1(y-x) + x\phi_2(y-x) + \frac{1}{25}\, e^{2x+3y}$
(2) $z = x\phi_1(y-x) + y\phi_2(y-x) + \frac{1}{5}e^{2x+3y}$
(3) $z = \phi_1(y-x) + x\phi_2(y-x) + \frac{1}{5}\, e^{2x+3y}$
(4) $z = x\phi_1(y-x) + y\phi_2(y-x) + \frac{1}{25}\, e^{2x+3y}$

98. The radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$ is
(1) 3
(2) $\sqrt{3}$
(3) $4\sqrt{5}$
(4) 80

99. The equation of right circular cylinder, which has guiding curve as the circle $x^2 + y^2 + z^2 - \alpha^2 = 0z - b, 0 < b < a$ and generators are parallel to z-axis is
(1) $x^2 + y^2 = a^2 - b^2$
(2) $x^2 + y^2 + z^2 = b^2$
(3) $x^2 + y^2 = b^2$
(4) $x^2 + y^2 + z^2 = a^2 - b^2$

100. The locus of the point of intersection of two mutually perpendicular tangent lines to the curve $\frac{l}{r} = 1 + cos\theta$ is
(1) a line
(2) a parabola
(3) a circle
(4) an ellipse

101. The number of independent components of skew-symmetric covariant tensor of order two in an n −dimensional space is at most
(1) n
(2) n^2
(3) $\frac{n(n+1)}{2}$
(4) $\frac{n(n-1)}{2}$

102. The arc length of the curve $\gamma: [0,2\pi] \to R^3, \gamma(t) = (3\cos t, 3\, sint, 4t)$ is
(1) 3π
(2)6π
(3) 10π
(4) 12π

103. The torsion of the curve $\gamma: \mathbb{R} \to \mathbb{R}^3 \gamma(t) = (a\, cost, a\, sint, at)$ at point t is
(1) $\frac{at}{a^2+c^2}$
(2) $\frac{c}{a^2+c^2}$
(3) $\frac{at}{a^2+c^2}$
(4) $\frac{a}{a^2+c^2}$

104. Which one of the following surface has negative Gaussian curvature at some of its points ?
(1) a plane
(2) a Sphere
(3) an ellipsoid
(4) a torus

105. Which one of the following curve is not a regular curve?
(1) $\gamma(t) = (t, t^2)$
(2) $\gamma(t) = (e^t, t^4)$
(3) $\gamma(t) = (t^2, t^4)$
(4) $\gamma(t) = (2t + 1, e^{-t})$

106. If k_1, k_2 be principal curvatures of a surface at a point, then the normal curvature of the surface at the same point along a direction making an angle with the principal direction is given by
(1) $k_1 \frac{\sqrt{3}}{2}$
(2) $k_2 \frac{\sqrt{3}}{2}$
(3) $\frac{3k_1+k_2}{4}$
(4) $\frac{k_1+3k_2}{4}$

107. Which one of the following curves is parametrized by its arc length ?
(1) $\gamma(t) = (a\, cost, a \sin t)$
(2) $\gamma(t) = (t, t)$
(3) $\gamma(t) = \left(a\cos\frac{t}{a}, a\sin\frac{t}{a}\right)$
(4) $\gamma(t) = \left(\cos\frac{t}{a}, \sin\frac{t}{a}\right)$

108. The sum of interior angles of a geodesic triangle on the surface of a sphere radius R is
(1) less than π
(2) π
(3) greater than π
(4) not constant

109. If Γ_y^h denotes the Chirstoffel symbols of second Kind, then the values of Γ_{ij}^i is
(1) $\frac{\partial g}{\partial x^j}$
(2) $\frac{1}{2}\frac{\partial g}{\partial x^j}$
(3) $\frac{1}{2}\frac{\partial}{\partial x^j}(\log g)$
(4) $\frac{\partial}{\partial x^j}(\log g)$

110. Consider the equation $Ax = b$ where $A = \begin{bmatrix} 2 & 3 & -1 -1 \\ 4 & 1 & 1 -2 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 9 \end{bmatrix}$, which one of the following is a basic solution ?
(1) $\left[0 - \frac{11}{5} 0 - \frac{28}{5}\right]$
(2) $[2\ 7\ 0\ 0\ ]$
(3) $\left[\frac{11}{5}\frac{4}{5} 0\ 0\right]$
(4) $\left[\frac{4}{3}\frac{11}{3}\ 0\ 0\right]$

111. The set of all feasible solution of a linear programming problem is always a
(1) convex set
(2) open set
(3) closed set
(4) unbounded set

112. Which one of the following sets is not a convex set?
(1) $X = \{(x_1, x_2): 4x_1^2 + x_2^2 \leq 36\}$
(2) $X = \{(x_1, x_2): x_1 \leq 4, x_2 \geq 2\}$
(3) $X = \{(x_1, x_2)x_1, x_2 \leq 1, x_1, x_2 \geq 0\}$
(4) $X = \{(x_1, x_2): x_1^2 + x_2 - 3 \geq 0, x_1 \geq 0, x_2 > 0\}$

