Physics Paper
2007

IMPORTANT NOTE FOR CANDIDATES

- **Attempt ALL the 25 questions.**
- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-25 (subjective questions) carry *twenty one* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 11 only.**

1. $$(x \quad y)\begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 15$$

The matrix equation above represents

(A) a circle of radius $\sqrt{15}$

(B) an ellipse of semi major axis $\sqrt{5}$

(C) an ellipse of semi major axis 5

(D) a hyperbola

2. $f(x)$ is a periodic function of x with a period of 2π. In the interval $-\pi < x < \pi$, $f(x)$ is given by

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

In the expansion of $f(x)$ as a Fourier series of sine and cosine functions, the coefficient of $\cos(2x)$ is

(A) $\dfrac{2}{3\pi}$

(B) $\dfrac{1}{\pi}$

(C) 0

(D) $-\dfrac{2}{3\pi}$

3. The speed of an electron, whose de Broglie wavelength is equal to its Compton wavelength, is (c is the speed of light)

(A) c

(B) $c/\sqrt{2}$

(C) c/2

(D) c/3

4. A satellite moves around a planet in a circular orbit at a distance R from its centre. The time period of revolution of the satellite is T. If the same satellite is taken to an orbit of radius $4R$ around the same planet, the time period would be

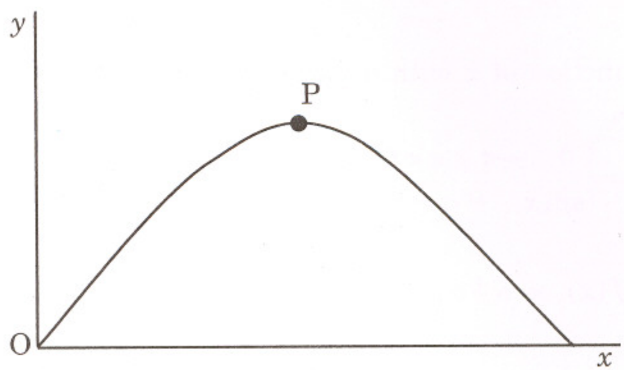
(A) 8T

(B) 4T

(C) T/4

(D) T/8

5. A projectile is fired from the origin O at an angle of 45° from the horizontal. At the highest point P of its trajectory the radial and transverse components of its acceleration in terms of the gravitational acceleration g are



(A) $a_r = \frac{2g}{\sqrt{5}}, \quad a_\theta = \frac{g}{\sqrt{5}}$

(B) $a_r = \frac{-2g}{\sqrt{5}}, \quad a_\theta = \frac{-g}{\sqrt{5}}$

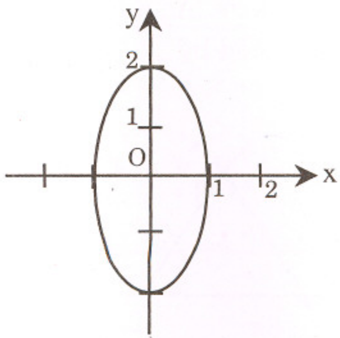
(C) $a_r = \frac{g}{\sqrt{5}}, \quad a_\theta = \frac{2g}{\sqrt{5}}$

(D) $a_r = \frac{-g}{\sqrt{5}}, \quad a_\theta = \frac{-2g}{\sqrt{5}}$

6. When two simple harmonic oscillations represented by

$x = A_0 \cos(\omega t + \alpha)$ and $y = B_0 \cos(\omega t + \beta)$

are superposed at right angles, the resultant is an ellipse with its major axis along the y-axis as shown in the figure. The conditions which correspond to this are



(A) $\beta = \alpha + \frac{\pi}{2}; A_0 = 2B_0$

(B) $\beta = \alpha - \frac{\pi}{4}; A_0 = B_0$

(C) $\beta = \alpha + \frac{\pi}{2}; 2A_0 = B_0$

(D) $\beta = \alpha + \frac{\pi}{4}; A_0 = B_0$

7. In terms of the basic units of mass (M), length (L), time (T) and charge (Q), the dimensions of magnetic permeability of vacuum (μ_o) are

(A) MLQ^{-2}

(B) $ML^2T^{-1}Q^{-2}$

(C) LTQ^{-1}

(D) $LT^{-1}Q^{-1}$

8. The black body spectrum of an object O_1 is such that its radiant intensity (i.e., intensity per unit wavelength interval) is maximum at a wavelength of 200 nm. Another object O_2 has the maximum radiant intensity at 600 nm. The ratio of power emitted per unit area by O_1 to that of O_2 is

