Preparing for JEST (Theoretical Computer Science):
Information for candidates

Joint Entrance Screening Test (JEST) is a preliminary screening test conducted jointly by several premier research institutes. Among these institutes, **The Institute of Mathematical Sciences, Chennai** is the only one that offers a Ph.D. programme in Theoretical Computer Science.

The JEST screening test for entrance to the PhD programme in theoretical computer science will be held typically on a **Sunday** in the middle of **February** every year. The test focusses on the following areas:

Analytical Reasoning and Deduction, Combinatorics, Data Structures and Algorithms, Discrete Mathematics, Graph Theory, Principles of Programming.

In each of these areas, familiarity with the basics (including the necessary simple mathematics) is assumed. There are questions of both types – some requiring short answers as well as some involving detailed problem solving. A sample questionnaire is attached.

Some textbooks which may help you prepare for the test are listed below. You don't need to read all the books. There is no specified "portion" for the test; rather, the test is designed to check the applicant's understanding of foundational aspects of computing.

Suggested texts

1. *Elements of Discrete Mathematics*, C.L. Liu

2. *Discrete Mathematical Structures with Applications to Computer Science*,
   Jean-Paul Tremblay and Ram P. Manohar

3. *Compilers: Principles, Techniques and Tools*,
   Alfred V. Aho, Ravi Sethi and Jeffrey D. Ullman

4. *Fundamentals of Data Structures*, Ellis Horowitz and Sartaj Sahni

5. *An Introduction to Data Structures with Applications*,
   Jean-Paul Tremblay and P.G. Sorenson

6. *Fundamentals of Computer Algorithms*, Ellis Horowitz, Sartaj Sahni and S. Rajasekaran

7. *The Design and Analysis of Computer Algorithms*,
   Alfred V. Aho, John E. Hopcroft and Jeffrey D. Ullman

8. *Introduction to Algorithms,*
    Thomas H. Cormen, Charles E. Leiserson and Ronald L. Rivest

9. *How to solve it by computer*, R.G. Dromey

10. *Programming Languages, Concepts and Constructs*, Ravi Sethi

Subject: Theoretical Computer Science

1. Select the correct alternative in each of the following:

   (a) Let a and b be positive integers such that $a > b$ and $a^2 - b^2$ is a prime number. Then $a^2 - b^2$ is equal to
   (A) $a - b$ (B) $a + b$ (C) $a \times b$ (D) none of the above

   (b) When is the following statement true? $(A \cup B) \cap C = A \cap C$
   (A) If $\bar{A} \cap B \cap C = \phi$ (B) If $A \cap B \cap C = \phi$
   (C) always   (D) never

   (c) If a fair die (with 6 faces) is cast twice, what is the probability that the two numbers obtained differ by 2?
   (A) 1/12   (B) 1/6   (C) 2/9   (D) 1/2

   (d) $T(n) = T(n/2) + 2$; $T(1) = 1$
   When n is a power of 2, the correct expression for $T(n)$ is:
   (A) $2(\log n + 1)$ (B) $2 \log n$ (C) $\log n + 1$ (D) $2 \log n + 1$

2. Consider the following function, defined by a recursive program:

```
function AP(x,y: integer) returns integer;
   if   x = 0 then return y+1
   else if y = 0 then return AP(x-1,1)
        else   return AP(x-1, AP(x,y-1))
```

   (a) Show that on all nonnegative arguments x and y, the function AP terminates.

   (b) Show that for any x, $\text{AP}(x, y) > y$.

3. How many subsets of even cardinality does an n-element set have ? Justify answer.

4. A tournament is a directed graph in which there is exactly one directed edge between every pair of vertices. Let T_n be a tournament on n vertices.

   (a) Use induction to prove the following statement:
   T_n has a directed hamiltonian path (a directed path that visits all vertices).

   (b) Describe an algorithm that finds a directed hamiltonian path in a given tournament. *Do not* write whole programs; pseudocode, or a simple description of the steps in the algorithm, will suffice. What is the worst case time complexity of your algorithm?

5. Describe two different data structures to represent a graph. For each such representation, specify a simple property about the graph that can be more efficiently checked in that representation than in the other representation. Indicate the worst case time required for verifying both of your properties in either representation.

6. Two gamblers have an argument. The first one claims that if a fair coin is tossed repeatedly, getting two consecutive heads is very unlikely. The second, naturally, is denying this.

   They decide to settle this by an actual trial; if, within n coin tosses, no two consecutive heads turn up, the first gambler wins.

   (a) What value of n should the second gambler insist on to have more than a 50% chance of winning?

   (b) In general, let $P(n)$ denote the probability that two consecutive heads show up within n trials. Write a recurrence relation for $P(n)$.

   (c) Implicit in the second gambler's stand is the claim that *for all* sufficiently large n, there is a good chance of getting two consecutive heads in n trials; i.e. $P(n) > 1/2$. In the first part of this question, one such n has been demonstrated. What happens for larger values of n? Is it true that $P(n)$ only increases with n? Justify your answer.

7. Consider the following program:

```
function mu(a,b:integer) returns integer;
   var i,y: integer;
   begin
     ---------P----------
     i = 0; y = 0;
     while (i < a) do
       begin --------Q------------
         y := y + b ;
         i = i + 1
       end
     return y
   end
```

   Write a condition P such that the program terminates, and a condition Q which is true whenever program execution reaches the place marked Q above.