113. Consider the LPP:
Maximize $(3x_1 + x_2 + 3x_3)$ subject to
$2x_1 + x_2 + x_3 \leq 2$
$x_1 + 2x_2 + 3x_3 \leq 5$
$2x_1 + 2x_2 + x_3 \leq 6$
$x_1, x_2, x_3 \geq 0$
In solving this problem by revised simplex method, the basic feasible solution is
(1) $[0 \quad 0 \quad 0 \; 2 \quad 5 \quad 6]$
(2) $[2 \quad 5 \quad 6 \; 0 \quad 0 \quad 0]$
(3) $[0 \quad 2 \quad 5 \; 6 \quad 0 \quad 0]$
(4) $[2 \quad 0 \quad 5 \; 0 \quad 6 \quad 0]$

114. The dual of the following LPP's :
Maximize $2x + 3y + 4z$ subject to
$x - 5y + 3z = 7$
$2x - 5y \leq 3$
$3y - z \geq 5$
$x, y \geq 0, z$ is unrestricted
(1) Minimize $7\lambda + 3\mu - 5v$
Subject to
$\lambda + 2\mu \geq 2$
$-5\lambda - 5\mu - 3v \geq 3$
$3\lambda + v = 4$
$\mu, v \geq 0, \lambda$ is unrestricted
(2) Minimize $(-7\lambda + 3\mu - 5v)$
Subject to
$\lambda + 2\mu \geq 2$
$-5\lambda - 5\mu - 3v \geq 3$
$3\lambda + v = 4$
$\lambda\mu, v \geq 0$, is unrestricted
(3) Minimize $(-7\lambda + 3\mu - 5v)$
Subject to
$\lambda + 2\mu \geq 2$
$-5\lambda - 5\mu - 3v \geq 3$
$3\lambda + v = 4$
$\mu, v \geq 0, \lambda$ is unrestricted
(4) Minimize $(7\lambda + 3\mu - 5v)$
Subject to
$\lambda + 2\mu \geq 2$
$-5\lambda - 5\mu - 3v \geq 3$
$3\lambda + v = 4$
$\lambda, \mu, v \geq 0$, is unrestricted

115. For a balance transportation problem, which one of the following is false
(1) There are at least $m+n-1$ basic variables
(2) There are at most $m+n-1$ basic variables
(3) There are at least $m+n+1$ basis variables
(4) There are at most $m+n+1$ basic variables

116. The initial basic feasible solution of the following balanced transportation problem using lowest cost entry method is :

Destination

		D_1	D_2	D_3	D_4	Capacity
Origin	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
		6	10	15	4	Demand

(1) $x_{13}=14, x_{21}=6, x_{22}=9, x_{23}=1, x_{32}=1, x_{34}=4$
(2) $x_{13}=14, x_{21}=6, x_{22}=10, x_{23}=15, x_{32}=5, x_{34}=4$
(3) $x_{13}=14, x_{21}=6, x_{22}=10, x_{23}=1, x_{32}=5, x_{34}=1$
(4) $x_{13}=14, x_{21}=6, x_{22}=9, x_{23}=5, x_{32}=5, x_{34}=1$

117. Consider the following minimal assignments problem:

Men

		1	2	3	4
Jobs	I	12	30	21	15
	II	18	33	9	31
	III	44	25	24	21
	IV	23	30	28	14

Which one of the following is true solution?
(1) $I \rightarrow 1\ II \rightarrow 3\ III \rightarrow 2\ IV \rightarrow 4$
(2) $I \rightarrow 2\ II \rightarrow 1\ III \rightarrow 4\ IV \rightarrow 3$
(3) $I \rightarrow 4\ II \rightarrow 2\ III \rightarrow 2\ IV \rightarrow 1$
(4) $I \rightarrow 1\ II \rightarrow 4\ III \rightarrow 2\ IV \rightarrow 3$

118. The Fourier series of function $f(x)=|sinx|on|-\pi,\pi]$ will not contain
(1) constant term
(2) sine terms
(3) cosine terms
(4) Both sine and cosine terms

119. The Laplace transform of function $f(t) = \frac{\sin t}{t}$ is

(1) $tan^{-1}s$

(2) $sin^{-1}s$

(3) $\frac{1}{s^2+1}$

(4) $tan^{-1}\frac{1}{s}$

120. The inverse Laplace transform of the function $F(s) = \frac{s+1}{(s^2+2s+2)^2}$ is

(1) $\frac{1}{2}e^{-t}t\sin t$

(2) $\frac{1}{2}e^{-t}\sin t$

(3) $\frac{1}{2}e^{-t}t\cos t$

(4) $\frac{1}{2}e^{-t}t^2\sin t$