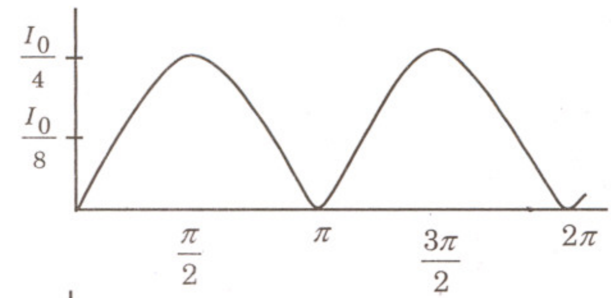
(A) $\frac{1}{81}$

(B) $\frac{1}{9}$

(C) 9

(D) 81

9. Three polarizers P, Q and R are placed parallel to each other with their planes perpendicular to the z-axis. Q is placed between P and R. Initially the polarizing directions of P and Q are parallel, but that of R is perpendicular to them. In this arrangement when unpolarized light of intensity I_0 is incident on P, the intensity coming out of R is zero. The polarizer Q is now rotated about the z-axis. As a function of angle of rotation, the intensity of light coming out of R is best represented by

(A)


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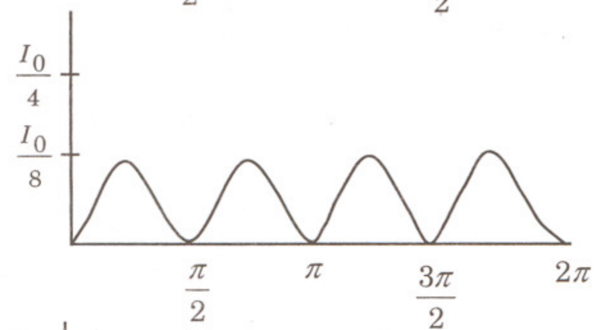
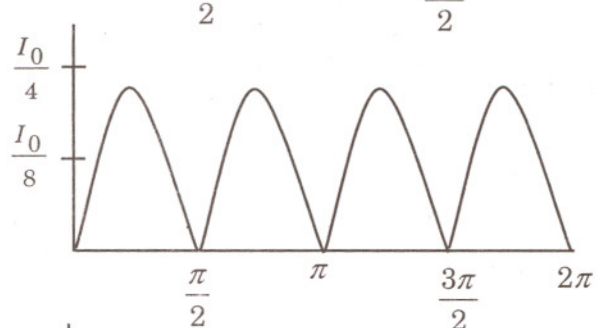


(C)


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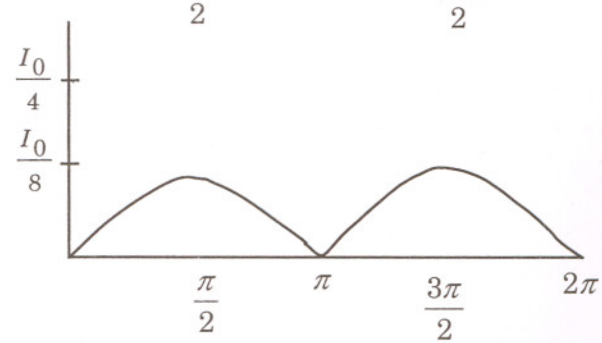
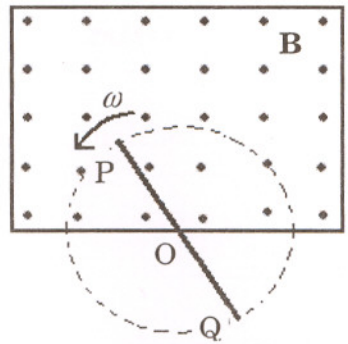
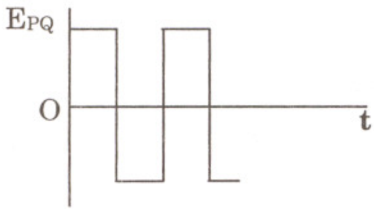


10. A uniform and constant magnetic field **B** coming out of the plane of the paper exists in a rectangular region as shown in the figure. A conducting rod PQ is rotated about O with a uniform angular speed ω in the plane of the paper. The emf E_{PQ} induced between P and Q is best represented by the graph



(A)



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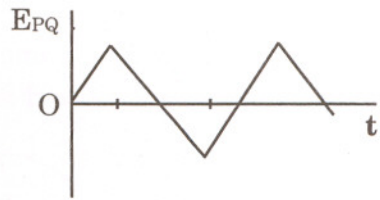
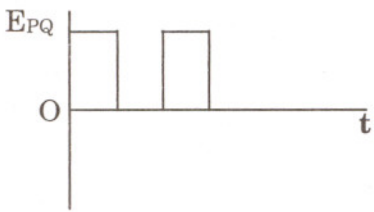
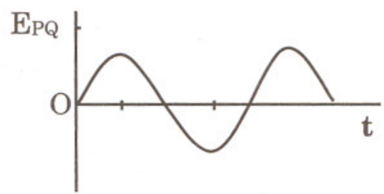


(C)



(D)



11. Experimental measurements of heat capacity per mole of Aluminium at low temperatures show that the data can be fitted to the formula, $C_V = aT + bT^3$, where $a = 0.00135$ J K^{-2} $mole^{-1}$, $b = 2.48 \times 10^{-5}$ J K^{-4} $mole^{-1}$ and T is the temperature in Kelvin. The entropy of a mole of Aluminium at such temperatures is given by the formula

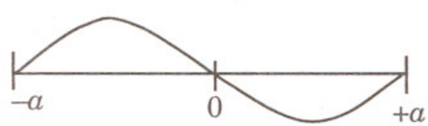
(A) $aT + \frac{b}{3}T^3 + c$, where $c > 0$ is a constant

(B) $\frac{aT}{2} + \frac{b}{4}T^3 + c$, where $c > 0$ is a constant

(C) $aT + \frac{b}{3}T^3$

(D) $\frac{aT}{2} + \frac{b}{4}T^3$

12. A particle is confined in a one dimensional box with impenetrable walls at $x = \pm a$. Its energy eigenvalue is 2 eV and the corresponding eigenfunction is as shown below.



The lowest possible energy of the particle is

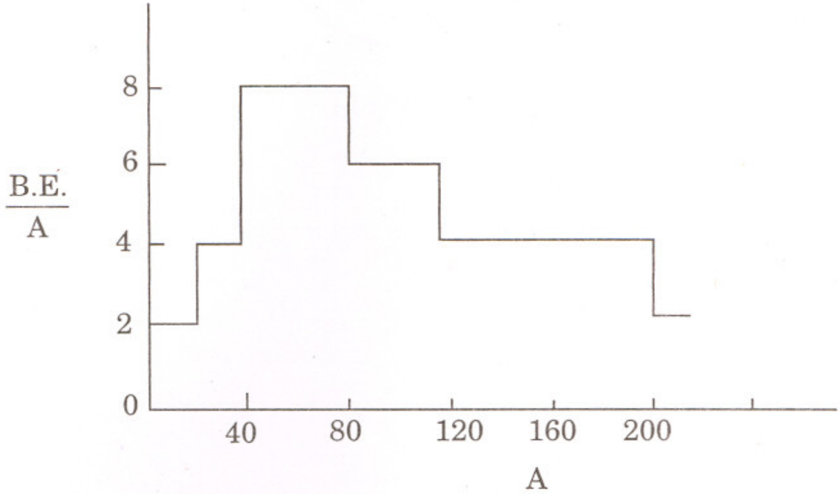
(A) 4 eV

(B) 2 eV

(C) 1 eV

(D) 0.5 eV

13. The following histogram represents the binding energy per particle (B.E./A) in MeV as a function of the mass number A of a nucleus.



A nucleus with mass number A = 180 fissions into two nuclei of equal masses. In the process

(A) 180 MeV of energy is released

(B) 180 MeV of energy is absorbed

(C) 360 MeV of energy is released

(D) 360 MeV of energy is absorbed

14. Fermi energy of a certain metal M_1 is 5 eV. A second metal M_2 has an electron density which is 6% higher than that of M_1. Assuming that the free electron theory is valid for both the metals, the Fermi energy of M_2 is closest to

(A) 5.6 eV

(B) 5.2 eV

(C) 4.8 eV

(D) 4.4 eV

15.

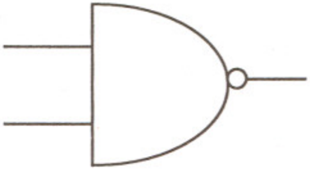


Fig (i)

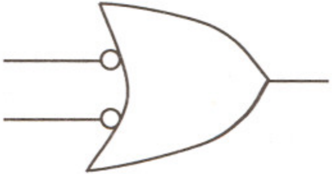


Fig (ii)

Figures (i) and (ii) represent respectively,

(A) NOR, NOR

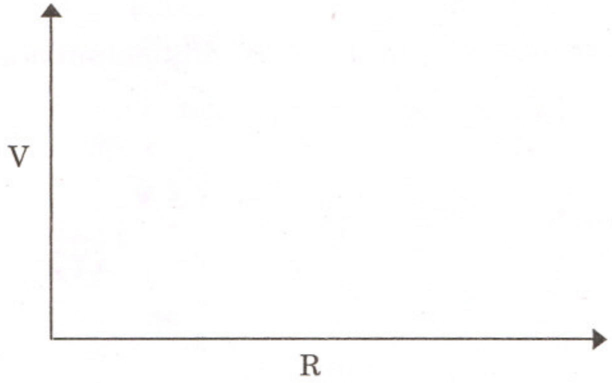
(B) NOR, NAND

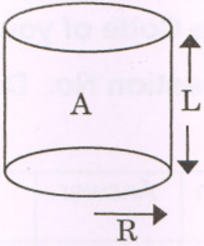
(C) NAND, NAND

(D) OR, NAND

16. If the total surface area (including the area of the top and bottom ends) of a cylinder is to be kept fixed (=A), what is its maximum possible volume?

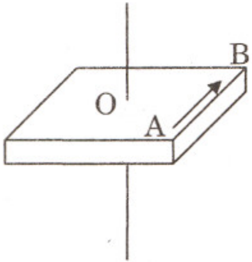
For such cylinders of fixed total area, plot in the axes shown below their volume (V) versus the radius (R) clearly indicating the values of R for which the volume is maximum and zero.





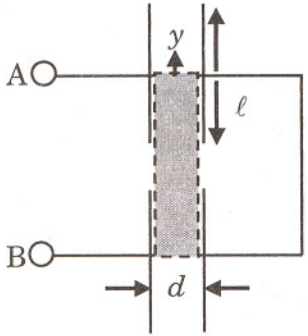
[21]

17. A horizontal square platform of mass m and side a is free to rotate about a vertical axis passing through its centre O. The platform is stationary and a person of the same mass (m) as the platform is standing on it at point A. The person now starts walking along the edge from A to B (see figure). The speed v of the person with respect to the platform is constant. Find the time the person takes to reach B. Also find his distance $r(t)$ from O as a function of time. Further find the angle through which the platform has rotated by the time the person reaches B.



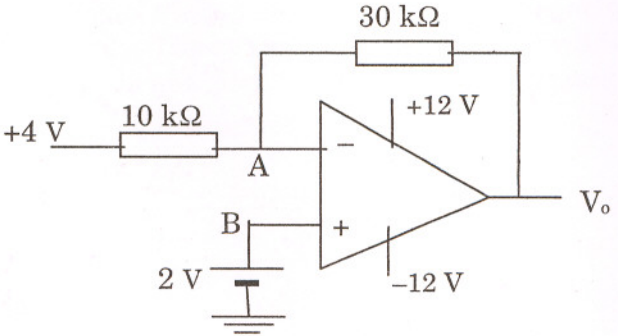
[21]

18. Two identical parallel plate capacitors are connected across terminals A and B as shown. Each of the capacitors is made of square plates of side ℓ with a distance d between them. A dielectric slab (relative permittivity k) of thickness d is kept between the plates. The slab covers only half of the length of the plates in each of the capacitors as shown. Find the total capacitance of the assembly. The capacitors are charged by a battery and then the battery is disconnected. If the slab is now displaced slightly by a distance $y[(y/\ell)<<1]$, show that it will perform simple harmonic oscillations.



[21]

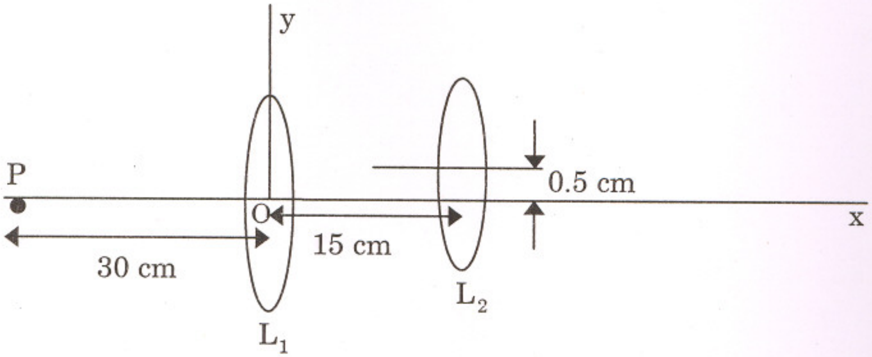
19. For the circuit shown below, calculate the output voltage V_o. What would V_o be if the polarity of 2 V battery is reversed at terminal B? (Assume the operational amplifier to be ideal).



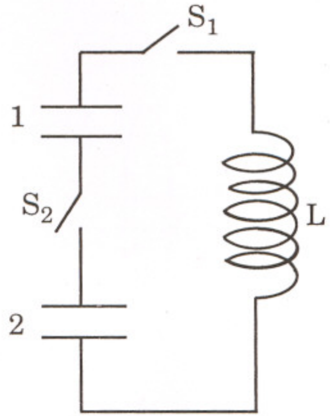
[21]

20. A beam of light of wavelength 400 nm and power 1.55 mW is directed at the cathode of a photoelectric cell. (given: hc = 1240 eV nm, e = 1.6×10^{-19} C). If only 10% of the incident photons effectively produce photoelectrons, find the current due to these electrons. If the wavelength of light is now reduced to 200 nm, keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. What are the values of the stopping potentials for the two wavelengths? [21]

21. Two thin lenses L_1 and L_2 of focal lengths 15 cm and 10 cm respectively, are kept 15 cm apart from each other. Their axes are separated by 0.5 cm as shown in figure (not to scale). If a point object P is placed on the axis of L_1 to its left at a distance of 30 cm, find the x and y coordinates (origin O) of the image formed by the combination.



22. The circuit shown consists of two identical capacitors of capacitance C each and an inductor of inductance L. Initially, both switches are open and capacitor 1 is charged with a charge Q_0 while the second capacitor has no charge. Switches S_1 and S_2 are closed simultaneously at $t = 0$. The circuit now becomes oscillatory.

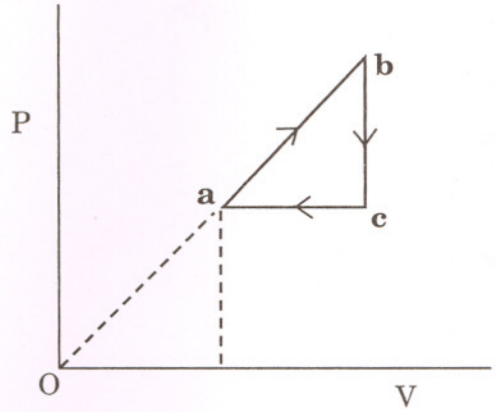


(a) Calculate the maximum current in the circuit. [12]

(b) Obtain expressions for the charge on the capacitors 1 and 2 as a function of time. [9]

23. A particle travels along the diameter of the earth at a relativistic speed. It crosses the earth in a time 3×10^{-2} s in its own frame. An observer, located on the earth, measures the same time interval to be 5×10^{-2} s. Find the speed of the particle with respect to the earth and the diameter of the earth. [21]

24. 1 m^3 of an ideal gas with $\gamma = C_P/C_V = 1.5$ is at a pressure of 100 kPa and a temperature of 300 K. Initially the state of the gas is at the point **a** of the PV diagram shown. The gas is taken through a reversible cycle **a**$\rightarrow$ **b**$\rightarrow$ **c**$\rightarrow$ **a**. The pressure at point **b** is 200 kPa and the line **ba**, when extended, passes through the origin.



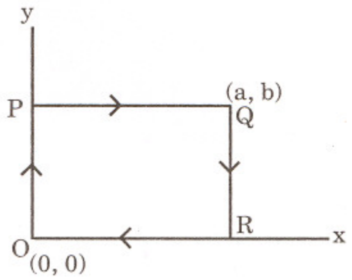
(a) Calculate the work done by the gas in each of the steps **a**$\rightarrow$ **b**, **b**$\rightarrow$ **c** and **c**$\rightarrow$ **a**. [9]

(b) Calculate the change in entropy of the gas in each of the three steps above. [12]

25. How much work is done when an object moves from O→ P→ Q→ R→ O in a force field given by

$$F(x,y)=(x^2-y^2)\hat{i}+2xy\,\hat{j}$$

along the rectangular path shown. Find the answer by evaluating the line integral and also by using the Stokes' theorem.



[21]